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Scattering mechanisms in $Hg_{1-x}Cd_xTe$

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The calculation of electron-transport properties of $Hg_{1-x}Cd_xTe$ has been generalized by using an accurate energy dispersion relation for the conduction electrons and by including the effects of electron-wave-function admixture, screening of lattice vibrations, and the degeneracy of the electron distribution. The drift and the Hall mobilities in the material are calculated for various values of x at 300 as well as at 77°K. It is found that there remains a large discrepancy between the theory and the experimental data on mobility if one considers polar scattering by the longitudinal-optical vibrations of CdTe and HgTe in addition to scattering by acoustic modes (both deformation potential and piezoelectric types), ionized impurities, and heavy holes. Incorporation of alloy scattering improves agreement between theory and experiment at moderately large (but not small) values of x. The experimental results for all values of x at 300 and also at 77°K can, on the other hand, be fitted with theory if in addition to the above scattering mechanisms, interaction with the transverse-optical phonons of HgTe is assumed to take place.

I. INTRODUCTION

Electron transport in $Hg_{1-x}Cd_xTe$ is receiving increasing attention in recent years in view of the variation of band parameters with composition and the technological interest in this material. Calculations of electron-drift mobility in this material at room temperature have been performed,[1] considering polar scattering by the longitudinal-optical phonons characteristic of CdTe and HgTe and also scattering by deformation-potential acoustic phonons, ionized impurities, and heavy holes. A comparison of the calculated results with experiments[2] shows that the lattice scattering in $Hg_{1-x}Cd_xTe$ occurs mainly through polar coupling to the longitudinal-optical vibrations of the end members CdTe and HgTe. This conclusion is, however, subject to criticism since these earlier calculations[1] have the following limitations: (i) The assumed conduction-band energy dispersion relation is not of general validity when the band-gap energy E_g is comparable to the spin-orbit splitting energy Δ. The relation may not, therefore, yield accurate results for all values of the composition x. (ii) The effect of electron wave-function admixture was not included. This effect would be significant particularly in the region $x \sim 0.2$, where band nonparabolicity is remarkable. (iii) Screening of lattice vibrations and degeneracy of the electron distribution which is important for $x \lesssim 0.2$ were not considered. (iv) The calculated values give the drift mobility while the experimental results refer to the Hall mobility.

In the present paper we have reconsidered the problem by removing the above limitations. The electron-drift and Hall mobilities in semiconducting $Hg_{1-x}Cd_xTe$ are computed for all values of x at room temperature as well as at 77 °K, incorporating the complexities of the band structure and of the scattering probabilities. Important information on the prevailing scattering mechanisms is obtained by comparing our theoretically calculated results with experiments.

II. GENERAL FORMULATION

A general form of the conduction-band energy dispersion relation that holds without restriction on the relative magnitudes of E_g and Δ has been given by Kane.[3] It has been shown that this relationship may be simplified as[4]

$$\hbar^2 k^2/2m^* = \gamma(E) = E(1+\alpha E)\,, \qquad (1)$$

where

$$\alpha = \frac{1}{E_g}\left(1 - \frac{2m^*}{m_0}\right)\left(1 - \frac{E_g \Delta}{3(E_g+\Delta)(E_g+\frac{2}{3}\Delta)}\right), \qquad (2)$$

$\hbar$ is the Dirac constant, m^* is the band-edge effective mass, m_0 is the free-electron mass, and E is the carrier energy for wave vector $\vec{k}$. The simplified E-$\vec{k}$ relation as given by Eq. (1) agrees within 3% with the exact relation[3] for energies up to 0.1 eV. This simplified relation thus causes very little error, and has been conveniently used in the present calculations for all values of the mole fraction x.

