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Light scattering from semi-infinite media for non-normal incidence

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Kröger and Kretschmann and Maradudin and Mills have reported conflicting theoretical results for p-polarization non-normally incident light scattered from semi-infinite rough surfaces. Both of these works calculate scattering from roughness-dependent δ-function polarization currents, whereas the author has used a coordinate transformation approach where the scattering currents extend throughout both (upper and lower) media. The author's results are in agreement with those of Kröger and Kretschmann.

Recently there has been a surge of interest in light scattering associated with a multitude of applications. Most of these first-order perturbation treatments have dealt only with normally incident light. However, it is also important to predict scattering from non-normal incidence as well. Kretschmann[1] presented such a calculation which was later corrected by Kröger and Kretschmann.[2] More recently, Maradudin and Mills[3] presented a similar classical calculation. References 2 and 3 are similar in that both assume δ-function scattering currents confined to the interface separating the vacuum and scattering medium. The author has made a scattering calculation whereby a coordinate transformation, defined by $u_1 = x$, $u_2 = y$, $u_3 = z - \zeta(x, y)$, where $\zeta(x, y)$ is the roughness profile function, maps the rough surface into a plane. This method yields scattering currents which are not δ function in character rather they extend throughout both media. The method given previously by Elson and Ritchie[4] for normal incidence has been modified were to include non-normal incidence and to utilize the vector potential $\vec{A}$ rather than the Hertz vector potential. In any case, when going from normal incidence to non-normally incident p-polarized light, it is especially important to take care that the proper boundary conditions at the actual rough surfaces are fulfilled.

The δ-function surface currents used in Refs. 2 and 3 are derived somewhat differently, yet yield nearly identical polarization-scattering currents. Kroger and Kretschmanns's[2] (hereinafter referred to as KK) currents are phenomenological whereas Maradudin and Mills[3] (MM) derive their currents by expanding the dielectric constant about the median surface plane. The strength of the currents used by KK is clearly defined at the scattering surface, whereas in MM the inherent discontinuity of their scattering currents across the boundary leaves some ambiguity. By satisfying the boundary conditions to first order in $\zeta(x, y)$, KK showed that certain field discontinuities which appear across the boundary could be caused by δ-function source currents of the type assumed in their work. Further, by analyzing Maxwell's equations with similar currents it became apparent that these currents should be located in vacuum and at the boundary. MM solve the wave equation by using Green's-function techniques. Integration over the sources include the δ function and because of the ambiguous discontinuity across the boundary, the average value of both (upper and lower media) sources evaluated at the boundary is taken. Further, the boundary conditions on the first-order fields are assumed to be analogous to those for the zero-order field. At any rate the results of KK and MM for the case of nonnormal p-polarized incident light do not agree.

In the present work, the source currents emerge as a result of the nonorthogonal coordinate transformation on the wave equation. Consider the wave equation in the form $LA = 0$, where the operator

$$L = \nabla x \nabla x + \frac{\epsilon(z)}{c^2} \frac{\partial^2}{\partial t^2}$$

and $A = (A_x, A_y, A_z)$. Transforming the equation as indicated above yields $L = L^{(0)} + L^{(1)}$ to first-order accuracy in ζ. Also, $A = (A_1, A_2, A_3)$ since there is a new basis set. With this the wave equation may be written $L^{(0)}A = -L^{(1)}A$ and this suggests solution by iteration. Letting $A = A^{(0)} + A^{(1)}$, where the superscripts (0) and (1) refer to zero and first order, respectively, yields $L^{(0)}A^{(0)} = 0$ and $L^{(0)}A^{(1)} = -L^{(1)}A^{(0)}$. The latter equation may be solved by Green's-function methods to yield $A^{(1)}$ where the former equation yields $A^{(0)}$ for a general zero-order field. Since the solution is expected to be accurate to first order, care must be taken to ensure that the boundary conditions will also be satisfied to first order. In this work, the standard boundary conditions are considered for continuity of the tangential components of the electric $\vec{E} = \vec{E}^{(0)} + \vec{E}^{(1)}$ and magnetic $\vec{H} = \vec{H}^{(0)} + \vec{H}^{(1)}$ fields [see Eq. (2.8) of Ref. 4]. Also considered is the continuity of the normal components of the displacement vector $\vec{D} = \epsilon\vec{E}$ and magnetic field (nonmagnetic media). The coordinate transformation is applied to the boundary conditions and by keeping the terms to first order in ζ, the continuity conditions are derived. These continuity conditions, when satisfied, will ensure that the appropriate boundary conditions will be satisfied, to first order, across the boundary. It is necessary to add an homogeneous solution to $A^{(1)}$ adjusted in such a way that these boundary conditions will be fulfilled. The source current of KK is initially adjusted so as to provide for the correct boundary conditions.

