PHYSICAL REVIEW B VOLUME 12, NUMBER 9 1 NOVEMBER 1975

Magnon-photon coupling in antiferromagnets in the presence of an external static magnetic field

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(Received 3 March 1975)

The coupling of photons with long-wavelength magnons in antiferromagnetic MnF_2 in the presence of an externally applied magnetic field in the body c direction is investigated. It is found that two pure, circularly polarized modes can propagate in the crystal along the direction of the magnetic field. A study of the reflection spectra for normally incident plane-polarized radiation reveals that an antiferromagnetic crystal in an applied magnetic field can be used as an optical device to obtain pure, circularly polarized radiation.

I. INTRODUCTION

In analogy to the well-known phenomenon of coupling between photons and phonons in an ionic crystal, the possibility of coupling between photons and magnons in a ferromagnet has been discussed by various authors.[1-8] The problem of such a coupling in antiferromagnetic MnF_2 has recently been discussed by Manohar and Venkataraman.[9] Formally, their method is similar to that previously used to study the spin-resonance problem, except that in their analysis it is assumed that there exists an oscillating photon field associated with the spin fluctuations in the crystal. With this assumption, they discuss the dispersion relations for the coupled magnon-photon modes in MnF_2 for various directions of the incident wave vector $\vec{k}$. They also discuss the reflectivity spectrum $R(\omega)$ for normally incident plane-polarized radiation. The reflectivity, of course, shows a characteristic flat-topped band extending from ω_1, the antiferromagnetic resonance frequency, to its longitudinal counterpart ω_2—a result exactly analogous to the reflectivity bands associated with the infrared vibrational frequencies in an ionic crystal.

In this paper, we study the problem of magnon-photon coupling in an antiferromagnetic crystal in the presence of an externally applied static magnetic field. Our method of approach is similar to that employed by Manohar and Venkataraman,[9] except that here we introduce an applied magnetic field H_0 acting along the c axis, in the mathematical formulation. It is found that, in the presence of H_0, two pure circularly polarized modes can propagate through the crystal, and the dispersion relations for these modes are presented. The reflectivity of the crystal for normally incident, linearly polarized electromagnetic radiation is calculated, and is shown to yield two flat-topped bands each followed by a dip—a well-known characteristic phenomenon observed in the reflectivity spectrum of an ionic crystal. A numerical evaluation of the frequency dependence of the phase of the reflected wave is presented. A careful examination of the phase angle reveals that the aforementioned dips in the reflectivity curve correspond to the reflection of only a single pure right or left circularly polarized radiation. Thus, the possibility of using such an antiferromagnet crystal as an optical device to obtain circularly polarized radiation is discussed. Since the reflection spectra are magnetic field dependent, such an optical device will have the advantage of being tunable in frequency.

In Sec. II, we discuss the dispersion relations and the reflectivity spectra of an antiferromagnetic crystal in an applied magnetic field. Section III contains a brief discussion and the concluding remarks.

II. THEORY

It is well known that MnF_2 is of rutile structure and becomes antiferromagnetically coupled only below 67 °K, the spins of Mn^{++} ions being oriented parallel to the c axis.[10] The spin Hamiltonian for this system is

$$\mathcal{H} = -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j - H_A g\mu_B\Big(\sum_i S_{iZ} - \sum_j S_{jZ}\Big) - g\mu_B\Big(\sum_i \vec{S}_i + \sum_j \vec{S}_j\Big)\cdot(\vec{H}_0+\vec{H}) . \tag{1}$$

Here J represents the nearest-neighbor exchange integral, μ_B is the Bohr magneton, and g is the gyromagnetic ratio. H_A denotes the anisotropic field which is directed along the Z axis taken parallel to the c direction. $\vec{H}_0$ is the external static magnetic field, which is also chosen to be along the c axis. $\vec{H}$ is the fluctuating photon field associated with the spin motion and obviously satisfies Maxwell's equations. Thus the term containing $\vec{H}$ in Eq. (1) corresponds to the interaction of the photon field with the spins. The subscripts i and j denote, respectively, the spins on the two sublattices. The notation $\langle ij\rangle$ indicates summation over nearest-neighbor pairs.

