

Stopping powers of several composite materials for 2.4 and 3.5 MeV deuterons and 5.5 MeV α particles*

C. L. Shepard[†] and L. E. Porter[‡]

University of Montana, Missoula, Montana 59801

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Stopping powers of the alloys Havar, Mu Metal, and Permalloy 4750 have been determined for 2.4- and 3.5-MeV deuterons. Similar measurements were made with foils of Havar, Mu Metal, Teflon, and Mylar for ^{241}Am α particles. Mean excitation energies extracted from the data were compared with values obtained from Bragg's rule. Whereas results for Havar, Mu Metal, and Teflon generally agree with additivity predictions, the experiment-based mean excitation energy of Mylar lies 10% above, and that of Permalloy 4750 6% below, the Bragg's-rule calculation.

I. INTRODUCTION

In order to conduct experiments in atomic and nuclear physics, it is often necessary to know with considerable accuracy the energy losses of massive charged particles in various materials. In the case of an elemental material the energy loss generally can be calculated from Bethe-Bloch theory, provided that the mean excitation energy and shell corrections for the subject element have been determined from previous range or stopping-power measurements. Moreover, applicability of the Bethe-Bloch formula¹ relies on validity of the first Born approximation for the projectile velocities concerned. The stopping-power formula thus derived features dependence on the square of the projectile charge. Several observed deviations from this dependence²⁻⁵ led ultimately to quantitative theoretical formulations of a correction, proportional to the cube of the projectile charge, to be included for very low^{6,7} and very high⁷ velocities.

In the case of composite materials also, the energy loss can be calculated from Bethe-Bloch theory, provided that Bragg's rule⁸ for the additivity of stopping effects is invoked and properly applied.⁹ However, the energy losses thus obtained have in a number of instances been found to differ from experimental values, despite the fact that the additivity rule can be expected to obtain exactly for mixtures and to within a few per cent for compounds.^{9,10}

Deviations for compounds are expected to be greatest where the average atomic number \bar{Z} is small.¹⁰ Yet observed violations of the additivity rule for composite materials have been confined neither to compounds, nor to low- \bar{Z} compounds. In particular, one set of stopping-power measurements with 2.5-7.0-MeV deuterons and a medium- \bar{Z} alloy as target indicated a deviation of 15% between experimental and additivity-based values of mean excitation energy.¹¹ Hence it appeared that if energy losses were to be determined with great accuracy, the only reliable course of action was to conduct appropriate measurements. In the experi-

ment now reported, stopping powers of several medium- and low- \bar{Z} materials were determined for 2.42- and 3.50-MeV deuterons and for 5.48-MeV α particles. Target selection was based chiefly on availability of uniform thin foils of materials frequently utilized in physics experiments. The resulting set of targets included medium- \bar{Z} alloys both identical with, and close in, average atomic number to that investigated previously.¹¹ The medium- \bar{Z} alloy which had formerly appeared to violate the additivity rule¹¹ evinced consistency with additivity predictions in the present measurements, as noted in preliminary reports of the experiment.^{12,13} A completely independent measurement of the stopping power of this target for 2.9-6.0-MeV protons and 0.8-3.9-MeV deuterons¹⁴ has very recently corroborated obedience to the additivity rule.

II. APPARATUS

Measurements of energy loss were carried out with two distinct sources of projectiles. Deuteron beams emanated from a Van de Graaff accelerator whereas α particles originated in a thin layer of americium dioxide. Target-foil preparation was the same for both sets of measurements, however. Masses of foils of predetermined areas were established with a Cahn electrobalance prior to mounting the foils on holders. Foil thicknesses thus found were remeasured with another electrobalance subsequent to energy-loss determinations. Energy losses for both types of projectile were obtained by observing in multichannel analyzers the retardation of pulse-height spectra caused by placement of target foils in the paths of collimated particle beams. The detector and preamplifier used for deuteron detection were duplicates of those used for α -particle detection.

An ^{241}Am α -particle source was prepared by deposition of americium dioxide onto a tantalum backing. The source mass of 0.53 μg was distributed in a thin-line configuration with approximate dimensions of 4.6 mm in length, 0.025 mm

in width, and 0.40 μm in depth. The flat tantalum sheet was mounted in a vertical plane with the line source horizontal. The source was positioned 3.8 cm in front of the detector, whose cover slits were in turn centered on the source and opened to 6.4-mm vertical height and 5.1-mm horizontal width. The silicon surface barrier detector possessed a depletion depth of 100 μm , a sensitive area of 100 mm^2 , and a resolution of 15-keV full width at half-maximum for 5.5-MeV α particles. Both source and detector were situated on the interior of a horizontal circular base plate with a pumping port at its center. A hollow cylinder, open and flanged on one end, was placed on the base plate, which held an O-ring near its circumference. The chamber thus formed was evacuated to a pressure of about 7 μm . A target-holder assembly was attached to a vertical rod which could be raised and lowered through an O-ring-protected hole in the closed top of the hollow cylinder. Thus, any one of three targets could be emplaced, about halfway between source and detector, without loss of vacuum. Signals which originated in the detector from the stopping of α particles passed from the preamplifier to a linear amplifier, and thence to a 1024 channel pulse-height analyzer. Target foils comprised the following materials with cited nominal thicknesses: Ni of 0.6, 1, 2, and 4 μm ; Al of 5 μm ; Havar of 2 and 4 μm ; Mu Metal of 3 μm ; Mylar of 6 μm ; and Teflon of 6 μm . The Ni and Al foils were included for energy calibration of the detection system. A detailed description of the calibration technique appears below.

