Bernoulli potential in superconductors*

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(Received 10 March 1975)

The Bernoulli potential in a superconductor is calculated using the Gorkov equations. General expressions for the potential are derived. In the high-κ limit, the expression reduces to the result of the two-fluid model augmented by a term due to the spatial variation of the order parameter. The temperature dependence of the potential is computed, in the case of clean superconductors, for various values of κ, and, in the case of impure superconductors, for $\kappa = \infty$.

I. INTRODUCTION

In a superconductor, the chemical potential increases with the supercurrent because of the kinetic energy associated with it. In the steady state, any spatial variation of the chemical potential must be compensated by a corresponding change in the electric potential so that the electrochemical potential remains constant throughout the superconductor. Hence a uniform current in a superconductor results in an electric field, analogous to the Bernoulli pressure variation associated with the nonuniform flow of a classical fluid. Such an electric field has been measured in a number of experiments.[1,2]

This Bernoulli field was first predicted by London,[3] who using the model of a charged superfluid, showed that the electric field is given by

$$\vec{E} = -(m/2e)\nabla v_s^2, \tag{1}$$

where $\vec{v}_s$ is the superfluid velocity. This equation was later modified by Van Vijfeijken and Staas,[4] who took into account the normal fluid component. They obtained the result

$$\vec{E} = -(n_s/n_0)(m/2e)\nabla v_s^2, \tag{2}$$

where n_s is the superfluid density and n_0 is the total fluid density.

Calculations based on the BCS model have been made by Adkins and Waldram,[5] and Rickayzen.[6] Adkins and Waldram showed that band structures may have important effects on the Bernoulli potential. However, for a spherical Fermi surface, their result agrees with Eq. (1). Their calculation was for $T = 0\,^\circ$K, but they indicated that at higher temperatures, especially near T_c, the variation of the order parameter with the current plays an important role. This feature is borne out by the present work. Rickayzen extended the calculation to include temperature dependence, but he assumed a constant order parameter. Both works are performed in the local limit.

Time-dependent Ginzburg-Landau equations have also been used in the calculation of the Bernoulli potential. Jakeman and Pike[7] obtained an expression which differs from Eq. (2) by a factor of 2. They also took into account of the Thomas-Fermi screening, but because the screening length is extremely small, this has negligible effect on the potential, although the electric field and the charge distribution near the surface of the superconductor are modified. Rieger[8] explicitly calculated the spatial variation of the order parameter, but his result is different from the other works.

In this paper, we treat the problem using the BCS-Gorkov theory. We consider a superconductor with a spherical Fermi surface. In Sec. II, expressions for the Bernoulli potential are obtained. In Sec. III, we show that in the local limit, our expression reduces to Eq. (2), augmented by a term resulting from the spatial variation of the order parameter. In Sec. IV, the Bernoulli potential induced in a semi-infinite superconductor by an external magnetic field is calculated. Numerical results are obtained for clean materials with $\kappa = 0.1$, 1, and 10, and for high-κ materials with various values of the mean free path.

II. EQUATIONS FOR THE BERNOULLI POTENTIAL

We start with Gorkov's equations for a superconductor in a static magnetic field. It is convenient to use a gauge in which the order parameter is real. The vector potential in this gauge is identified as the superfluid velocity,[9] i.e., $\vec{v}_s = e\vec{A}/m$ [in an arbitrary gauge, $\vec{v}_s = (e\vec{A} + \nabla W)/m$, where $2W$ is the phase of the order parameter]. Gorkov's equations may now be written as ($\hbar = c = 1$),

$$\hat{L}\hat{G}(\omega_l, \vec{r}, \vec{r}\,') = \delta(\vec{r} - \vec{r}\,')\hat{I}, \tag{3}$$

