Flux flow in nearly pure low-κ superconductors*

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A theory of flux flow useful for low-κ superconductors is given where the electrodynamics is nonlocal. Fields and chemical potential gradients are induced by a transport current which causes the vortex to move with velocity $\vec{v}_L$. These are determined self-consistently by the requirement that they generate a supercurrent to correspond to the displacement of a static vortex and compensate for scattering of thermally excited quasiparticles. Making use of an approximate analytic solution for a static vortex by Kramer and Pesch, it is shown that for the case of pure ($l \gg \xi_0$) superconductors at low temperatures ($T \lesssim 0.5\,T_c$) there is no backflow. The component of $\vec{v}_L$ in the direction of the transport current is the transport velocity, a result of the Hall effect on the bound states in the vortex core.

I. INTRODUCTION

Although the theory of the motion of vortices in superconductors has received considerable attention over the years, no satisfactory solution from microscopic theory has been given for relatively pure superconductors with a mean free path large compared with the coherence distance ($l \gg \xi_0$). As a contribution to this problem, we consider here the motion of an isolated straight vortex (H close to H_{c1}) resulting from a uniform transport current flowing normal to the axis of the vortex. The temperature is assumed sufficiently low so that only bound states in the core can be thermally excited (in practice, $T <\sim 0.5\ T_c$). We make use of a solution for the energies of the bound states of a static vortex line as derived by Kramer and Pesch[1] from solutions of the Eilenberger equations.

We show how the effective force on an electron resulting from the electrochemical gradient and the magnetic field may be derived in a self-consistent manner. As first shown by Caroli *et al.*[2] a vortex may be considered to have a normal core with a radius roughly equal to the coherence distance ξ_0. We assume that the transport current J_T, is sufficiently small so that the distance a vortex moves during a relaxation time is small compared with ξ_0. No attempt is made to treat the magnetic field of the transport current self-consistently.

One may regard the present theory as a nonlocal generalization of the local model of Stephen and one of the authors.[3] In the local model, it is assumed that the current density $\vec{J}_s(\vec{v}_s)$ depends only on the local value of the superfluid velocity $\vec{v}_s$. The free-energy density $F(\vec{P})$ is a local function of the kinetic momentum $\vec{P} = \vec{p} - (e/c)\vec{A}$ and $\vec{v}_s = \partial F(\vec{P})/\partial \vec{P}$. It was shown that fields are set up so as to drive the transport current through the normal core in such a way that the total current density

$$\vec{J} = \vec{J}_s + \vec{J}_n = \vec{J}_T + \vec{J}_0(\vec{r} - \vec{v}_L t), \qquad (1)$$

where $\vec{J}_T$ is the uniform transport current, $\vec{J}_0(\vec{r})$ is the circulating current around a stationary vortex, and $\vec{v}_L$ is the velocity of the vortex relative to the lattice. This was found to be true regardless of the details of the local model.

Nozières and Vinen[4] have discussed vortex motion in pure superconductors on the basis of a general hydrodynamical model. A critical discussion and extension of this model as well as that of the local model has been given by Vinen and Warren.[5] They conclude that the component of $\vec{v}_L$ in the direction of the transport current should be equal to the transport velocity $\vec{v}_T$ of the electrons. If, as illustrated in Fig. 1, $\vec{v}_T$ is in the x direction, then $v_{Lx} = |\vec{v}_T| \equiv v_T$. The component v_{Ly} in the transverse direction is such as to make up for the losses in the core. The circulating currents are assumed to be in the clockwise direction so that the magnetic field H_z is in the negative z direction, assuming e is positive. According to the hydrodynamical model, the vortex should move at the Hall angle α relative to $\vec{v}_T$, such that $\tan\alpha = v_T/v_{Ly}$. This is contrary to the prediction of the local model of Bardeen and Stephen[3] for which the Hall angle should be equal to that of the normal metal for a magnetic field equal to that in the core.

