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Observation of a cosϕ term in the current-phase relation for "Dayem"-type weak link contained in an rf-biased superconducting quantum interference device

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The presence of a cosϕ term in the current–quantum-mechanical phase relation for a superconducting weak link can be established from the frequency response of an rf-biased superconducting quantum interference device containing the structure. For narrow constrictions, the sign of the coefficient of the cosine term was determined to be *negative*.

The response to an applied flux of a superconducting ring containing a narrow constriction weak link has been studied in detail in the regime where the critical current of the weak link is very small compared to $\varphi_0/2\pi L$, where φ_0 is the flux quantum $h/2e$, and L is the inductance of the ring. In this regime the response is very sensitive to the form of the current-phase relation. For all weakly coupled superconductors, the quantum-mechanical phase varies spatially across the weakened region and the current flowing across the region is a function of this quantum-mechanical phase difference. In the case of a dielectric barrier the current-phase relation has been calculated from microscopic tunneling theory and has the form[1]

$$I=I_0\sin(\varphi_1-\varphi_2)+GV[1+\gamma\cos(\varphi_1-\varphi_2)], \qquad (1)$$

where I_0 is the critical current, φ_1 and φ_2 are quantum-mechanical phases on either side of the barrier, G is the conductance of the junction, and γ is a dimensionless parameter which is a measure of the amplitude of the $\cos(\varphi_1-\varphi_2)$ term (quasiparticle interference term).

The last term—the cosine term—has only recently been invoked to explain the linewidth of the Josephson plasma resonance in oxide junctions.[2] However, the sign of the deduced coefficient was *negative* and thus opposite in sign to that predicted by the microscopic theory for the conditions of the experiment. A number of other experiments have been performed on other types of weak-link structures for which the microscopic tunneling theory is probably not valid—for example, point contacts, and proximity-effect barriers—and the form of the current-phase relation illustrated in Eq. (1) was again consistent with the experimental observations.[3,4] In these experiments the sign of γ was again determined to be negative. However, in these latter experiments the apparent disagreement with the predictions of the microscopic tunneling theory is not too disturbing since there are no reasons to assume that the details of the microscopic tunneling theory for dielectric barriers are also appropriate for these weak-link structures.

A number of phenomenological models have been proposed which give a current phase relationship similar in form to Eq. (1). The model of Vincent and Deaver[4] which is appropriate for a small metallic bridge yields a coefficient γ which is negative and consistent with their very limited results. (They only presented a value of γ at one temperature.) On the other hand, Notarys, Yu, and Mercereau have proposed a model for proximity-effect barriers under high-current-density conditions in which a cosine term is present but has a completely different physical interpretation than the term from the microscopic theory.[5]

The form of the current-phase relation for a weak-link structure can be experimentally studied if the terminals of the structure are shorted together to form a superconducting quantum interference device (SQUID) configuration. The variations of the electrical impedance of a SQUID are sensitive to the form of the I-φ response of the weak link, especially when the shielding currents flowing circumferentially in the SQUID are very small compared to φ_0/L. Under these circumstances the shielding current is a single-valued function of the applied magnetic flux. It has been experimentally demonstrated that for very narrow constriction weak links, a current-phase relation of the form $I=I_0\sin(\varphi_1-\varphi_2)$ is a good approximation for very small critical currents in the limit of zero applied voltage.[6] If one assumes such a current-phase relationship the impedance of the weak link is a function of applied magnetic flux and can be written as an effective inductance

$$\mathcal{L}(I)=\frac{\varphi_0}{2\pi I_0}\frac{1}{[I+(I/I_0)^2]^{1/2}},$$

where I is the imposed current flowing through the weak link and I_0 is the critical current of the weak link.[7] If such a SQUID were coupled to a resonant tank circuit, and the drive current were tuned to the resonant frequency of this circuit, the voltage across the tank would not depend (to first order) on an applied (low-frequency) magnetic flux. On the other hand, if the drive current were slightly detuned from the resonant frequency of the tank circuit, the voltage across the tank circuit would be modulated by the ambient flux with a magnitude of the signal as a function of the drive frequency resembling a derivative of a resonance curve.