$$\hat{G}(\omega_l, \vec{r}, \vec{r}\,') = \begin{pmatrix} G(\omega_l, \vec{r}, \vec{r}\,') & -F(\omega_l, \vec{r}, \vec{r}\,') \\ F^\dagger(\omega_l, \vec{r}, \vec{r}\,') & G(-\omega_l, \vec{r}\,', \vec{r}) \end{pmatrix}, \tag{4}$$

$$\hat{L}=\begin{pmatrix} i\omega_l-(1/2m)[i\nabla-m\vec{v}_s(\vec{r})]^2+e\phi(\vec{r})+\mu & \Delta(\vec{r}) \\ -\Delta(\vec{r}) & -i\omega_l-(1/2m)[i\nabla+m\vec{v}_s(\vec{r})]^2+e\phi(\vec{r})+\mu \end{pmatrix}. \quad (5)$$

Here $\omega_l=(2l+1)\pi k_B T$, $\hat{I}$ is the identity matrix, and $-e\phi$ is the self-consistent Coulomb interaction

$$-e\phi(\vec{r})=\int d^3r'\,\frac{e^2\delta n(\vec{r}')}{|\vec{r}-\vec{r}'|}. \quad (6)$$

It turns out that this is exactly the electric potential one would measure in the metal. In this calculation, therefore, we are treating the Coulomb interaction in the Hartree approximation, while the phonon-mediated interaction is treated in the usual manner.

A perturbative solution to Eq. (3) may be written down immediately. Writing

$$\Delta(\vec{r})=\Delta_0+\Delta_1(\vec{r}), \quad (7)$$

$$\mu_1(\vec{r})=e\phi(\vec{r})-\tfrac{1}{2}mv_s^2(\vec{r}), \quad (8)$$

and defining the Fourier transform

$$\hat{G}(\omega_l,\vec{r},\vec{r}')=\int\frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}e^{i(\vec{p}\cdot\vec{r}-\vec{p}'\cdot\vec{r}')}\hat{G}(\omega_l,\vec{p},\vec{p}') \quad (9)$$

we obtain (suppressing the variable ω_l)

$$\hat{G}(\vec{p},\vec{p}')=(2\pi)^3\delta(\vec{p}-\vec{p}')\hat{G}^0(\vec{p})+\sum_{N=1}^{\infty}\hat{G}^{(N)}(\vec{p},\vec{p}'), \quad (10)$$

$$\hat{G}^{(N)}(\vec{p},\vec{p}')=\int\frac{d^3k_1\cdots d^3k_N}{(2\pi)^{3N}}(2\pi)^3\delta\Big(\sum_{i=1}^{N}\vec{k}_i-\vec{p}+\vec{p}'\Big)\times\hat{G}^0(\vec{P})\hat{A}(\vec{k}_1)\hat{G}^0(\vec{P}-\vec{k}_1)\cdots\hat{A}(\vec{k}_N)\hat{G}^0(\vec{p}'). \quad (11)$$

Here

$$\hat{G}^0(\vec{p})=-\frac{1}{2}\sum_{\sigma=\pm1}\frac{\hat{I}-\sigma(\vec{u}\cdot\hat{\vec{\tau}})}{\xi_p+i\sigma W_l}, \quad (12)$$

$$\hat{A}(k)=\begin{pmatrix}\vec{p}\cdot\vec{v}_s(\vec{k})-\mu_1(\vec{k}) & -\Delta_1(\vec{k}) \\ \Delta_1(\vec{k}) & -\vec{p}\cdot\vec{v}_s(\vec{k})-\mu_1(\vec{k})\end{pmatrix}. \quad (13)$$

As usual, $\xi_p=p^2/2m-\mu$, $W_l=(\omega_l^2+\Delta_0^2)^{1/2}$, $\hat{\tau}_i$ $(i=1, 2, 3)$ are the Pauli matrices, $\vec{u}=(0,\Delta_0,\omega_l)/W$, and the condition $\operatorname{div}\vec{V}_s=0$ is assumed.