Let us assume that a magnetic induction $\vec{B}$ is applied in the z direction. If the total electric field $\vec{\mathcal{E}}$ (resulting from the applied and Hall fields) is taken to be in the x direction, the carrier distribution function may be expressed as[5]

$$f(\vec{k}) = f_0(E) - \frac{e\hbar\mathcal{E}}{m^*}\left(k_x\phi_x(E) + \frac{eB}{m^*}k_y\phi_y(E)\right)\frac{\partial f_0}{\partial E}, \qquad (3)$$

where e is the electron charge, ϕ_x, ϕ_y are the perturbation terms, and $f_0(E)$ is the equilibrium

Fermi-Dirac distribution function given by

$$f_0(E)=\{\exp[(E-E_F)/k_BT_L]+1\}^{-1}, \tag{4}$$

where k_B is the Boltzmann's constant, T_L is the lattice temperature, and E_F is the Fermi energy. For given values of the free-electron concentration n, E_F can be obtained by numerically solving the relation[6]

$$n=\frac{\sqrt{2}\,m^{*3/2}}{\pi^2\hbar^3}\int_0^\infty \sqrt{\gamma}\,\gamma' f_0\,dE, \tag{5}$$

where

$$\gamma'=\frac{d\gamma}{dE}.$$

Substituting (3) in the Boltzmann transport equation, retaining only the first-order terms in $\mathcal{E}$ and B, and equating the coefficients of k_x and k_y, one obtains

$$L\phi_x=1/\gamma' \tag{6}$$

and

$$L\phi_y=-\phi_x/\gamma', \tag{7}$$

where L is the collision operator. The drift mobility is given by

$$\mu_d=\frac{2e}{3m^*k_BT_L}\langle\phi_x\rangle, \tag{8}$$

where

$$\langle\phi_\alpha^n\rangle=\left(\int_0^\infty \gamma^{3/2}\phi_\alpha^n f_0(1-f_0)\,dE\right)\bigg/\int_0^\infty \gamma^{1/2}\gamma' f_0\,dE;$$

$$\alpha=x,y \quad\text{and}\quad n=1,2. \tag{9}$$

The Hall mobility is expressed as

$$\mu_H=-(e/m^*)\langle\phi_y\rangle/\langle\phi_x\rangle. \tag{10}$$

Using the self-adjoint property of the collision operator L, Eq. (10) may be reduced to[7,8]

$$\mu_H=(e/m^*)\langle\phi_x^2\rangle/\langle\phi_x\rangle. \tag{11}$$

It is obvious from Eqs. (8) and (11) that both μ_d and μ_H may be determined if only Eq. (6) is solved for ϕ_x. Considering polar interaction with the two types of longitudinal-optical vibrations, characteristic of CdTe and HgTe, and also scattering by ionized centers[9] and acoustic modes through both deformation potential and piezoelectric coupling, Eq. (6) can be written

$$\begin{aligned}[(1-x)\lambda_{1\text{po}}+x\lambda_{2\text{po}}+\lambda_{\text{ac}}+\lambda_{\text{pz}}+\lambda_{\text{ion}}]\phi_x(E)\\ =1/\gamma'+(1-x)S_{1\text{po}}\phi_x(E+k_B\Theta_1)+xS_{2\text{po}}\phi_x(E+k_B\Theta_2)\\ +(1-x)R_{1\text{po}}\phi_x(E-k_B\Theta_1)+xR_{2\text{po}}\phi_x(E-k_B\Theta_2).\end{aligned} \tag{12}$$

The subscripts 1 and 2 refer to the two types of longitudinal-optical vibrations characteristic of HgTe and CdTe, respectively; Θ_1 and Θ_2 are the corresponding phonon Debye temperatures, and po, ac, pz, and ion, respectively, stand for polar optical, deformation potential acoustic, piezoelectric, and ionized center scattering.

The expressions for λ's, S's, and R's including the effects of electron screening and wave-function admixture in cases when $\Delta\gg E_g$ and the electron distribution is nondegenerate, may be found in Ref. 10. In our case, however, Δ may be comparable to E_g, and also the electron distribution may be degenerate. The expressions in our case incorporating the effects of degeneracy in addition to those of electron screening and wave-function admixture without restriction on relative magnitudes of E_g and Δ are given in the Appendix.

