Electromagnetic wave propagation at the interface between two conductors*

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We predict a new plasmon-polariton mode—supported by a biconducting interface in a certain frequency "window" (ω_1, ω_s). The limiting frequencies are defined by $\epsilon_1(\omega_1)=0$ and $\epsilon_1(\omega_s)+\epsilon_2(\omega_s)=0$, where $\epsilon_i(\omega)$, $i=1,2$, are the dielectric functions of the two media. If the Drude approximation for $\epsilon_i(\omega)$ is applicable then $\omega_s{}^2=(\epsilon_{L1}\omega_1^2+\epsilon_{L2}\omega_2^2)/(\epsilon_{L1}+\epsilon_{L2})$, where ω_i and ϵ_{Li} are the plasma frequencies and lattice dielectric constants of the conductors. These "interface plasmons" may propagate at a metal-metal, or semiconductor-semiconductor, or metal-semiconductor interface. The dispersion, damping, and polarization of the modes are analyzed in the retardation region. Optical methods of excitation and detection are discussed.

Recent years have seen a boom of publications[1] on surface polaritons. The study of these electromagnetic excitations was, however, all but limited to dielectric-dielectric, or dielectric-conductor interfaces. The first to consider a conductor-conductor interface were Stern and Ferrell[2] in 1960. They predicted that two free-electron plasmas bounding one another and characterized by plasma frequencies ω_1 and ω_2, will resonate at the interface at a frequency $(\omega_1^2+\omega_2^2)^{1/2}/\sqrt{2}$. This result was deducted from very simple considerations, neglecting retardation on one hand and nonlocal or quantum effects on the other hand. Six years later Kunz[3] confirmed the above prediction by measuring the energy loss of electrons shot through Mg-Al foils in contact. There have been very few[4,5] subsequent developments, and with one execption,[5] these are not concerned with the retardation or polariton region $\lambda \gtrsim c/\omega_p$. This region, however, seems to be very promising for the study of interfaces, e.g., electron density profiles.

We consider a plane interface of two semi-infinite conducting media. Each is characterized by an isotropic homogeneous and local dielectric function $\epsilon_i(\omega)$, $i=1,2$. We mean by "homogeneous" that the electron-density profile at the interface has the form of a step function, and by "local" that $\epsilon_i(\omega)$ does not depend explicitly on the wave vector $\vec{q}$. The z axis is normal to the interface, and the y axis is along the direction of propagation of the wave in the plane of the interface. Then the wave fields in each medium may be described by plane waves of the form $\exp[i(q_y y+q_z z-\omega t)]$, $i=1,2$. In each medium, $q_y^2+q_{zi}^2=\epsilon_i(\omega)\omega^2/c^2$. From the usual boundary conditions at the interface, we find $q_{z1}/q_{z2}=\epsilon_1(\omega)/\epsilon_2(\omega)$. The solutions of these three equations are

$$q_y^2=\frac{\omega^2}{c^2}\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2},\quad q_{zi}^2=\frac{\omega^2}{c^2}\frac{\epsilon_i^2}{\epsilon_1+\epsilon_2},\ i=1,2. \tag{1}$$

For the time being, we take $\mathrm{Im}\,\epsilon_i(\omega)=0$, i.e., we ignore damping effects. For an interface mode, one must have real propagation along the interface (real q_y) and exponential decay away from it (imaginary q_{zi}). It is then clear from Eqs. (1) that interface waves may propagate only in a region of frequencies where the dielectric functions of the two media have opposite signs and their sum is a negative number. We may choose

$$\epsilon_1(\omega)>0,\quad \epsilon_2(\omega)<0,\quad \epsilon_1(\omega)<|\epsilon_2(\omega)|. \tag{2}$$

As a simple illustration, we assume that $\epsilon_i(\omega)$ are monotonically increasing functions (see Fig. 1). The zeroes of these two functions, ω_1 and ω_2, define the bulk plasma frequencies of the two conductors. From the conditions (2) and inspection of Fig. 1, we may conclude that *a biconducting interface supports a plasmon-polariton mode in a frequency "window"*[6] defined by $\omega_1<\omega<\omega_s$, where ω_s is given by