Since the time those papers were written, a great deal has been learned about structure of static vortex lines from nonlocal microscopic theory.[1,6-8] For values of κ of the order of unity, a large part of the circulation comes from bound states in the core. The pair potential $\Delta(r)$ that confines the flow acts like an effective magnetic field which is in addition to the true magnetic field. We shall show that at low temperatures the

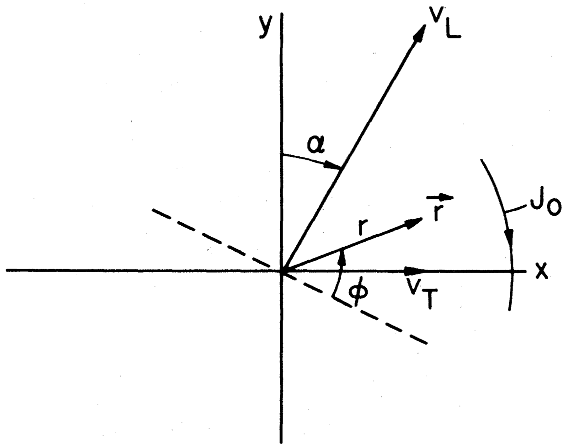

FIG. 1. Coordinates used to discuss vortex motion. The current circulates in the clockwise direction and the magnetic field is in the negative z direction.

values of the energy of the bound states determined by Kramer and Pesch are consistent both with Eq. (1) and also with $v_{Lx}=v_T$ as suggested by Nozières and Vinen. We also determine a viscosity coefficient and flux-flow resistivity for motion in the transverse direction v_{Ly}. In Sec. II, we discuss the nature of the effective fields and forces on the electrons. The relations between them as derived from momentum conservation are described in Sec. III. In Sec. IV, we make use of the microscopic theory to derive the area of the core and the effective Hall field. Finally, in Sec. V, we apply the results to derive the viscosity coefficient for flow normal to the transport current and the flux-flow resistivity. We also discuss possible future applications of the theory.

II. EFFECTIVE FIELDS

In this section we discover how the effective fields that move the vortex are determined. Let us first consider a static vortex about which currents circulate that give the magnetic field along the axis. We consider only vortex lines that carry a single flux quantum $\Phi_0=hc/2e$, or in units such that $\hbar=c=1$, π/e. As illustrated in Fig. 1, we use cylindrical coordinates, r, ϕ, z, where z is along the vortex axis and ϕ is measured from the normal to $\vec{v}_L$. The static magnetic field may be described by a vector potential that has only a ϕ component $A_\phi(r)$ which is a function only of the distance r from the axis. For a line with a single quantum one may take a gauge $A'_\phi(r)$,

$$A'_\phi(r)=A_\phi(r)+\Phi_0/2\pi r, \tag{2}$$

where $A_\phi(r)\to 0$ as $r\to 0$ and $A'_\phi(r)\to 0$ as $r\to\infty$ more rapidly than $1/r$. This insures that the total flux corresponds to one quantum. The magnetic field

$$H_z(r)=r^{-1}\frac{d(rA_\phi)}{dr}=r^{-1}\frac{d(rA'_\phi)}{dr}. \tag{3}$$

If the vortex is moving at a velocity $\vec{v}_L$ relative to the lattice, all quantities in steady-state flow will be functions of $\vec{r}-\vec{v}_L t$. The moving magnetic field will give rise to an electric field normal to $\vec{v}_L$,

$$\vec{E}_H=-\vec{v}_L\times\vec{H}=-\vec{\nabla}(\vec{v}_L\cdot\vec{A})+(\vec{v}_L\cdot\vec{\nabla})\vec{A}. \tag{4}$$