$\phi_x(E)$ is determined from Eq. (12) by the numerical iteration technique. The method has originally been developed by Rode[11] in cases when only one type of optical phonons are present. The method of calculations when more than one type of optical phonons are operative (as it is in the present situation) is described in great detail by Nag.[8] The results presented here, are obtained using the method described in Ref. 8.

III. CALCULATIONS AT 300°K

The values of the spin-orbit-splitting energy Δ for various values of x were taken from Chadi and Cohen.[12] c_t and e_{14} were assigned the values of $1.55\times10^{10}\ \text{N/m}^2$ and $0.0336\ \text{C/m}^2$, respectively, which are the same as in CdTe.[13] Possible variations of c_t and e_{14} with x were not considered. Such variations in any case would affect our results insignificantly since piezoelectric scattering contributes very little to the total scattering. The other material constants were taken from the sources mentioned in Ref. 1.

The effect of ionized impurity scattering was estimated by taking the samples to be uncompensated.[2] At room temperature, the samples[2] have an ionized impurity concentration (n_D) of $2\times10^{15}\ \text{cm}^{-3}$. For $x\geqslant0.4$, at this temperature, the intrinsic carrier concentration[14] (n_i) is less than $3\times10^{14}\ \text{cm}^{-3}$ so that electron-hole scattering is much weaker than ionized impurity scattering for such compositions and is neglected. At $x=0.15$ and 0.2, however, $n_i=6\times10^{16}\ \text{cm}^{-3}$ and $2.8\times10^{16}\ \text{cm}^{-3}$, respectively, at room temperature.[14] These are much larger than n_D; the contribution of ionized impurity scattering is thus much less than that of electron-hole scattering at these x values and is ignored.

The calculated values of drift and Hall mobilities

TABLE I. Room-temperature mobility ($m^2\,\mathrm{V}^{-1}\,\mathrm{sec}^{-1}$) in n-$\mathrm{Hg}_{1-x}\mathrm{Cd}_x\mathrm{Te}$. [a]

x	μ_{d1}	μ_{H1}	μ_{t1}	μ_{obs} (Ref. 2)	μ_{d2}	μ_{H2}	μ_{t2}
0.15	4.4	4.74	4.03	1.9	2.17	2.395	2.07
0.2	2.68	2.71	2.35	1.03	1.22	1.27	1.15
0.4	0.678	0.694	0.48	0.274	0.333	0.342	0.275
0.6	0.305	0.313	0.19	0.155	0.181	0.185	0.133
0.8	0.17	0.175	0.113	0.116	0.125	0.1286	0.091

[a] μ_{d1}, μ_{H1} represent, respectively, the drift and Hall mobilities considering polar interaction with the longitudinal-optical vibrations characteristic of CdTe and HgTe, and also scattering by ionized centers and acoustic modes through both deformation potential and piezoelectric coupling. μ_{t1} is the over-all mobility due to the combination of the alloy scattering-limited mobility μ_a and μ_{d1}. μ_{d2} and μ_{H2} are, respectively, the drift and Hall mobilities due to scattering by the transverse-optical phonons of HgTe together with the scattering mechanisms used in calculating μ_{d1} and μ_{H1}. μ_{t2} is the over-all mobility obtained by combining μ_{d2} and μ_a. μ_{obs} is the experimental value of mobility.

(μ_{d1} and μ_{H1}) at room temperature for different values of x are given in Table I. It is found that Hall-to-drift mobility ratio is not much larger than unity, and hence no serious error is incurred by replacing the Hall mobilities by drift mobilities. The calculated values, however, are found to differ markedly from the experimental values of mobility (μ_{obs}) given by Scott.[2] Thus we find that when the shortcomings of the previous calculations[1] are removed, the experimental results cannot be explained with the help of the scattering processes considered. It is concluded that scattering mechanisms other than those considered, do play a significant role.