For the problem of the Bernoulli potential, we need only calculate $\hat{G}^{(1)}$ and $\hat{G}^{(2)}$, and it is easily seen that both μ_1 and Δ_1 are of order v_s^2, so that in the calculation of $\hat{G}^{(2)}$, we may replace $\hat{A}$ by $(\vec{p}\cdot\vec{v}_s)\hat{\tau}_3$.

To take into account the effect of impurities, we have to make the following replacements in Eq. (12):

$$\omega_l\to\tilde{\omega}_l=\omega_l\eta_l, \quad (14)$$

$$\Delta_0\to\tilde{\Delta}_0=\Delta_0\eta_l, \quad (15)$$

$$\eta_l=1+1/2\tau W_l, \quad (16)$$

and hence,

$$W_l\to\tilde{W}_l=W_l+1/2\tau, \quad (17)$$

where τ is the scattering time. Furthermore, we have to evaluate the various vertex corrections to Eq. (11). Considering only s-wave scattering, there are no corrections for the individual $\vec{v}_s$ vertices (but there is an impurity bridge across the two $\vec{v}_s$'s in $\hat{G}^{(2)}$). From $\hat{G}^{(1)}$, we obtain the usual relation between the current density $\vec{j}$ and $\vec{v}_s$,[10]

$$\vec{j}(\vec{k})=-Q(k)n_0e\vec{v}_s(k), \quad (18)$$

where

$$Q(k)=\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2}{W_l^2\tilde{W}_l}F\Big(\frac{v_F k}{2\tilde{W}_l}\Big), \quad (19)$$

$$F(x)=\frac{3}{2x^2}\Big[\Big(\frac{1}{x}+x\Big)\tan^{-1}x-1\Big]. \quad (20)$$

The changes in the density and the order parameter are given by the self-consistency equation

$$\begin{pmatrix}\tfrac{1}{2}\delta n(\vec{k}) \\ (1/V)\Delta_1(\vec{k})\end{pmatrix}=k_B T\sum_{l=-\infty}^{\infty}\sum_{N=1}^{2}\int\frac{d^3p}{(2\pi)^3}\hat{G}_1^{(N)}(\omega_l,\vec{p},\vec{p}-\vec{k}), \quad (21)$$

where V is the BCS interaction constant and $\hat{G}_1^{(N)}$ is the first column of $\hat{G}^{(N)}$. Substituting Eq. (11) into Eq. (21) and correcting for impurity bridging, we find that all the corrected quantities M satisfy the equation

$$\hat{M}=\hat{M}_0+\frac{1}{2\pi N(0)\tau}\int\frac{d^3p}{(2\pi)^3}\hat{G}^0(\vec{p})\hat{M}G^0(\vec{p}-\vec{k}), \quad (22)$$

where $\hat{M}_0$ is the uncorrected matrix (both $\hat{M}$ and $\hat{M}_0$ are matrices independent of $\vec{p}$) and $N(0)$ is the density of states at the Fermi surface. As the Bernoulli potential is of the order $\frac{1}{2}mv_s^2$ and it is well known[11] that near T_c, $\Delta_1\sim mv_s^2\mu/\Delta_0$, we must therefore calculate the various coefficients up to order Δ_0/μ, i.e., we have to keep the first two terms in the expansion of the density of states about the Fermi surface. Thus

$$\int\frac{d^3p}{(2\pi)^3}\to N(0)\int\frac{d\Omega}{4\pi}\int d\xi(1+\xi/2\mu). \quad (23)$$