In the course of a study of the response of SQUID as a function of electrical-circuit parameters, it was observed that in the regime of vanishingly small critical current ($I \ll \varphi_0/2\pi L$), very near the critical temperature of the film, there was a substantial modulation signal when the drive current was tuned to the resonant frequency of the tank and a zero in the modulation signal was observed only when the drive current was tuned to some frequency below this resonant frequency. This observed behavior is obviously not consistent with a current-phase relationship for the weak link of the form

$$I = I_0 \sin(\varphi_1 - \varphi_2) .$$

Recently, Hansma[8] has calculated the response of a SQUID inductively coupled to a resonant circuit if the weak link in the SQUID has a current-phase relation of the form shown in Eq. (1). In particular, for $\epsilon \equiv 2\pi I_0 L/\varphi_0 \ll 1$, where the effects of the screening currents are small, the magnitude of the ambient-field modulated signal from the SQUID would be zero for the drive frequency below, coincident with, or above the resonant frequency depending on whether to first order the sign of $\gamma - 2\epsilon$ (where γ is the coefficient of the cosine term) was negative, zero, or positive, respectively. The dependence of the magnitude of the modulated signal as a function of drive frequency was given as a function of the electrical characteristics of the SQUID and the tank circuit to which the SQUID was coupled.

The devices studied were made from superconducting materials that had transition temperatures below 4.2 K so that the temperature of the helium bath surrounding the SQUID could be varied to determine the appropriate temperature range for which the condition on critical current was satisfied. The general procedure and techniques for measuring the characteristics of SQUID in the low-critical-current regime have been described elsewhere.[9]

A few comments will be made here on the procedure used to determine the parameters necessary to make a quantitative comparison with Hansma's calculation. These required parameters are $\omega L/R$ (the ratio of the inductive reactance of the SQUID to the normal resistance of the weak link), the Q of the tank circuit, and the coupling coefficient k^2 between the SQUID and the resonant circuit to which the SQUID is coupled. The ratio $\omega L/R$ is obtained directly from the loading of the tank circuit as the device in the normal state (at temperatures slightly above T_c) is inserted into the tank circuit. The Q used is the unloaded Q_0 of the tank circuit at the temperature of the measurements and k^2 is determined from the normalized frequency shift observed when the device is inserted into the tank circuit at a temperature well below the superconducting transition temperature of the device, and is defined as

$$k^2 = 2(\omega_0 - \omega_s)/\omega_s ,$$

where ω_0 is the frequency of the empty tank circuit and ω_s is the frequency of the tank when the device is inserted.

The final quantity that must be determined is the critical current. This must be accurately known for two reasons, first to ensure that at the temperature of the measurement the quantity $\epsilon = 2I_0\pi L/\varphi_0$ was not only less than 1 so that Hansma's theory applies but small enough ($\epsilon < 0.1$) so that higher-order corrections to the theory in powers of ϵ are negligible; in addition I_0 is needed to fit the frequency dependence of the signal to the theory.[8]

The critical current at the temperature at which the measurements were made was determined in the following manner: According to the theory for SQUID operation[8,10] the spacing between Bessel maxima (the difference between the applied rf magnetic flux for which the modulated signal has its maximum values) is $\varphi_0/2$. Thus the values of the applied magnetic flux corresponding to these maxima will be given by

$$LI_0 + \tfrac{1}{4}\varphi_0, \quad LI_0 + \tfrac{3}{4}\varphi_0, \quad LI_0 + \tfrac{5}{4}\varphi_0, \text{ etc.}$$

The experimental quantity that is measured is the voltage across the tank circuit and this is related to the applied magnetic flux at the SQUID:

$$V_{\text{1st max}} \approx (\omega Q_0 M/L)(LI_0 + \tfrac{1}{4}\varphi_0) ,$$

$$V_{\text{2nd max}} \approx (\omega Q_0 M/L)(LI_0 + \tfrac{3}{4}\varphi_0),$$

where the coefficient shown is for the case of loose coupling between the SQUID and the tank circuit ($Qk^2 < 1$), and where M is the mutual inductance between the SQUID and the tank circuit, and Q_0 is the unloaded Q of the tank circuit. With the device at a temperature below T_c, the temperature is slowly raised and the tank voltages corresponding to the various Bessel maxima are measured as I_0 decreases. The ratio between the voltage corresponding to the first Bessel maxima to the spacing between successive maxima is given by

$$V_1/(V_n - V_{n-1}) = (2LI_0/\varphi_0) + \tfrac{1}{2}, \quad n = 2, 3, 4 \ldots .$$