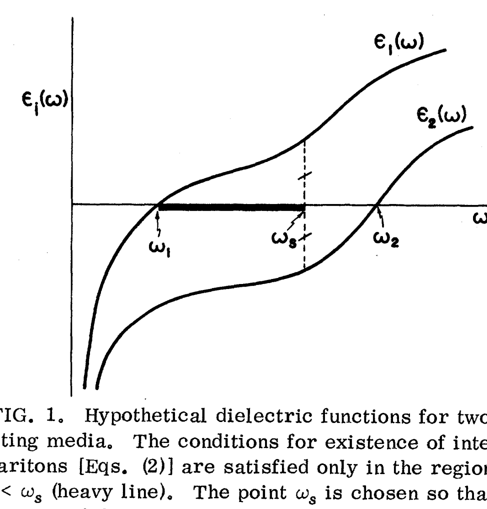

FIG. 1. Hypothetical dielectric functions for two conducting media. The conditions for existence of interface polaritons [Eqs. (2)] are satisfied only in the region $\omega_1<\omega<\omega_s$ (heavy line). The point ω_s is chosen so that the two parts of the dashed line are equal. Damping effects are ignored.

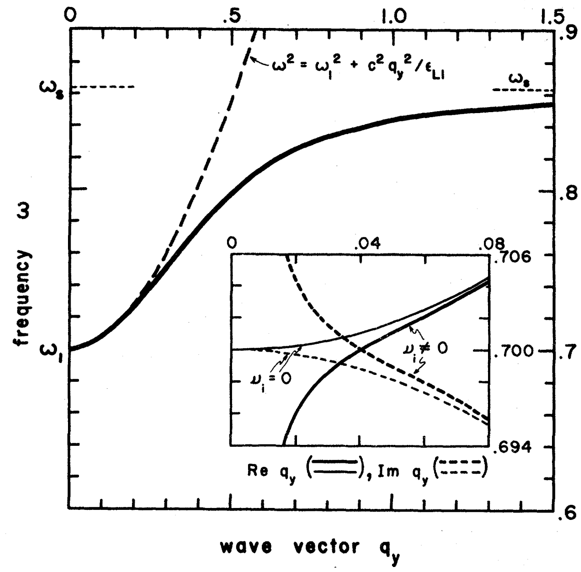

FIG. 2. Dispersion relation of a plasmon polariton at the interface between two conducting media (damping neglected). The frequencies ω_1 and ω_s delineate the propagation "window." The dashed line is the dispersion relation for bulk transverse waves in medium 1. The inlay displays the effect of damping in the neighborhood of ω_1. Numerical values correspond to a Mg-Al interface at room temperature: $\omega_1/\omega_2 = 0.7$, $\nu_1/\omega_1 = 0.0065$, $\nu_2/\omega_2 = 0.0059$, $\epsilon_{L1}/\epsilon_{L2} = 1$. The frequency and wave vector are normalized to ω_2 and $\omega_2 \epsilon_{L2}{}^{1/2}/c$, respectively.

$$\epsilon_1(\omega_s) + \epsilon_2(\omega_s) = 0. \tag{3}$$

We will now limit the discussion to a simple model based on the Drude theory of conductivity. The dielectric functions are

$$\epsilon_i(\omega) = \epsilon_{Li}\{1 - \omega_i^2/[\omega(\omega + i\nu_i)]\}, \quad i = 1, 2, \tag{4}$$

where ω_i, ν_i, and ϵ_{Li} are the plasma frequencies, collision frequencies, and optical lattice dielectric constants, respectively, of the two media. If we neglect damping ($\nu_i = 0$), then it follows from Eq. (3) that the limiting frequency of the interface plasmon is given by

$$\omega_s^2 = (\epsilon_{L1}\omega_1^2 + \epsilon_{L2}\omega_2^2)/(\epsilon_{L1} + \epsilon_{L2}). \tag{5}$$

In the special case that medium 1 is a dielectric ($\omega_1 = 0$), and medium 2 a metal ($\epsilon_{L2} = 1$), we get $\omega_s = \omega_2/(1 + \epsilon_{L1})^{1/2}$. If both media are metals ($\epsilon_{L1} = \epsilon_{L2} = 1$), Eq. (5) reduces to $\omega_s = (\omega_1^2 + \omega_2^2)^{1/2}/\sqrt{2}$. Both special cases were first derived by Stern and Ferrell.[2] Thus Eq. (5) is the interface plasmon frequency that one would calculate neglecting retardation, as well as nonlocal and quantum effects.