This expression is gauge invariant, so that one could use either $\vec{A}$ or $\vec{A}'$ to describe the magnetic field; we shall use $\vec{A}$. In addition to this field, there may be electrochemical gradients whose potential can be added to $\vec{v}_L\cdot\vec{A}$, giving a field of the form

$$\vec{f}=-\vec{\nabla}W+(\vec{v}_L\cdot\vec{\nabla})\vec{A}. \tag{5}$$

One may regard this field, as in the local model of Bardeen and Stephen, as arising in part from the time rate of change of the canonical momentum P_0 which has only a ϕ component given by

$$P_\phi(r)=e\Phi_0/2\pi r-eA_\phi(r). \tag{6}$$

The force from the time rate of change of $P_\phi(\vec{r}-\vec{v}_L t)$ is, at $t=0$,

$$e\vec{f}=-(\vec{v}_L\cdot\vec{\nabla})P_\phi(r)\vec{1}_\phi, \tag{7}$$

with components

$$f_r=v_L\cos\phi(P_\phi/r), \tag{8a}$$

$$f_\phi=-v_L\sin\phi\left(\frac{\partial P_\phi}{\partial r}\right), \tag{8b}$$

where $\vec{1}_\phi$ is a unit vector in the ϕ direction.

In the local model,[3] it was assumed that there is a normal core of radius a outside of which supercurrents circulate to give a magnetic field in the core. From the term $e\Phi_0/2\pi r$ in P_ϕ, there is a surface charge density $\sigma(a)=-\Phi_0 v_L\cos\phi/4\pi a^2$ at the core boundary, a dipolar field outside of the core normal to v_L given by

$$ef_c=-e\Phi_0 v_L/2\pi a^2. \tag{9}$$

In the nonlocal theory, the core boundary and charge density will be spread out over a distance of the order of the coherence distance from the axis of the vortex. We superimpose fields with a continuous distribution of a values.

The normal component comes from the nonpaired bound states in the core.[6] At a distance r, a fraction $g(r)$ of the density at the Fermi surface will arise from bound states, where $g(r)$ increases from zero at $r=\infty$ and approaches unity as $r\to 0$.

A fraction $-[dg(r)/dr]da$ changes from normal to superconducting in the interval da. One may then, in analogy with the local model, take for the term $-\vec{\nabla}W$ in (5) an integral of fields over different a values weighted by the factor $-(dg/dr)_a$,

$$-\frac{\partial W}{\partial r}=-\frac{\Phi_0 v_L\cos\phi}{2\pi r^2}\left(1-g(r)+r^2\int_r^\infty \frac{dg}{da}\frac{da}{a^2}\right), \tag{10a}$$

$$-\frac{1}{r}\frac{\partial W}{\partial\phi}=-\frac{\Phi_0 v_L\sin\phi}{2\pi r^2}\left(1-g(r)-r^2\int_r^\infty \frac{dg}{da}\frac{da}{a^2}\right). \tag{10b}$$

The terms with a factor $1-g(r)$ correspond to the bipolar field outside the core of the local model while those with the integral represent the integral of the uniform fields similar to (9) from the term $e\Phi_0/2\pi r$ for $a>r$. One may readily verify that $\nabla^2 W=-4\pi\rho$, where ρ is the charge density $-\sigma(r)(dg/dr)$.

The integral throughout space of $-\vec{\nabla}W$ and of $(\vec{v}_L\cdot\vec{\nabla})\vec{A}$ have components perpendicular to $\vec{v}_L$ of equal magnitude and vanishing components parallel to $\vec{v}_L$. Their sum is equal to the integral of $-\vec{v}_L\times\vec{H}$, or since H is in the negative z direction, $(\vec{v}_L\times\vec{1}_z)\Phi_0$.

More generally, one may regard W as a function to be determined self-consistently so that it gives rise to a steady-state motion of the vortex with velocity $\vec{v}_L$. It is required that W be regular at the origin and approach the values indicated in Eq. (10) as $r\to\infty$. Then $\vec{v}_L$ is determined by the condition that the fields compensate for scattering within the core. At higher temperatures, when quasiparticle states with energies greater than Δ_∞ may be present, one would have to determine the steady-state quasiparticle distribution as well. This latter is a difficult problem because one must consider both elastic and inelastic scattering as the quasiparticles relax to form ground-state pairs.