In general,

$$\hat{M}_0=a_0\hat{I}+\vec{b}_0\cdot\hat{\vec{\tau}}. \quad (24)$$

We may solve Eq. (22) by writing

$$\hat{M}=a\hat{I}+\vec{b}\cdot\hat{\vec{\tau}}. \quad (25)$$

Substitution in Eq. (22) yields

$$a = a_0 + \frac{i}{2\mu}\frac{1}{2\tau}(\vec{b}_0\cdot\vec{u}),$$

$$\vec{b} = \frac{\vec{b}_0 - J_k(\vec{b}_0\cdot\vec{u})\vec{u}}{1-J_k} + \frac{i}{2\mu}\frac{1}{2\tau}\left(a_0\vec{u} + \frac{i(1-I_k)}{(1-J_k)^2}(\vec{b}_0\times\vec{u})\right), \quad (26)$$

$$I_k = \frac{2\tilde{W}_l}{v_F k}\tan^{-1}\left(\frac{v_F k}{2\tilde{W}_l}\right), \quad (27)$$

$$J_k = (1/2\tau\tilde{W}_l)I_k. \quad (28)$$

Evaluating the various M_0 in Eq. (21) and using the above formulae, a lengthy but straightforward calculation gives the following results:

$$\delta n(\vec{k}) = 2N(0)\left(e\phi(\vec{k}) + \frac{\Delta_0}{2\mu}\frac{1}{N(0)V}\Delta_1(\vec{k}) - \frac{m}{2}\int\frac{d^3q}{(2\pi)^3}v_{si}(\vec{q})v_{sj}(\vec{k}-\vec{q})K_{ij}(\vec{q},\vec{k}-\vec{q})\right), \quad (29)$$

$$\Delta_1(\vec{k}) = -4\mu m\int\frac{d^3q}{(2\pi)^3}\frac{v_{si}(\vec{q})v_{sj}(\vec{k}-\vec{q})H_{ij}(\vec{q},\vec{k}-\vec{q})}{D(k)}, \quad (30)$$

where we have used the replacement

$$\pi k_B T\sum_{l=-\infty}^{\infty}\frac{1}{W_l} \to \frac{1}{N(0)V}, \quad (31)$$

and with $\vec{p} = \vec{k} - \vec{q}$, $\vec{n}$ a unit vector, v_F = Fermi velocity,

$$K_{ij}(\vec{q},\vec{p}) = 4\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2(2\tilde{W}_l + W_l)}{W_l^2}A_{ij}(\omega_l,\vec{q},\vec{p}), \quad (32)$$

$$H_{ij}(\vec{q},\vec{p}) = \pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0}{W_l}\frac{4\tilde{\omega}_l^2 B_{ij}(\omega_l,\vec{q},\vec{p}) - A_{ij}(\omega_l,\vec{q},\vec{p})[1 - J_k(\Delta_0^2/W_l^2)]}{1-J_k}, \quad (33)$$

$$D(k) = \pi k_B T\sum_{l=-\infty}^{\infty}\frac{1}{W_l}\left(1 - \frac{I_k}{1-J_k}\frac{\omega_l^2}{\tilde{W}_l W_l}\right), \quad (34)$$

$$A_{ij}(\omega_l,\vec{q},\vec{p}) = \int\frac{d\Omega(\vec{n})}{4\pi}n_i n_j\frac{4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{q})(\vec{n}\cdot\vec{p})}{[4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{q})^2][4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{p})^2]}, \quad (35)$$

$$B_{ij}(\omega_l,\vec{q},\vec{p}) = \int\frac{d\Omega(\vec{n})}{4\pi}n_i n_j\frac{12\tilde{W}_l^2 + v_F^2[(\vec{n}\cdot\vec{q})^2 + (\vec{n}\cdot\vec{q})(\vec{n}\cdot\vec{p}) + (\vec{n}\cdot\vec{p})^2]}{[4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{q})^2][4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{p})^2][4\tilde{W}_l^2 + v_F^2(\vec{n}\cdot\vec{k})^2]}. \quad (36)$$

From Eq. (6), we get

$$-e\phi(\vec{k}) = (4\pi e^2/k^2)\delta n(\vec{k}). \quad (37)$$

Since the Thomas-Fermi screening length $\lambda_{TF}^2 = 1/8\pi N(0)e^2$ is extremely small (of the order of interatomic spacing), we have $k^2\lambda_{TF}^2 \ll 1$. Eq. (29) therefore becomes

$$e\phi(\vec{k}) = \frac{m}{2}\int\frac{d^3q}{(2\pi)^3}v_{si}(\vec{q})v_{sj}(\vec{k}-\vec{q})K_{ij}(\vec{q},\vec{k}-\vec{q}) - \frac{\Delta_0}{2\mu}\frac{1}{N(0)V}\Delta_1(\vec{k}). \quad (38)$$