The dispersion relation $\omega = \omega(q_y)$ may be readily found from Eqs. (1) and (4). With $\nu_i = 0$, the result is

$$\omega^2 = \Omega^2 - (\Omega^4 - \omega_1^2\omega_2^2 - \omega_s^2 c^2 Q^2)^{1/2}, \tag{6}$$

where $2\Omega^2 = \omega_1^2 + \omega_2^2 + c^2Q^2$ and $Q = (\epsilon_{L1}^{-1} + \epsilon_{L2}^{-1})^{1/2} q_y$. This function is plotted in Fig. 2 (solid line). The limiting frequencies of the polariton mode are ω_1 and ω_s, as one may expect from the foregoing discussion. For sufficiently large q_y, the behavior is quite similar to the plasmon dispersion at a conductor-dielectric interface.[1] On the other hand, for small q_y the behavior is completely different: instead of a linear dependence $\omega \propto q_y$, we have a frequency gap followed by a parabolic dependence of ω on q_y. As a matter of fact, an expansion of Eq. (6) shows that initially the dispersion relation of the surface wave follows that of a bulk transverse electromagnetic wave in medium 1 (dashed line). Numerical values in Figs. 2 and 3 correspond to a Mg-Al interface; these materials are known to satisfy quite well the Drude theory of conductivity.

If the collision frequencies are finite ($\nu_i \neq 0$), then q_y and q_{zi} [Eqs. (1)] become complex. Two qualitative changes in the dispersion relation will appear. One is a sharp backbending of the dispersion curve at a wave vector $q_y \cong 2.1\,\omega_2/c \cong 16 \times 10^5$ cm^{-1} (not shown in Fig. 1). An effect of the same nature was found by Arakawa *et al.*[7] in experiments on a silver-air interface. Their results were subsequently analyzed by Alexander *et al.*[8]

Another effect of the damping appears at very small q_y (inlay of Fig. 1). As Re q_y approaches

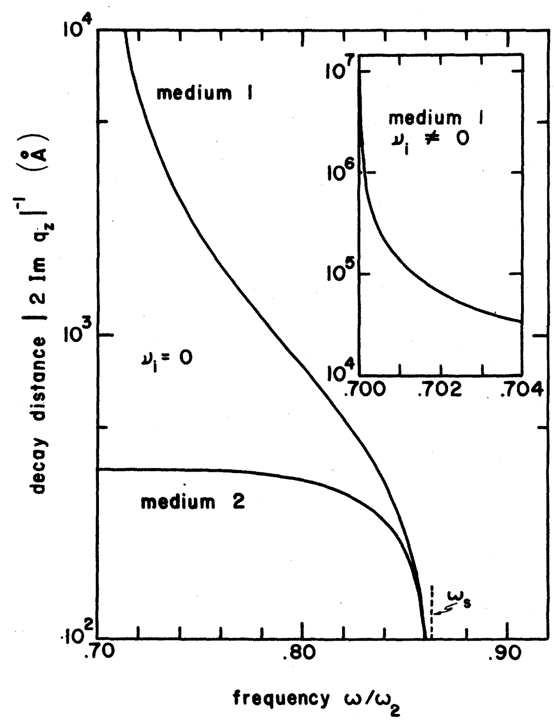

FIG. 3. Exponential decay distances into the two media (damping neglected). Inlay shows "cutoff" of decay distance in medium 1 due to damping effects. For numerical values of the parameters, see caption to Fig. 2.

zero, ω does not tend to the limit ω_1, but keeps decreasing. At $\omega=\omega_1$, the real part of q_y has a finite value equal to $\mathrm{Im}q_y$. For $\omega<\omega_1$, damping effects predominate ($\mathrm{Re}q_y<\mathrm{Im}q_y$). Therefore we expect that under the right experimental conditions, a wave-number gap in the spectrum will appear. For a Mg-Al interface, this gap is of the order of 0.04 $\omega_2/c\cong 0.3\times10^5$ cm^{-1}.

In Fig. 3, we plot the exponential decay distances into the two conductors defined by $|2\,\mathrm{Im}q_{zi}|^{-1}$. If damping is neglected, the decay distance into medium 1 is infinite at the "threshold" frequency ω_1. In real media this distance is limited; for the Mg-Al interface, we find a penetration of ~1 mm into the Mg metal at its own plasma frequency (see inlay). The decay distance into medium 2, on the other hand, is quite independent of ν_i and is given by $0.5|\epsilon_2(\omega_1)|^{-1/2}c/\omega_1$ in the limit $\omega\to\omega_1$. As the frequency approaches ω_s, both decay distances display minima (not shown in Fig. 3), which are related to the backbending of the dispersion curve.