III. MOMENTUM CONSERVATION

Before discussing the details of the microscopic theory we consider the physics of the problem. The key is the nature of the bound quasiparticle states with energies less than Δ_∞. These are not paired, as are the states in the bulk, so that transitions between them do not have the usual coherence factors. States of positive energy may be regarded as having equal amplitudes of particles above and holes below the Fermi sea. The circulation of current around the axis is similar to that in an effective magnetic field, in part due to the real magnetic field and in part due to the pair potential which confines the states, as discussed in Ref. 6. We show that the Hall effect from this effective field is just such as to make $v_{Lx}=v_T$ and that the transport current flows directly through the core with no backflow.

The bound-state wave functions drop off exponentially with distance from the axis so that the change from supercurrent flow to normal flow in the core is a gradual one, as discussed in Sec. II. When there is a transport current with velocity v_T, the Fermi surface is shifted by an amount v_T. If there is no backflow, the Fermi surface of the bound states is also shifted by v_T. However, the quasiparticles tend to relax to the lattice so that a field is required to maintain the distribution in steady state. The Hall effect on the bound states implies that this field acts at an angle α relative to the transport current and the vortex moves in a direction normal to this field, as illustrated in Fig. 1.

The density of bound states at the Fermi surface is the normal density multiplied by an area A_c, which may be regarded as the area of the normal core. The value may be derived from the bound-state energies of Kramer and Pesch, but our value differs slightly from theirs because of a difference in physical interpretation of the result.

Let us first consider an ideally pure superconductor. If the core were at rest relative to the lattice, and a transport velocity $\vec{v}_T$ were applied, the supercurrent would flow around the normal core region. This might be described by a velocity $-\vec{v}_T$ of the core, with associated backflow outside the core, on which a uniform $\vec{v}_T$ is superimposed. The velocity of the core would then be zero. Now suppose that the core moves with the transport current, so that the core states have an additional velocity $\vec{v}_T$. There would then be no backflow. In a frame moving with transport current, the vortex would be at rest, but the lattice would be moving with a velocity $-\vec{v}_T$. If we ignore the magnetic field produced by the moving lattice, the vortex would move with the electron fluid, as suggested by the hydrodynamical model of Nozières and Vinen.[4]

If we go back to the lattice frame, electric fields would be generated by the moving magnetic field of the lattice but there would be no net force because this field is just balanced by the effect of the magnetic field on the transport current

$$\vec{f}=\vec{E}+\vec{v}_T\times\vec{H}=0. \tag{11}$$

Let us now assume impurity scattering of the core electrons with a transport relaxation time $\tau=v_F l$. We assume that there is no backflow, as described in Eq. (1), and show that this solution is self-consistent if we allow an appropriate velocity v_{Ly} perpendicular to $\vec{v}_T$ in addition to $v_{Lx} = v_T$.

We determine the force ef required to maintain the flow as follows: The normal density of states of one spin of positive energy is $2N(0)$ since both electrons above and holes below the Fermi surface are counted as positive excitation. If $N_n(\vec{r})$ is the corresponding density of bound states at the Fermi surface, the normal fraction is

$$g(r) = \tfrac{1}{2} N_n(r)/N(0). \tag{12}$$

If the total electron density is n, the rate of loss of momentum from scattering is per unit volume for unit length of line:

$$g(r) n m v_T/\tau. \tag{13}$$

Since all of the electrons are accelerated by the force, momentum is gained at a rate

$$nm\frac{dv}{dt} = nef_x, \tag{14}$$

which must be equal to (13) if the two are in balance. The integral of $g(r)$ over the r, ϕ plane is the effective area of the core:

$$A_c = 2\pi \int_0^\infty r g(r)\, dr. \tag{15}$$

The integral of f_x over the same plane is, from (5) with boundary conditions for large r as in (10),

$$v_{Ly} \Phi_0. \tag{16}$$

Thus we find, equating the integrals of (13) and (14),

$$v_{Ly}/v_T = m A_c/\pi\hbar\tau. \tag{17}$$

There remains to determine A_c and v_{Lx} from the microscopic theory of the bound states in the core, which is done in Sec. IV. The power dissipation is compensated for by the motion of the vortex line in a direction transverse to $\vec{v}_T$. The line moves down a free-energy gradient $-\vec{J}_T \times \vec{\Phi}_0$ giving a net force

$$ne \int \vec{f}\, d^2r = -\vec{J}_T \times \vec{\Phi}_0 = \pi n \hbar v_T \vec{1}_y. \tag{18}$$

In the lattice frame there is an additional change in momentum from the passage of the circulating current, $J_0(\vec{r} - \vec{v}_L t)$. The time rate of change of momentum per unit volume $\vec{F}(r)$ multiplied by e is

$$\frac{e}{m}\vec{F} = \frac{\partial \vec{J}_0}{\partial t} = -(\vec{v}_L \cdot \vec{\nabla}) J_0. \tag{19}$$

The components in the r and ϕ directions are

$$\frac{e}{m} F_r = v_L \cos\phi \left(\frac{J_0(r)}{r}\right), \tag{20a}$$

$$\frac{e}{m} F_\phi = -v_L \sin\phi \left(\frac{dJ_0(r)}{dr}\right). \tag{20b}$$

The component normal to $\vec{v}_L$ is

$$\frac{e}{m} F_\perp = v_L \cos^2\phi \left(\frac{J_0}{r}\right) + v_L \sin^2\phi \left(\frac{dJ_0}{dr}\right). \tag{21}$$

The integral over space is

$$e \int F_\perp d^2r = \pi v_L m \int_0^\infty \left[J_0 + r\left(\frac{dJ_0}{dr}\right)\right] dr = \pi v_L m (rJ_0)\Big|_0^\infty = 0. \tag{22}$$

Thus there is no net force associated with the passage of the line, as is evident from the fact that v_T is assumed to remain constant as the vortex line passes. This would not be true unless there were some external source of power to keep v_T constant, as the power that drives the line comes ultimately from this external source.

In superconductors with large values of κ, the peak in rJ_0 will be near the boundary of the core. If we calculate the integral to some lower limit for r, we would have $-\pi v_L r J_0(r)$. Since we have assumed that J_0 is negative, this expression is positive. In regions where $|rJ_0(r)|$ is increasing $F_\perp$ is positive; in regions where it is decreasing $F_\perp$ is negative on the average. Thus there would have to be compensating forces in the superfluid outside of the core. However the net force in the core would still be given by Eq. (16) if $rJ_0(r)$ is small at the core boundary. Thus although the microscopic theory to be given later applies only to small κ superconductors, Eq. (1) may still be valid for large κ as well.

IV. MICROSCOPIC THEORY

Accurate calculations of the structure of pure static vortex lines have been derived by Pesch and Kramer[8] from the Eilenberger version of the Gorkov equations. They have derived the pair potential $\Delta(r)$ and the current density $J(r)$ for various values of the Ginzburg-Landau parameter κ and of the reduced temperature $t = T/T_c$. More recently, the same authors[1] have derived analytic expressions for the quasiparticle energies of the bound states for small κ superconductors. The approximations involve keeping only the low-frequency terms in the Eilenberger equations, corresponding to keeping only the bound-state terms in the Bogoliubov equations, which should be appropriate for values of κ near one. They also consider values of κ for type-I superconductors where vortex lines may be formed in thin films subjected to transverse fields.