III. THE LOCAL LIMIT

To make contact with the two-fluid model, we consider a clean superconductor in the local limit where the various quantities do not vary appreciably over distances of the order $\xi_0 \sim \hbar v_F/k_B T_c$. In this case, we have $\tau \to \infty$ and $W_l \gg v_F k$, so that

$$Q(k) = \pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2}{W_l^3} \equiv \frac{n_s}{n_0}, \quad (39)$$

which defines the superfluid density n_s. Also

$$K_{ij}(\vec{q},\vec{p}) = (n_s/n_0)\delta_{ij}.$$

Eq. (38) then becomes, in real space,

$$e\phi(\vec{r}) = \frac{1}{2}\frac{n_s}{n_0}mv_s^2(\vec{r}) - \frac{\Delta_0}{2\mu}\frac{1}{N(0)V}\Delta_1(\vec{r}). \quad (40)$$

The first term on the right-hand side is just what one would expect from the hydrodynamic model [Eq. (2)]. The origin of the second term may be traced to the shift in chemical potential due to the pair interaction energy. To see this, we note that the chemical potential μ in the superconductor differs from the normal state value μ_N by an amount

$$\mu - \mu_N = -\frac{1}{4\mu}\left(\frac{1}{N(0)V} - \gamma(T)\right)\Delta^2. \quad (41)$$

Here $\gamma(T)$ is a function which goes from $\frac{1}{2}$ at $T=0$ to 1 at $T=T_c$, but these finer details are omitted in our calculation. In the absence of other fields, a spatial variation of Δ must be compensated by a change in density so that μ remains constant, i.e.,

$$0=\left(\frac{\partial\mu}{\partial n}\right)\delta n+\left(\frac{\partial\mu}{\partial\Delta}\right)\delta\Delta\,, \tag{42}$$

or

$$\delta n=2N(0)\frac{\Delta}{2\mu}\frac{1}{N(0)V}\delta\Delta\,. \tag{43}$$

This is just the contribution we find in Eq. (29), and it agrees with an expression derived by Adkins and Waldram.[5]

For the case of arbitrary mean free path but still in the local limit, we find

$$Q(\vec{k})=\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2}{W_l^2\tilde{W}_l}\equiv Q(T)\,, \tag{44}$$

$$K_{ij}(\vec{q},\vec{p})=\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2(2\tilde{W}_l+W_l)}{W_l^2\tilde{W}_l^2}\frac{1}{3}\delta_{ij}\equiv K(T)\delta_{ij}\,, \tag{45}$$

$$H_{ij}(\vec{q},\vec{p})=\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0}{W_l^3\tilde{W}_l}\left(\frac{\omega_l^2}{W_l}-\frac{\Delta_0^2}{2\tilde{W}_l}\right)\frac{1}{6}\delta_{ij}\equiv H(T)\delta_{ij}\,, \tag{46}$$

$$D(k)=\pi k_B T\sum_{l=-\infty}^{\infty}\frac{\Delta_0^2}{W_l^3}\left[1+\tfrac{1}{2}\xi^2(T)k^2\right]\equiv D(T)\left[1+\tfrac{1}{2}\xi^2(T)k^2\right], \tag{47}$$

$$\xi^2(T)=\frac{1}{6}\left(\frac{v_F}{\Delta_0}\right)^2\sum_{l=-\infty}^{\infty}\frac{\omega_l^2}{W_l^4\tilde{W}_l}\bigg/\sum_{l=-\infty}^{\infty}\frac{1}{W_l^3}\,. \tag{48}$$