IV. ALLOY SCATTERING

It has been recently found[15] that disorder or alloy scattering is of importance in determining the transport properties of mixed semiconducting crystals. An expression for mobility due to this type of scattering for a simple parabolic band has been given by Brooks.[16] It is found that the mobility depends on the factor $x(1-x)$. Consequently, the alloy scattering is expected to have effect for intermediate values of x and would be relatively ineffective for very small or very large values of x. Furthermore, the alloy scattering limited mobility depends on $m^{*5/2}$. Since m^* diminishes steadily as x is reduced, alloy scattering would be still more ineffective in the low-x samples. The discrepancy between theory and experiment for low values of x cannot thus be expected to be much reduced by alloy scattering. But we may hope for some effect of the alloy scattering at relatively larger (but not very large) values of x.

Incorporating the effect of nonparabolicity, the relaxation rate for alloy scattering can be written

$$\lambda_a = \frac{\sqrt{2}\, m^{*3/2} x(1-x)\,|\Delta E|^2}{\pi \hbar^4 N_0} \times \sqrt{\gamma}\,\gamma' \int_{-1}^{+1} O(z)(1-z)\,dz\,, \tag{13}$$

where ΔE is the scattering potential and N_0 is the number of atoms per unit volume. The corresponding $\phi_x(E)$ can be expressed from the Boltzmann equation as

$$\phi_x = 1/\gamma' \lambda_a\,, \tag{14}$$

whence the alloy scattering-limited mobility μ_a is determined by the same equation (8).

The value of ΔE is not precisely known, but may approximately be taken[15] as the difference in the energy gaps of the two pure materials CdTe and HgTe. Because of the approximate nature of calculations it is sufficient to determine the over-all mobility (μ_{t1}) due to alloy scattering and other scattering mechanisms by the formula

$$\mu_{t1} = \mu_{d1}\mu_a/(\mu_{d1} + \mu_a)\,. \tag{15}$$

Values of μ_{t1} at room temperature are included in Table I. It is found by comparing μ_{d1} and μ_{t1} that alloy scattering affects the mobility mostly at $x = 0.6$ and also significantly at $x = 0.4$ and 0.8. The effects at $x = 0.15$ and 0.2 are, however, much less, as expected. A comparison of μ_{t1} and μ_{obs} shows that inclusion of alloy scattering improves agreement with experiment at moderately high values of x. The agreement at low values of x is, however, poor. In fact, at $x = 0.15$ and 0.2, values of μ_{t1} are more than two times the values of μ_{obs}.

V. SCATTERING BY TRANSVERSE-OPTICAL PHONONS OF HgTe

It is found from the above results that the discrepancy between theory and experiment becomes remarkable when x is reduced, i.e., when the fractional content of HgTe is increased. It would thus appear that some scattering mechanism associated with HgTe must be operative, the neglect of which gives rise to the above disparities. Recent experiments[17] on oscillatory photoelectric effects in $Hg_{0.76}Cd_{0.24}Te$ indeed indicate that electrons may be coupled to the transverse-optical vibrations of HgTe. Such a coupling is not surprising, since these vibrations have a polar character in the mixed crystal[17] due to the asymmetry in the arrangement of atoms. In this section we have incorporated the polar interaction with the transverse-optical phonons of HgTe in addition to the scattering mechanisms considered before. However, the coupling constant for interaction with the transverse-optical phonons of HgTe is not known. We have, therefore, thought it judicious to first find the value of the coupling constant by fitting the calculated results with experiment for a particular value of x at room temperature. The same coupling constant is then used to calculate the mobilities for different values of x at room temperature and also at 77 °K to find if agreement with experiments is obtained in all the cases. A positive answer would definitely establish the role played by the transverse-optical phonons of HgTe.

Let Θ_3 denote the Debye temperature of the transverse optical phonons in HgTe. Incorporation of interaction with these phonons will introduce an additional term $(1-x)\lambda_3\phi_x(E)$ in the left-hand side, and two additional terms $(1-x)S_3\phi_x(E+k_B\Theta_3)$ and $(1-x)R_3\phi_x(E-K_B\Theta_3)$ in the right-hand side[18] of Eq. (12). The resulting equation may again be solved for $\phi_x(E)$ by the numerical iteration technique as described by Nag,[8] and the corresponding drift and the Hall mobilities (μ_{d2} and μ_{H2}) can be computed. It should be noticed that μ_{d2} and μ_{H2} include the effects of scattering by acoustic modes, ionized centers, longitudinal-optical phonons of CdTe and HgTe as well as by the transverse-optical phonons of HgTe.