The Bogoliubov amplitudes $u_n(r)$ and $v_n(r)$ may be designated by the wave vector $k_z = k_F \cos\theta$, the magnetic quantum number μ, which takes half-odd integral values, and the spin. Generally, there is only one bound state for each μ, with positive energies corresponding to positive μ, for each value of the spin. One may regard states of negative μ as occupied in the ground state, giving a circulation in the counterclockwise direction. For values of κ near one, most of the circulation comes from the bound states in the core.[1,6] The gap parameter $\Delta(r)$ varies linearly with μ for small r and is to be determined self-consistently from the equation

$$\Delta(r) = V \sum_n u_n(r) v_n(r) \tanh(\tfrac{1}{2}\beta\epsilon_n). \tag{23}$$

According to Caroli *et al.*,[2] for r small,

$$u_\mu(r) = (k_F \Delta_\infty / 2v_F)^{1/2} e^{i(\mu+1/2)\phi} J_{\mu+1/2}(k_F r \cos\theta), \tag{24a}$$

$$v_\mu(r) = (k_F \Delta_\infty / 2v_F)^{1/2} e^{i(\mu-1/2)\phi} J_{\mu-1/2}(k_F r \cos\theta). \tag{24b}$$

Here μ is one-half an odd integer and the normalization is for unit length of vortex line. The quasiparticle energies are given by

$$\epsilon_\mu = \frac{\mu}{k_F \cos\theta} \int_0^\infty dr \frac{\Delta(r)}{r} e^{-U(r)} \Big/ \int_0^\infty dr\, e^{-U(r)}, \tag{25}$$

where

$$U(r) = \frac{2}{\hbar v_F \cos\theta} \int_0^r dr\, \Delta(r). \tag{26}$$

If only small μ contribute to the sum in Eq. (23), Kramer and Pesch find by the Bogoliubov method for $T_c \gg T \gg \Delta_\infty^2/E_F$ (in Gaussian units)

$$\epsilon_\mu = \frac{2\mu\hbar^2 \ln(\xi_0/\xi_1)}{\pi^2 m \xi_0^2 \cos^2\theta}, \tag{27}$$

where $\xi_1 \sim \xi_0(T/T_c)$. This expression applies when μ is small so that one is in the region where ϵ_μ varies linearly with μ. They found the same expression with use of the Eilenberger method.

The factor $\cos^2\theta$ arises from the geometry of the Fermi surface, as shown in Fig. 2(a). The density of states in the interval $d\theta$ is for transverse excitations

$$N_{\dagger\mathrm{tr}}(\theta)\, d\theta = \frac{d\mu}{d\epsilon_\mu} \frac{dk_z}{2\pi} = \frac{\pi m \xi_0^2 k_F \cos^3\theta\, d\theta}{4\hbar^2 \ln(\xi_0/\xi_1)}. \tag{28}$$

A similar expression with $\cos^2\theta$ replaced by $\sin^2\theta$ comes from excitation of k_z, with the Fermi surface $k_{zF}(\theta)$ unchanged. In the over-all density, excitations of k_z contribute equally to those in the x and y directions. The factor $\cos^3\theta$ in $d\mu/d\epsilon_\mu$ accounts for the decrease in phase space and number of electrons with increase in k_z, as illustrated in Fig. 2(a). It is the Fermi energy that determines k_r such that $k_r^2 + k_z^2 = k_F^2$. One can have excitations in k_z without changing k_{zF}. The total density is obtained by replacing $\cos^3\theta$ in Eq. (28) with $\cos\theta$ and integrating over θ between $-\frac{1}{2}\pi$ and $\frac{1}{2}\pi$. Setting this equal to $2N(0)A_c$, we find

$$A_c = \pi^3 \xi_0^2 / 2\ln(\xi_0/\xi_1) \tag{29}$$

since the normal density $2N(0) = mk_F/(\pi\hbar)^2$. This value for A_c is $\frac{3}{4}$ the value given by Kramer and Pesch[1]; a factor $\frac{3}{2}$ comes from including excitations in k_z and a factor $\frac{1}{2}$ comes from comparing the density with $2N(0)$ rather than $N(0)$ for the normal metal.