It is easy to verify that an antiferromagnetic substance whose Hamiltonian is given by Eq. (1) is anisotropic for the propagation of electromagnetic radiation and allows two circularly polarized modes (left and right) to propagate as pure modes. The permeability of the medium corresponding to the right or left circularly polarized modes can be expressed as

$$\mu_{\pm} = 1 - \frac{\alpha}{(\omega \pm \gamma H_0)^2 - \omega_1^2} , \tag{2}$$

where

$$\alpha = 8\pi H_E H_A / nJz , \quad \gamma = g\mu_B$$

and

$$\omega_1 = \gamma [H_A (H_A + 2H_E)]^{1/2} .$$

Note that n, z, and ω_1 correspond to the density of magnetic ions, the number of nearest neighbors, and the antiferromagnetic resonance frequency, respectively.

(*a*) *Dispersion relation.* The dispersion relation satisfied by the right and left circularly polarized radiation can be obtained by solving the wave equation as

$$\frac{c^2 k^2}{\omega^2 \epsilon} = \frac{(\omega \pm \gamma H_0)^2 - \omega_2^2}{(\omega \pm \gamma H_0)^2 - \omega_1^2} , \tag{3}$$

where

$$\omega_2^2 = \omega_1^2 + \alpha .$$

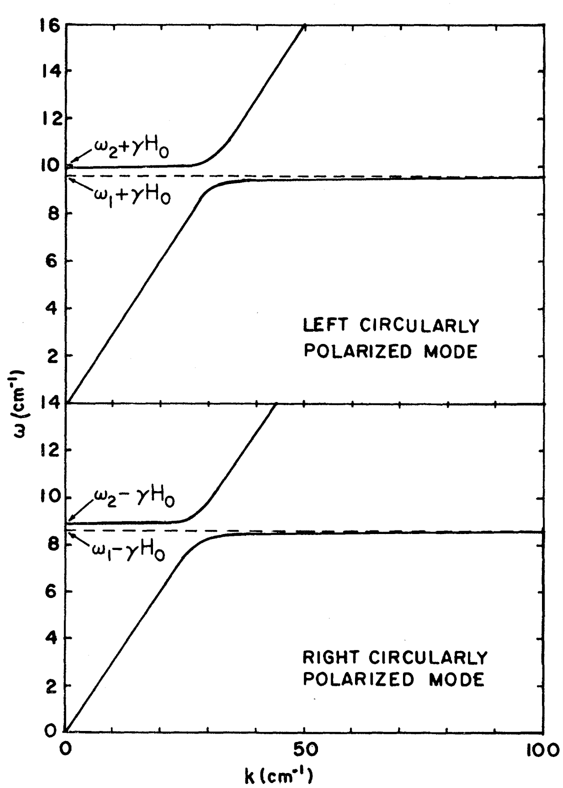

FIG. 1. Dispersion curves of the left and right circularly polarized modes for MnF_2 in an applied magnetic field $\gamma H_0 = 0.5$ cm^{-1}. The frequencies ω_1 and ω_2 for MnF_2 are 9.10 and 9.2 cm^{-1}, respectively.

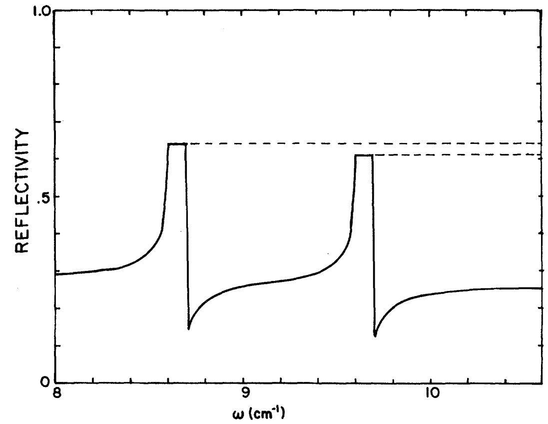

FIG. 2. Reflectivity spectrum $R(\omega)$ for linearly polarized radiation incident normally on a semi-infinite specimen of MnF_2 having the c axis perpendicular to its face. Here, γH_0 has the value of 0.5 cm^{-1}.