Values of μ_{d2} and μ_{H2} calculated with a ratio of 5:8 between the coupling constants for the longitudinal and transverse phonons of HgTe are shown in Table I. The over-all mobility μ_{t2} is obtained by combining μ_{d2} with the alloy scattering limited mobility μ_a; the formula for the combination being Eq. (15) with the subscript 1 replaced by the subscript 2. Values of μ_{t2} are also presented in Table I. It is noted that μ_{t2} agrees well with μ_{obs} for all values of x, the maximum discrepancy between the two values being about 20% at $x=0.8$. This amount of disagreement is tolerable considering the uncertainties in the parameter values, the approximate way in which the effect of alloy scattering is estimated, and the possible errors in reading the values of μ_{obs} from the published curve.

TABLE II. Mobility ($m^2\,V^{-1}\,sec^{-1}$) in n-$Hg_{1-x}Cd_xTe$ at 77 °K. [a]

x	μ_{d2}	μ_{H2}	μ_{t2}	μ_{obs}
0.195	20.6	25.8	23.1	17.5 (Ref. 19)
0.41	2.95	3.70	1.92	1.85 (Ref. 2)
0.59	1.69	2.1	0.73	0.79 (Ref. 2)

[a] μ_{d2} and μ_{H2} have the same significance as in Table I. μ_{t2} is the over-all mobility obtained by combining μ_{H2} with the alloy scattering-limited mobility μ_a. μ_{obs} is the experimental value of Hall mobility.

We thus find that the disagreement between theory and experiment that existed at low values of x prior to the inclusion of scattering by transverse phonons of HgTe is removed by its incorporation. The general good agreement with experiments for all values of x leads us to conclude that electron transport in $Hg_{1-x}Cd_xTe$ at room temperature is determined by alloy scattering and interaction with the transverse-optical phonons of HgTe combined with scattering by the longitudinal-optical phonons of HgTe and CdTe, by acoustic modes and by ionized centers.

VI. CALCULATIONS AT 77°K

In order to examine whether all the above scattering mechanisms can satisfactorily account for the experimentally observed mobilities at other temperatures, calculations have also been carried out at 77 °K. A value of 5:8 for the ratio between the coupling constants for the longitudinal- and transverse-optical phonons of HgTe is assumed, this value being identical to that at room temperature. Calculations have been performed for $x=0.195$, 0.41, and 0.59, the experimentally measured Hall mobilities (μ_{obs}) for these x values being taken from Refs. 2 and 19 given in Table II. The carrier concentrations in the experimental samples are, respectively, 4.6, 6, and 3×10^{14} cm^{-3}. The intrinsic carrier concentrations[14] at 77 °K for the considered x values are much lower. Consequently, electron-hole scattering is insignificant with respect to ionized impurity scattering and is neglected. The ionized impurity scattering is included by assuming that the samples are uncompensated.[2]

In Table II we give the calculated values of μ_{d2} and μ_{H2} at 77 °K, these symbols having the same meanings as at room temperature. It is noticed that in this case the Hall-to-drift mobility ratio ($\mu_{H2}:\mu_{d2}$) is larger than 1.2. This is contrary to the case at room temperature where this ratio is not larger than 1.1. The comparison between theory and experiment at 77 °K should therefore be made with some caution. In fact, we should pay attention to μ_{H2}, the experimental values referring to the Hall mobilities.