We obtain, therefore,

$$e\phi(\vec{r})=K(T)\tfrac{1}{2}mv_s^2(\vec{r})-\frac{\Delta_0}{2\mu}\frac{1}{N(0)V}\Delta_1(\vec{r})\,, \tag{49}$$

$$\Delta_1(\vec{k})=\frac{-2\mu}{3\Delta_0}C(T)\frac{m\int[d^3q/(2\pi)^3]\vec{v}_s(\vec{q})\cdot\vec{v}_s(\vec{k}-\vec{q})}{1+\frac{1}{2}\xi^2(T)k^2}\,, \tag{50}$$

$$C(T)=6\Delta_0 H(T)/D(T)\,. \tag{51}$$

Close to T_c, $C(T)$ reduces to Gorkov's impurity function

$$C(T_c)=\chi(1/\tau)=\frac{8}{7\zeta(3)}\sum_{l=0}^{\infty}\frac{1}{(2l+1)^2(2l+1+1/2\pi k_B T_c\tau)}\,, \tag{52}$$

and $\xi^2(T)$ becomes the Ginzburg-Landau coherence length

$$\xi^2(T)=\tfrac{1}{6}(v_F/\Delta_0)^2\,\chi(1/\tau)\,, \tag{53}$$

and Eq. (50) is exactly what one would obtain from the Ginzburg-Landau equation. In particular, for high-κ materials, $\xi^2k^2\ll 1$, so that Eq. (50) gives

$$\Delta_1(\vec{r})=-(2\mu/3\Delta_0)\chi(1/\tau)mv_s^2(\vec{r})\,, \tag{54}$$

which is a result we have referred to in Sec. II.

IV. SEMI-INFINITE CASE

As a concrete example, we consider the geometry of a superconductor occupying the half-space $x\geq 0$ and determine the Bernoulli potential arising from the nonuniform Meissner current induced by an external magnetic field in the y direction.

Assuming specular scattering at the boundary, then for a solution valid in the region $x\geq 0$, we may write down Maxwell's equation in the form

$$-\frac{d^2}{dx^2}A(x)=2H_0\delta(x)+4\pi j(x)\,, \tag{55}$$

and extend the superconductor to occupy the whole space by means of a reflection. Here H_0 is the magnetic field at the surface. From this we obtain the standard result

$$v_s(k)=\frac{(2e/m)H_0}{k^2+R(k)}\,, \tag{56}$$

$$R(k)=(4\pi n_0e^2/m)\,Q(k)\,. \tag{57}$$

Equations (30) and (38) then yields an expression for the Bernoulli potential.

This calculation may be carried out analytically in the local limit. In this limit,

$$R(k)=(4\pi n_0e^2/m)\quad Q(T)=1/\lambda^2(T)\,, \tag{58}$$

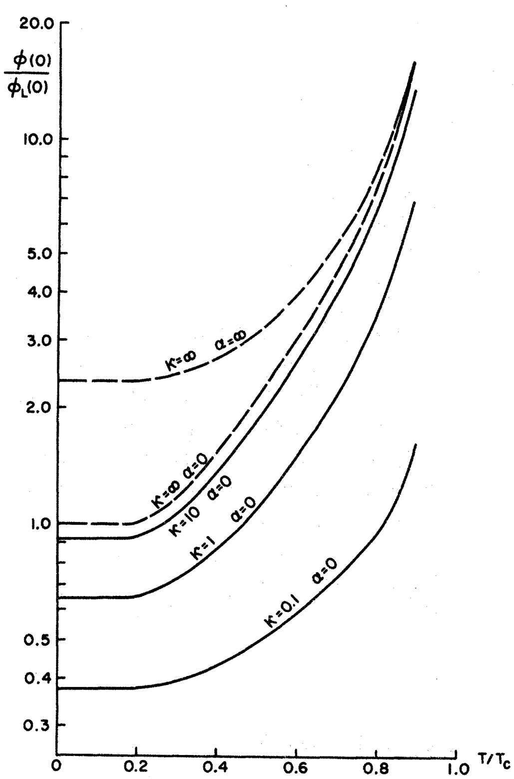

FIG. 1. Bernoulli potential relative to the temperature-independent result of the two-fluid model as a function of T/T_c. $\alpha=0.882\xi_0/l$.