Here, $\vec{k}$ is the wave vector parallel to the c axis, and ϵ is the dielectric constant of the medium and its value appropriate to MnF_2 is 9.9. The dispersion curves appropriate to Eq. (3) are sketched in Fig. 1 for $\gamma H_0 = 0.5$ cm^{-1}, the other constants being the same as those used in Ref. 9. Clearly, there are two dispersion curves corresponding to left circularly polarized radiation and right circularly polarized radiation. For each of these modes there are photonlike and magnonlike parts of the dispersion curves, which can be easily identified. We also note that because of the magnon-photon coupling there is a frequency gap generated between $\omega_2 - \gamma H_0$ and $\omega_1 - \gamma H_0$ in the right circularly polarized mode, and another frequency gap generated between $\omega_1 + \gamma H_0$ and $\omega_2 + \gamma H_0$ in the left circularly polarized mode. The appearance of such gaps has previously been pointed out by Pincus[3] and Kittel[5] in connection with their study of ferromagnets.

(*b*) *Reflectivity.* We now consider the reflection of linearly polarized electromagnetic radiation incident normally on a semi-infinite specimen having its c axis normal to the surface and in the presence of an external magnetic field H_0 along the c axis. The reflection coefficients r_R and r_L for the right and left circularly polarized light are found to be[11]

$$r_{R,L} = \frac{(\epsilon \mu_{\pm})^{1/2} - 1}{(\epsilon \mu_{\pm})^{1/2} + 1} . \tag{4}$$

Then the reflectivity $R(\omega)$ for a plane-polarized incident wave will be given by

$$R(\omega) = \tfrac{1}{2}(r_R^2 + r_L^2) . \tag{5}$$

A plot of the reflectivity $R(\omega)$ for $\gamma H_0 = 0.5$ cm^{-1} is shown in Fig. 2. Note that the $R(\omega)$ curve presents two flat-topped bands followed by two dips. The flat-topped bands occur at frequencies between ω_1

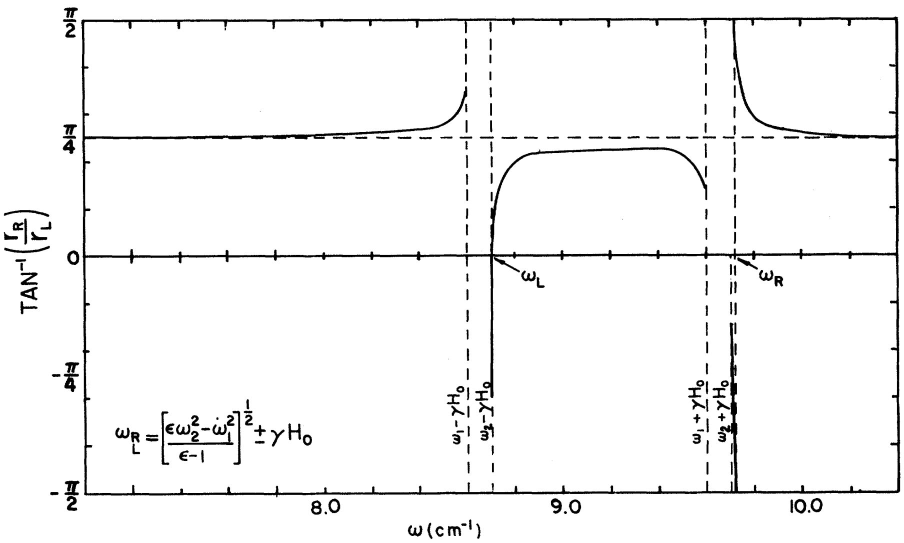

FIG. 3. Sketch of the phase angle ϕ, defined by Eq. (7), as a function of frequency. The crossover frequencies ω_R and ω_L correspond to reflected modes of right and left circular polarization, respectively.