To determine the effective magnetic field for the Hall effect, we would like to find the field such that there is one excitation for each spin and μ value in a volume A_c for unit length of vortex line. For a magnetic field, the frequencies are independent of k_r and thus of k_z. The phase space for an energy interval $\hbar\omega_c = eH_{\mathrm{eff}}/mc$, as indicated in Fig. 2(b), is such that

$$\delta\epsilon_\perp = (\hbar^2/m) k_r \delta k_r = \hbar\omega_c. \tag{30}$$

The phase space corresponding to one state of positive energy (counting both particles and holes as positive) is for a volume A_c given by

$$A_c k_r \delta k_r = \pi. \tag{31}$$

Thus from Eqs. (30) and (31), we have

$$\hbar\omega_c = \pi\hbar^2 / m A_c. \tag{32}$$

Note that the effective field H_{eff} is simply Φ_0/A_c. For this field, the Hall angle is

$$\tan\alpha = v_{Lx}/v_{Ly} = \omega_c \tau = \pi\hbar\tau/mA_c. \tag{33}$$

In Eq. (17), we found an identical expression for

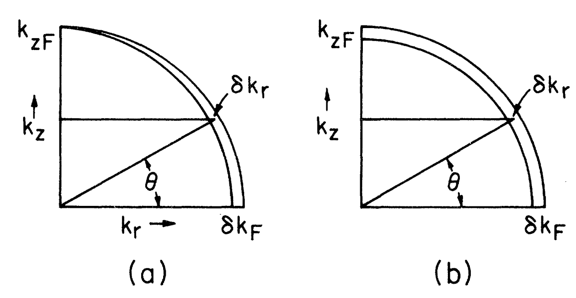

FIG. 2. (a) Phase space for bound state in the core of a vortex line; $\delta k_r = \delta k_F \cos\theta$. (b) Phase space for electrons in a normal metal in a magnetic field; $\delta k_r = \delta k_F / \cos\theta$.

v_T/v_{Ly}. Thus we find that $v_{Lx}=v_T$ as a result of the Hall effect on the electrons in the core. There is no backflow.

V. CONCLUSIONS

The effective field serves both to move the vortex line and to balance the effect of impurity scattering in the core, resulting in the simple form of Eq. (1). It is not certain whether or not Eq. (1) applies to dirty superconductors at low temperatures. It probably does not apply to pure type-I superconductors, which can have vortex lines penetrating a thin film in a transverse magnetic field. In this latter case, according to Kramer and Pesch,[1] the A_c as defined by Eq. (8) depends on θ, indicating that the Fermi surface is modified as the vortex moves.

The viscosity coefficient η for motion in the y direction, defined so that the retarding force is ηv_{Ly}, is given by

$$\eta = n\pi^2\hbar^2\tau/mA_c. \tag{34}$$

Except for the shrinkage of the core with decreasing temperature given by the logarithmic factor $\ln(\xi_0/\xi_1)$ in Eq. (8), this result is not far different from that given by the local model.[3] It would be of interest to confirm this shrinkage of the core experimentally.

One would expect that the same considerations would apply to a moving vortex lattice as long as the temperature is sufficiently low so that only bound states are excited. In practice, this implies $T \lesssim 0.5T_c$. Excluded are gapless superconductors for which the Eliashberg-Gorkov version of the time-dependent Ginzburg-Landau theory may be applied. Thompson and Hu[9] have shown that in this case one generally has backflow, contrary to Eq. (1).

The flux-flow resistivity ρ_f is that of a normal region of relative volume $(B/\Phi_0)A_c$, so that the ratio to the normal resistivity is

$$\rho_f/\rho_n = (B/\Phi_0)A_c.$$

For pure metals, this is approximately B/H_{c2}, as suggested by experiment.

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