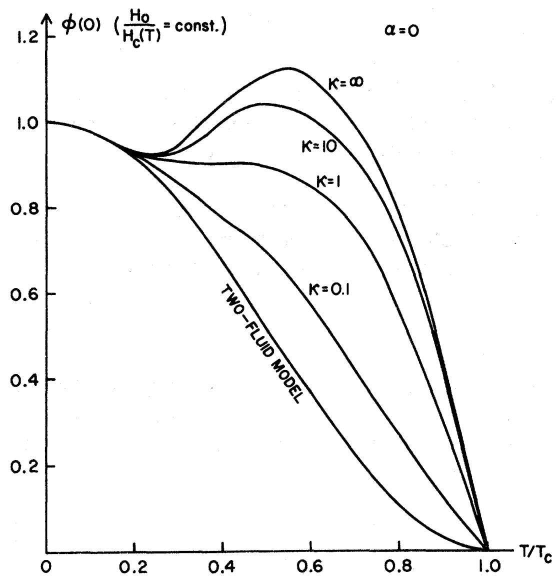

FIG. 2. Bernoulli potential in clean materials ($\alpha = 0$) with the field H_0 kept at a constant fraction of $H_c(T)$. The ordinate is in arbitrary units.

which defines the penetration depth λ. Hence

$$v_s(x) = \left(\frac{e\lambda}{m} H_0\right) e^{-x/\lambda} , \tag{59}$$

and Eq. (50) gives

$$\Delta_1(x) = -\frac{2m\mu}{3\Delta_0} C(T)\left(\frac{e\lambda}{m} H_0\right)^2 \times \frac{1}{1-2/\kappa^2}\left(e^{-2x/\lambda} - \frac{\sqrt{2}}{\kappa} e^{-\sqrt{2}x/\xi}\right) , \tag{60}$$

$$\kappa(T) = \lambda(T)/\xi(T) . \tag{61}$$

$\kappa(T_c)$ is just the Ginzburg-Landau parameter κ. Eq. (49) then gives the Bernoulli potential. In particular, the potential at the surface (relative to a point deep inside the superconductor) is given by

$$\phi(0) = \frac{H_0^2}{8\pi n_0 e}\left(A(T) + B(T)\frac{\kappa(T)}{\kappa(T)+\sqrt{2}}\frac{1}{N(0)V}\right), \tag{62}$$

$$A(T) = \frac{2}{3} + \frac{1}{3}\frac{S_{12}}{S_{21}} , \tag{63}$$

$$B(T) = \frac{1}{3}\left(\frac{\pi k_B T}{\Delta_0}\right)^2 \left[1 - \left(\frac{\Delta_0}{\pi k_B T}\right)^2 \frac{S_{41} + \frac{1}{2}S_{32}}{S_{21}}\right] \Big/ S_{30} , \tag{64}$$

$$S_{mn}(T) = (\pi k_B T)^{m+n} \sum_{l=0}^{\infty} \frac{1}{W_l^m \tilde{W}_l^n} . \tag{65}$$

This may be compared with the corresponding result of the two-fluid model, i.e., Eq. (2),

$$\phi_L(0) = H_0^2/8\pi n_0 e , \tag{66}$$

which is independent of temperature.