$-\gamma H_0$ and $\omega_2-\gamma H_0$, and also between $\omega_1+\gamma H_0$ and $\omega_2+\gamma H_0$, the frequency range in which the right and left circularly polarized lights have frequency gaps in their dispersion relations. At frequencies slightly higher than $\omega_2-\gamma H_0$ and $\omega_2+\gamma H_0$, the reflectivity passes through a minimum, increasing slowly again with a further increase of frequency. This type of reflectivity maximum followed by a minimum is not very uncommon and has indeed been observed in many ionic solids. One can estimate the frequency spacing between the maximum and the corresponding minimum to be

$$\Delta\omega \cong (\omega_2-\omega_1)/\epsilon \ . \tag{6}$$

In order to obtain a further understanding of the nature of the reflected radiation, we have also calculated the phase of this radiation, defined by

$$\phi = \arctan(r_R/r_L) \ . \tag{7}$$

The phase angle ϕ, expressed in units of radians, has been plotted as a function of ω in Fig. 3 for the case $\gamma H_0 = 0.5\ \mathrm{cm}^{-1}$. Note that at very low and high frequencies, $\phi \cong \frac{1}{4}\pi$, which implies that the reflected radiation is approximately plane polarized. At other intermediate frequencies, the reflected light is in general elliptically polarized. However, at a frequency ω_L slightly higher than $\omega_2-\gamma H_0$, ϕ goes through a zero and Eq. (7) implies that the reflected radiation is entirely left circularly polarized. A careful investigation of our numerical results indicates that this is also the frequency at which the reflectivity goes through the first of the aforementioned minima. Figure 3 also indicates at a frequency ω_R, slightly higher than $\omega_2+\gamma H_0$, the phase angle ϕ becomes exactly equal to $\frac{1}{4}\pi$. Equation (7) again indicates that at this frequency the reflected radiation is pure right circularly polarized. It is also found that at ω_R the reflectivity goes through its second minimum. Thus there are in fact two frequencies ω_L and ω_R which depend on $|\vec{H}_0|$ and at which normally incident linearly polarized radiation is reflected as purely left and right circularly polarized radiation, respectively.

The above interesting observation suggests the possible use of an antiferromagnetic crystal in the presence of an external static field as an optical device to obtain circularly polarized radiation at tunable frequencies. Such a device would be particularly useful since by varying the external applied dc field we can change the tuning frequency for the circularly polarized radiation. Thus, the same device could be used as a polarizer over a wide frequency band.

It is also seen in Fig. 4 that if the applied magnetic field is very weak, the two flat-topped peaks

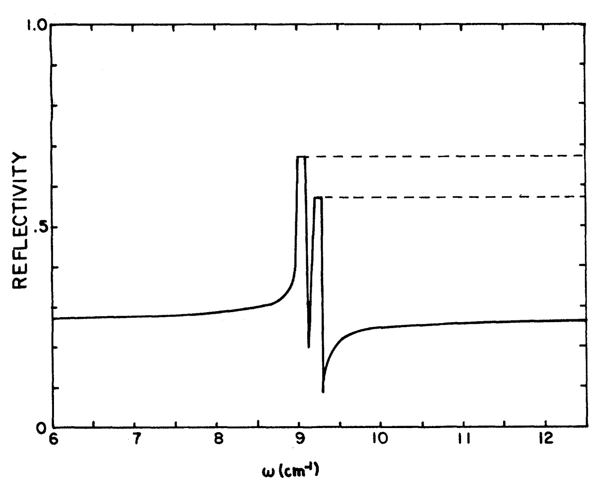

FIG. 4. Reflectivity spectrum, similar to Fig. 2, but for γH_0 having the value of $0.1\ \mathrm{cm}^{-1}$.

in the reflectivity spectrum are very close to each other and between these two maxima there exists a very sharp dip at ω_L where the reflected radiation is purely left circularly polarized. This implies that application of a low magnetic field will open a very narrow window for circularly polarized radiation—a fact which may also be of considerable practical importance.

III. CONCLUDING REMARKS

In this paper we have studied the possibility of magnon-photon coupling in an antiferromagnetic crystal in the presence of an externally applied magnetic field. It has been shown, that in the presence of a nonzero magnetic field, two pure circularly polarized modes of electromagnetic radiation can be excited in the crystal. The dispersion relations for these two pure modes present two frequency band gaps. The reflectivity spectra for a normally incident linearly polarized plane wave exhibits two flat-topped bands followed by two sharp dips. The evaluation of the phase of the reflected radiation reveals that each dip corresponds to a pure, circularly polarized radiation. This raises the interesting possibility of using an antiferromagnetic crystal in an applied magnetic field as an optical device to obtain circularly polarized radiation at frequencies which can be continuously varied by varying the applied magnetic field. Of course, there exists the other possibility, that at very low applied field H_0, the crystal will offer a narrow window for circularly polarized radiation.

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