In the present work, one calculates the first-order vector potential $A^{(1)}$ and subsequently the Poynting vector of scattered energy in the far zone.

By normalizing to the incident beam energy one may, by standard procedure, calculate the scattering probability per unit solid angle. Further, it is assumed that the incident beam illuminates an area L^2, is polarized at angle ϕ' relative to the plane of incidence and is incident at polar angle θ_0. The scattering media is characterized by a complex dielectric constant ϵ for incident frequency ω_0. The scattered light is at polar angle θ and azimuthal angle ϕ relative to the plane of incidence. The differential scattering probability into solid angle $d\Omega = \sin\theta\, d\theta\, d\phi$ is found to be

$$\frac{dP}{d\Omega} = \frac{(\omega_0/c)^4}{\pi^2} \cos\theta_0 \cos^2\theta \,|1-\epsilon|^2 \,\frac{|\zeta_{\vec{k}-\vec{k}_0}|^2}{L^2} \left(\frac{|\chi_\theta|^2}{|\nu - iq\epsilon|^2} + \frac{|\chi_\phi|^2}{|\nu - iq|^2}\right), \tag{1}$$

where

$$\chi_\theta = \frac{(\nu\nu_0 \cos\phi + kk_0\epsilon)\cos\phi'}{\nu_0 - iq_0\epsilon} - \frac{i(\omega_0/c)\nu \sin\phi \sin\phi'}{\nu_0 - iq_0} \tag{2}$$

and

$$\chi_\phi = \left(\frac{\omega_0}{c}\right)\left(\frac{\nu_0 \sin\phi \cos\phi'}{\nu_0 - iq_0\epsilon} + \frac{i(\omega_0/c)\cos\phi \sin\phi'}{\nu_0 - iq_0}\right) . \tag{3}$$

The component of the wave vector parallel (perpendicular) to the surface is $k_0 = (\omega_0/c)\sin\theta_0$ $\times(q_0 = (\omega_0/c)\cos\theta_0)$ and $k = (\omega_0/c)\sin\theta$ $(q = (\omega_0/c\cos\theta)$ for the incident and scattered light, respectively. Further, $\nu_0 = [k_0^2 - \epsilon(\omega_0/c)^2]^{1/2}$ and $\nu = [k^2 - \epsilon(\omega_0/c)^2]^{1/2}$. The Fourier transform of the surface profile defines

$$\zeta_{\vec{K}} = \int \zeta(\rho)\, e^{-i\vec{K}\cdot\vec{\rho}}\, d^2\rho \; .$$

By letting $\phi' = 0$ and $\frac{1}{2}\pi$, Eq. (1) may be specialized to the case of p- and s-polarized incident light, respectively. Averaging over ϕ' from $0 \to 2\pi$ yields the result for unpolarized incident light.

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Note added in proof. Maradudin and Mills have since corrected their work and the subsequent results agree with KK and the work here.

[1]E. Kretschmann, Z. Physik 227, 412 (1969).
[2]E. Kröger and E. Kretschmann, Z. Physik 237, 1 (1970), see footnote on p. 12.
[3]A. A. Maradudin and D. L. Mills, Phys. Rev. B 11, 1392 (1975).
[4]J. M. Elson and R. H. Ritchie, Phys. Status Solidi B 62, 461 (1974).