In Table II are also shown the values of the over-all mobility μ_{t2} These are obtained by combining μ_{H2} with the alloy-scattering-limited mobility μ_a by the formula (15), remembering that μ_{t1} and μ_{d1} in (15) are to be replaced by μ_{t2} and μ_{H2}, respectively. It is observed that μ_{t2} agrees with μ_{obs} very well at $x=0.41$ and 0.59. The agreement at $x=0.195$ is, however, not so good, the two values differing by about 30%. This discrepancy may arise from excess impurity scattering and also from the uncertainties of the parameters, particularly of m^* which is extremely small in this region ($m^* \sim 0.007m_0$).

VII. CONCLUSIONS

There remain discrepancies between the theoretically calculated and the experimentally measured mobilities in $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ when one considers polar scattering by the longitudinal-optical vibrations of CdTe and HgTe combined with scattering by acoustic modes (deformation potential and piezoelectric types), ionized impurities, and heavy holes. Agreement between theory and experiment at moderately large x values (but not small ones) is improved by the inclusion of alloy scattering. The experimental results for all values of x both at 300 and at 77 °K can, however, be brought into agreement with theory by assuming, in addition to the above scattering mechanisms, electron interaction with the transverse optical phonons of HgTe, and by properly choosing a coupling constant for such interaction.

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APPENDIX

Incorporating the effects of degeneracy, electron screening, and wave-function admixture, we have

$$\lambda_{\text{ac}} = \tfrac{1}{2} C_{\text{ac}} \sqrt{\gamma}\,\gamma' \int_{-1}^{+1} (1-z) O(z) S_c(z)\, dz \,,$$

$$\lambda_{\text{pz}} = \tfrac{1}{2} C_{\text{pz}} \gamma' \gamma^{-1/2} \int_{-1}^{+1} O(z) S_c(z)\, dz \,,$$

$$\lambda_{\text{ion}} = C_{\text{ion}} \gamma^{-3/2} \gamma' \int_{-1}^{+1} (1-z)^{-1} O(z) S_c(z)\, dz \,,$$

$$\lambda_{i\text{po}} = C_{i\text{po}} [F_{i+} + \exp(\Theta_i / T_L) F_{i-}] \,, \quad (i=1,2) \,,$$

$$F_{i\pm} = \left(\frac{1 - f_0(E_{i\pm})}{1 - f_0(E)} \right) \frac{\sqrt{\gamma(E_{i\pm})}}{\gamma(E)} (1 + 2\alpha E_{i\pm}) \times \int_{-1}^{+1} \psi_{i\pm} O_{i\pm}(z) S_{ci\pm}(z)\, dz \,,$$

$$S_{i\text{po}} = C_{i\text{po}} \left(\frac{1 - f_0(E_{i+})}{1 - f_0(E)} \right) \gamma(E_{i+}) \gamma^{-3/2}(E) \times (1 + 2\alpha E_{i+}) \int_{-1}^{+1} z \psi_{i+} O_{i+}(z) S_{ci+}(z)\, dz \qquad (i=1,2) \,,$$

$$R_{i\text{po}} = C_{i\text{po}} \left(\frac{1 - f_0(E_{i-})}{1 - f_0(E)} \right) \exp\left(\frac{\Theta_i}{T_L} \right) \gamma(E_{i-}) \gamma^{-3/2}(E) \times (1 + 2\alpha E_{i-}) \int_{-1}^{+1} z \psi_{i-} O_{i-}(z) S_{ci-}(z)\, dz \qquad (i=1,2) \,,$$

where

$$C_{\text{ac}} = \sqrt{2}\, E_1^2 m^{*3/2} k_B T_L / \pi \hbar^4 c_l \,,$$

$$C_{\text{pz}} = \frac{\sqrt{2}\, e^2 e_{14}^2 m^{*1/2} k_B T_L}{35 \pi \hbar^2 \epsilon_0^2} \left(\frac{3}{c_l} + \frac{4}{c_t} \right) ,$$

$$C_{\text{ion}} = e^4 N_i / 16 \sqrt{2}\, \pi m^{*1/2} \epsilon_0^2 \,,$$

$$C_{i\text{po}} = \frac{e^2 m^{*1/2} k_B \Theta_i (\epsilon_0 - \epsilon_\infty)}{4 \sqrt{2}\, \pi \hbar^2 \epsilon_0 \epsilon_\infty [\exp(\Theta_i / T_L - 1)]} \,,$$