We note that although the coefficient $B(T)$ diverges as $(T_c - T)^{-1}$ when T approaches T_c, the Bernoulli signal actually goes to zero because the critical field vanishes like $T_c - T$. This is demonstrated in Figs. 2 and 3. This divergence of $B(T)$ is in agreement with the thermodynamic result of Rickayzen.[6]

Equation (62) is valid for arbitrary κ only near T_c, and for a wider range of temperature in the case of materials with sufficiently high values of κ. For the general case of arbitrary κ and arbitrary T, $\phi(0)$ has to be computed numerically. The results for clean materials ($\tau = \infty$) with $\kappa = 0.1$, 1, and 10 are shown in Fig. 1. The factor $1/N(0)V$ has been chosen to be 5. The broken curves are obtained from Eq. (62) for the limit $\kappa = \infty$, the lower one being for the clean limit $\tau = \infty$ and the upper for the dirty limit $\tau = 0$.

In Fig. 2, the Bernoulli potential with the field H_0 kept at a constant fraction of the critical field $H_c(T)$ is plotted as a function of T. The ordinate is in arbitrary units, normalized to 1 at $T = 0\,°\mathrm{K}$. For the two-fluid model, this is just a plot of $H_c^2(T)/H_c^2(0)$. Figure (2) is for clean materials with various values of κ.

The scheme of Fig. 3 is the same as Fig. 2. But here the plots are for materials with $\kappa = \infty$ and various values of the impurity parameter $\alpha = 1/$

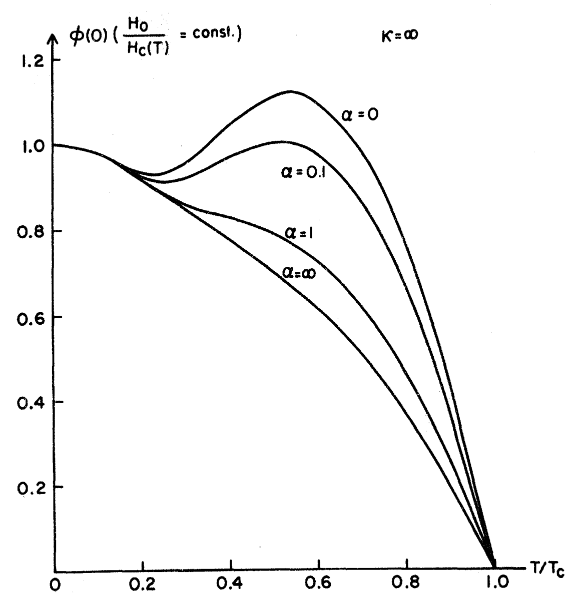

FIG. 3. Bernoulli potential in materials with $\kappa = \infty$ and with H_0 kept at a constant fraction of $H_c(T)$.

$2\pi k_B T_c \tau = 0.882\, \xi_0 / l$, where ξ_0 is the BCS coherence length and $l = v_F \tau$ is the mean free path.

Figures 2 and 3 bring out some prominent features of our results. The structures shown should be easily observed in experiments. Work with clean high-κ materials should then provide a ready check on this theory.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor A. E. Jacobs for his constant guidance during the course of this work, and to Professor E. Fawcett for suggesting the topic of this research.

*This research was supported by the National Research Council of Canada.

[1]J. Bok and J. Klein, Phys. Rev. Lett. 20, 660 (1968).

[2]T. D. Morris and J. B. Brown, Physica 55, 760 (1971).

[3]F. London, *Superfluids* (Wiley, New York, 1950), Vol. I, Sec. 8.

[4]A. G. Van Vijfeijken and F. A. Staas, Phys. Lett. 12, 175 (1964).

[5]C. J. Adkins and J. R. Waldram, Phys. Rev. Lett. 21, 76 (1968).

[6]G. Rickayzen, J. Phys. C 2, 1334 (1969).

[7]E. Jakeman and E. R. Pike, Proc. Phys. Soc. Lond. 91, 422 (1967).

[8]T. J. Rieger, Phys. Rev. B 3, 2253 (1971).

[9]M. J. Stephen, Phys. Rev. 139, A197 (1965).

[10]A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N. J., 1963), Chap. 7.

[11]J. Bardeen, Rev. Mod. Phys. 34, 667 (1962).