For loose coupling, as the temperature dependence is followed, the ratio can be measured with fairly good accuracy to values of I_0 less than $\frac{1}{10}(\varphi_0/2\pi L)$ which, according to Hansma's calculations, is the appropriate range of values for I_0 for which ϵ is only a small correction.

The results for a SQUID containing a 1-μm-wide Dayem bridge at a temperature where $\epsilon = 0.1$ are

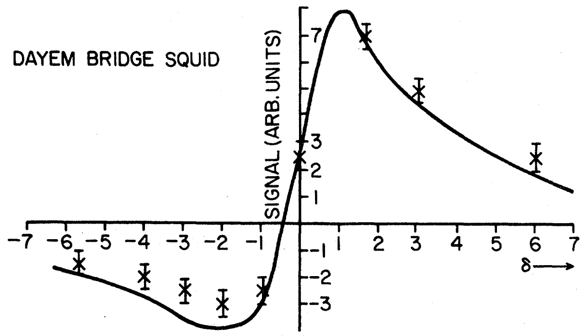

FIG. 1. First modulation signal maximum for a Dayem-bridge junction versus normalized frequency shift δ where $\delta = 2[(\omega-\omega_0)/\omega_0]Q$. The device was inductively coupled to a circuit resonant at 20 MHz. The smooth curve is based on Hansma's theory with $\beta \equiv (\varphi_0/2\pi L I_0)(\omega L/R) = 0.5$, $\Psi \equiv 2(Qk^2)2\pi L I_0/\varphi_0 = 0.44$, $\epsilon = 0.1$, and $\gamma = -0.5$.

shown in Fig. 1. For all such devices the zero crossings occurred at a frequency *below* the resonant frequency of the tank and the high-frequency maxima was larger in amplitude than the low-frequency maxima. This observed dependence of the modulation signal implies a negative value for $\gamma - 2\epsilon$. Using a value $\epsilon = 0.1$ and the other parameters determined as previously described, the fit shown in Fig. 1 was obtained for γ of -0.5. It must be kept in mind that while the quality of the fit is dependent on the choice of the *magnitude* of γ, the *sign* of γ for the known ϵ was determined by the experimental position of the zero amplitude of the modulation signal relative to the resonant frequency of the tank. The position of the zero crossing was observed to be a function of the quantity $Q_0 k^2$ and of the critical current for currents less than $\varphi_0/2\pi L$, but the zero crossing point was always consistent with the negative sign for γ, the coefficient of the cosine term.

Qualitatively similar results were also obtained for SQUID containing mechanically sculptured constrictions. For these latter devices only one lobe of the modulation signal could be observed. However, the shape of the curve was assymetrical with frequency about its maximum and the maximum was displaced above tank resonance again consistent with a negative value for γ for these devices (for the known value of ϵ).

In conclusion, it has been demonstrated that the response of an rf-based SQUID can be used to deduce the existence of a cosine term in the current-phase relation for the region of inhomogeneity contained in the SQUID. Using the technique described here, the sign and the magnitude of the coefficient of the cosine term can be calculated using experimentally obtained values for the parameters of the SQUID and of the tank circuit to which the SQUID is coupled.

For the SQUID studied in these experiments, the deduced sign of the coefficient of the cosine term in the current-phase relation was determined to be negative. This sign is opposite to that expected from microscopic tunneling theory for a dielectric barrier under the condition of applied dc voltage small compared to the superconducting energy gap. Since these measurements were made using alternating voltages, it may not even be appropriate to compare our results to the microscopic theory. Before any additional conclusions can be drawn from these experiments, calculations must be made of the current flowing through an inhomogeneous region of superconductor consisting of a dimensional constriction across which an alternating voltage has been applied.

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