The vertical Van de Graaff accelerator at the Los Alamos Scientific Laboratory furnished an analyzed deuteron beam for additional stopping-power determinations. The collimated beam was directed into a conventional cylindrical scattering chamber evacuated to a pressure of about 5×10^{-6} mm Hg. In order to reduce the counting rate to an acceptable level, the deuteron beam was forced to traverse a 200- $\mu\text{g}/\text{cm}^2$ gold leaf. Deuterons scattered at an angle of 39° above the horizontal incident-beam direction then passed through the target foil, which was positioned some 5 cm from the gold leaf. The foil was mounted in a holder which rested in a frame of about 5-cm length. The frame in turn was attached to a collimator 10 cm long, which extended through the chamber wall to the detector-preamplifier assembly. Deuterons which entered the detector cover slit aperture, 0.34 mm in diameter, stopped in the detector. Signals thus generated progressed from preamplifier to linear amplifier to 400 channel pulse-height analyzer. Target foils with cited nominal thicknesses consisted of: Havar of 4 and 8 μm ; Mu Metal of 6 μm ; and Permalloy 4750 of 6 μm . The two energies utilized were 2.42 and 3.50 MeV.

III. UNCERTAINTIES

Three measurements are required for a thin solid target stopping-power determination: foil mass, foil area, and energy loss of the projectile.

Foil masses showed a high degree of reproducibility when measured with two different Cahn electrobalances. The instrument is capable of 0.1% measurements when properly calibrated and used. Specific tests of the instruments utilized, with standard weights certified as class *M* by the National Bureau of Standards, indicated an achievable accuracy of 0.5%. Hence mass measurements were taken as accurate to within 0.5% except for Mu Metal and Permalloy 4750, in which cases larger dispersions in multiple measurements suggested 1% as a more reasonable assigned uncertainty.

The area of a given foil was established by placing the foil on a glass plate and overlaying the foil with semitransparent linear graph paper. Uncertainties in foil areas thus determined were estimated at 1%. Results of multiple measurements were entirely consistent with this estimate. Explicit tests of foil thickness uniformity were not conducted. However, in measurements with both types of projectile relatively large foil areas were exposed to the bombarding particles, so that minor nonuniformities in foil thickness would not have been observed in the determination of average energy loss. Such was the case especially for α -particle measurements. Internal consistency of data for deuteron measurements on Havar and Mu Metal, as well as consistency with α -particle measurements, indicate uniformity in foils of these materials. No such checks were possible for the single Permalloy 4750 foil, so that effects of nonuniformity in this case could not be conclusively ruled out.

Uncertainties in α -particle energy losses consisted of the basic uncertainty in peak locations because of detection system resolution, of uncertainty in the energy assigned to retarded particles due to uncertainty in the energy calibration of the detection system, and of uncertainty in retarded peak locations because of uncertainty in background subtraction. Resolution for the α -particle detection system, which featured a 1024 channel pulse-height analyzer, coincided essentially with that of the detector alone. The observed half-width at half-maximum of unretarded ^{241}Am α -particles was 7.5 keV. The basic uncertainty in each peak location was taken to be this value.

In order to calibrate the detection system in energy, the linear amplifier gain was adjusted so that unretarded ^{241}Am α -particles produced a spectrum near channel 1000 of the pulse-height analyzer. In order to establish the energy of these α particles, the stopping power of AmO_2 for 5.48-MeV α par-

ticles was calculated from tables⁹ based on Bethe-Bloch theory. The resulting value of 252 MeV cm²/g is consistent with expectation based on recent measurements of stopping powers of the nearby nuclear dioxides ThO₂, UO₂, and (U, Pu)O₂ for 1–9-MeV α particles.¹⁵ The energy loss suffered by α particles escaping from the center of the AmO₂ source, 59 keV, was assigned an uncertainty of 14%, primarily because of imprecise knowledge of the areal density of the source. Thus the energy of unretarded ²⁴¹Am α particles was 5.427 ± 0.008 MeV. When elemental foils were interposed between source and detector, the retarded spectra locations were recorded.