$$\psi_{i\pm} = \left[1 + \frac{\gamma(E_{i\pm})}{\gamma(E)} - 2 \left(\frac{\gamma(E_{i\pm})}{\gamma(E)} \right)^{1/2} z \right]^{-1} ,$$

$$E_{i\pm} = E \pm k_B \Theta_i \,.$$

E_1 is the acoustic deformation potential constant, c_l, c_t are the spherically averaged elastic constants for longitudinal- and transverse-acoustic modes, e_{14} is the single independent element of the piezoelectric tensor, N_i is the concentration of ionized centers, and ϵ_0, ϵ_∞ are the static and optic permittivities. The functions $O(z)$ and $O_{i\pm}(z)$ are the overlap factors representing the mixing of wave functions.[3,20] $S_c(z)$ and $S_{c_{i\pm}}(z)$ represent the effects of screening.[21] These functions are given by

$$O(z) = az^2 + bz + c \,, \quad O_{i\pm}(z) = a_{i\pm} z^2 + b_{i\pm} z + c_{i\pm} \,,$$

$$S_c(z) = (1-z)^2 / (1 - z + 2\xi)^2 \,,$$

$$S_{ci\pm}(z) = \left(\frac{1/\psi_{i\pm}}{1/\psi_{i\pm} + 4\xi} \right) \,,$$

z is the cosine of the angle between $\vec{k}$ and $\vec{k}'$, the electron wave vectors before and after scattering. The function ξ is expressed by

$$\xi = \hbar^2 \beta^2 / 8 m^* \gamma \,,$$

β is the inverse of the screening length, and is given for degenerate statistics and a nonparabolic band by[6]

$$\beta^2 = \frac{\sqrt{2}\, e^2 m^{*3/2}}{\pi^2 \hbar^3 \epsilon k_B T_L} \int_0^\infty \sqrt{\gamma}\,\gamma' f_0 (1 - f_0)\, dE .$$

Let E and E' be the energies corresponding to the wave vectors $\vec{k}$ and $\vec{k}'$. The functions a, b, and c are, respectively, given by the expressions of the functions $A(E,E')$, $B(E,E')$, and $C(E,E')$, quoted below,[20] by putting $E' = E$. The functions $a_{i\pm}$, $b_{i\pm}$, and $c_{i\pm}$ are also given, respectively, by the same expressions; but in the case of "+" sign E' has to be replaced by $E + k_B\Theta_i$, and in the case of "−" sign it has to be replaced by $E - k_B\Theta_i$:

$$A(E,E') = \tfrac{3}{4} b_k^2 b_{k'}^2 + c_k^2 c_{k'}^2 - \tfrac{1}{2}(b_k c_{k'} - b_{k'} c_k)^2 + (1/\sqrt{2}) b_k b_{k'} (b_k c_{k'} + b_{k'} c_k) ,$$

$$B(E,E') = 2 a_k a_{k'} (b_k b_{k'} + c_k c_{k'}) ,$$

$$C(E,E') = a_k^2 a_{k'}^2 + \tfrac{1}{4} b_k^2 b_{k'}^2 + \tfrac{1}{2}(b_k c_{k'} + b_{k'} c_k)^2 - (1/\sqrt{2}) b_k b_{k'} (b_k c_{k'} + b_{k'} c_k) ,$$

where

$$a_k^2 = (E + E_g)(E_g + \tfrac{2}{3}\Delta)(E_g + \Delta)/D ,$$

$$b_k^2 = \tfrac{2}{9}\Delta^2 E/D , \quad c_k^2 = E(E_g + \tfrac{2}{3}\Delta)^2/D ,$$

and

$$D = (2E + E_g)(E_g^2 + \tfrac{4}{3}\Delta E_g + \tfrac{2}{3}\Delta^2) + \tfrac{1}{3} E_g \Delta (E + E_g) .$$

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