The "propagation distance" of surface waves is defined[1,8] as $(2\,\mathrm{Im}q_y)^{-1}$. For bimetallic interfaces, this distance does not exceed a few μm at room temperature. Nevertheless, the fact that $\mathrm{Im}q_y \ll \mathrm{Re}q_y$ (for almost the entire frequency range of our propagation "window") ensures a well-defined electromagnetic excitation.

The interface polariton at a biconducting interface has a TM polarization, which is also the case for a conductor-dielectric interface. Thus the electromagnetic field is specified by the components H_x, E_y, and E_z. It is not difficult to show that

$$E_{z\,i}/E_y=-[\epsilon_1(\omega)\epsilon_2(\omega)]^{1/2}/\epsilon_i(\omega). \quad (7)$$

An interesting situation arises at $\omega\gtrsim\omega_1$. In medium 1, the electric field is approximately transverse ($E_y\sim0$), while in medium 2 it is approximately longitudinal ($E_{z2}\sim 0$). The longitudinal component E_y itself, however, must be very small, so as to satisfy the requirement of continuity across the boundary. In the limit $\omega\to\omega_s$, neglecting damping, we have $E_{zi}/E_y=\pm i$, i.e., circular polarization in the y-z plane.

The experimental methods of Otto[9] and Kretschmann[10] seem to be most suitable for the excitation and detection of polaritons at biconducting interfaces. In the Otto geometry,[9] medium 1 (of the lower-plasma frequency) has to be "sandwiched" between the prism and medium 2, i.e., medium 1 replaces the "air gap." In the Kretschmann geometry,[10] medium 2 (of the higher-plasma frequency) is "sandwiched" between the prism and medium 1. We expect that the interface plasmon-polariton mode predicted in this article (Fig. 2) may be readily verified experimentally for metal-metal, or metal-semiconductor, or semiconductor-semiconductor interfaces. For a bimetallic interface, the spectral region of interest is in general the visible or the ultraviolet. For example, the limiting wavelengths (corresponding to ω_1 and ω_s) of the propagation "window" for a Mg-Al interface are $\lambda_1\simeq 1170$ Å and $\lambda_s\simeq 940$ Å. For a metal-semiconductor interface, the spectral region depends on the amount of doping and temperature of the semiconductor. For instance, in the case of a Mg-InSb interface with a concentration of ~10^{19} carries/cm^3 in the InSb, the limiting wavelengths are $\lambda_1\sim 11$ μm and $\lambda_s\sim 0.5$ μm. An intriguing possibility is the case of an interface composed of two semiconductors of the same type (e.g., InSb on both sides), differing however in the amount of concentration of the charge carriers. If the two concentrations are close to each other, then $\omega_1\sim\omega_s$, and the "window" will be very narrow, resulting in very little dispersion ($\omega\sim$ const).

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Note added in proof: The values chosen for ν_1/ω_1 and ν_2/ω_2 (see caption to Fig. 2) correspond to the dc resistivities of Mg and Al. At optical frequencies these parameters may be roughly ten times larger. Therefore, our results for the damping effects should hold only qualitatively for a Mg-Al interface. Indeed, one may expect that in an optical experiment[9,10] these effects will be much more pronounced.

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plified model for studying resonant absorption of light in a laboratory plasma (of interest to laser fusion experiments). They find that, for given densities of the two plasmas, resonant absorption will occur for a certain angle of incidence of the light. Although they realize that this happens due to the excitation of a surface wave, the properties of this wave have not been worked out explicitly.

[6]Below the "threshold" frequency $\omega < \omega_1$, q_y and q_{zi} are imaginary. In the region $\omega_s < \omega < \omega_2$, q_{zi} are real but q_y is imaginary. These are domains of "virtual" excitations. For $\omega > \omega_2$, q_y and q_{zi} are real. The "angle of incidence" is determined by $\tan\theta = q_y/q_{z1} = (\epsilon_2/\epsilon_1)^{1/2}$ by Eqs. (2). Then θ is Brewster's angle. The "angle of refraction" is determined by $\tan\theta' = q_y/q_{z2} = (\epsilon_1/\epsilon_2)^{1/2}$. It is easy to see that θ and θ' satisfy Snell's law. Thus for $\omega > \omega_2$, Eqs. (2) are simply statements of Brewster's and Snell's laws.

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