In order to establish the energy of the retarded α particles, energy losses in the elemental absorbers were calculated from previous measurements—first from the stopping powers of Ni and Al for α particles as found by Nakata¹⁶ and second from the stopping power of Ni for α particles as found by Ward *et al.*,¹⁷ again using the Nakata data for the single Al point. Energy losses were simultaneously generated for either of the aforementioned sets of data and for the present measurements on Ni and Al by assuming initially a plausible channel width in energy. With the unretarded projectile energy held fixed, a single-parameter linear regression was performed to arrive at a best-fit slope. This new value of slope then became the assumed slope, and the procedure was repeated until convergence of initial and final values of slope was achieved for each of the sets of previous data. The slopes thus extracted were averaged arithmetically to provide an energy calibration factor of 5.617 ± 0.090 keV/channel. In both the foregoing and subsequent calculations, the energy E at which an energy loss occurred was taken as the energy of the projectile when half-way through the foil, essentially $E_0 - \Delta E/2$, with E_0 the incident energy and ΔE the total loss in the foil. Whereas corrections to this calculation of the central energy are necessary for energy losses that represent a large fraction of the incident projectile energy, such corrections were negligible for all losses sustained by α particles and deuterons in this experiment.¹⁸

Background subtraction generated an additional uncertainty in retarded peak locations. The two principal decay modes of ²⁴¹Am yield α particles of energies 5.486 and 5.443 MeV. The dominant activity is that of greater energy (86%), which constituted the α -particle source for the present study. The lesser activity (12.7%) of the lower-energy α particles represented a background in retarded spectra. Subtraction of these particles from the total spectrum introduced a further uncertainty of 3.0 keV into each retarded peak location.

Uncertainties in energies at which stopping pow-

ers were measured arose from two sources. The first of these uncertainties was the aforementioned uncertainty in the energy of the α particle emerging from the AmO₂ source. The second of these uncertainties stemmed from the uncertainty in the energy loss sustained by α particles in the target foil.

Deuteron-energy-loss uncertainties reflected the resolution of the deuteron detection system. Energy calibration of this system was based on observation of unretarded deuteron beams at the two different accelerator energies utilized. The channel width in energy was 15.1 keV. The basic uncertainty in peak location was then taken as one-half channel, or 7.5 keV.

Uncertainties in deuteron energies at which stopping powers were measured arose from three sources. The first of these uncertainties was 15 keV associated with the nominal energy of the analyzed deuteron beam. The second uncertainty was that of 3 keV associated with calculation of the energy loss incurred by deuterons during passage through the 200 $\mu\text{g}/\text{cm}^2$ gold leaf. These energy losses amounted to 25 and 27 keV for incident deuteron energies of 2.42 and 3.50 MeV, respectively. The third uncertainty was that associated with determination of the energy loss in the target foil, as was the case for α -particle measurements.

IV. EXPERIMENTAL RESULTS

The results of stopping-power measurements with α particles and with deuterons are summarized in Table I. The uncertainties in central energy E and in stopping-power $S_{\text{exp}}(E)$ contain all of the contributions discussed in Sec. III.

V. ANALYSIS AND DISCUSSION

Experimental stopping powers were fitted with Bethe-Bloch calculations, modified by incorporation of the projectile- z^3 effect,^{6,19,20} in essentially the same manner set forth in detail previously.^{11,13,21} This technique utilizes the computer code originally described by Bichsel²² and extensively revised by the authors so as to include the projectile- z^3 effect¹³ and to evaluate the effect of this inclusion on shell correction parameters.

In brief, the Bethe-Bloch formula¹ for stopping power as given in Ref. 22 was first modified¹³ to provide for the projectile- z^3 effect,^{6,19,20} and stopping-power measurements for several composite targets^{11,23,24} were studied in order to extract mean excitation energies.¹³ Neither the mean excitation energy I nor shell corrections C/Z for a target of atomic number Z can be calculated from first principles, in general, even in the case of an elemental target. Hence a fit of stopping-power measurements will yield a sum of terms in the stopping number, i. e., the sum $(\ln I + C/Z)$, unless the data

TABLE I. Stopping-power measurements.

Target material	Thickness M/A (mg/cm ²)	Projectile energy E_0 (MeV)	Energy loss ΔE (keV)	Central energy E (MeV)	Stopping power (MeV cm ² /g)
α -Particle measurements					
Havar	3.23	5.427	1511 ± 20	4.672 ± 0.010	469 ± 9
Havar	1.98	5.427	888 ± 14	4.983 ± 0.010	448 ± 10
Mu Metal	2.69	5.427	1294 ± 18	4.780 ± 0.010	481 ± 11
Mylar	0.90	5.427	741 ± 13	5.056 ± 0.010	825 ± 19
Teflon	1.44	5.427	976 ± 15	4.939 ± 0.010	678 ± 15
Deuteron measurements					
Havar	6.55	2.395	831 ± 11	1.979 ± 0.016	126.9 ± 2.2
Havar	3.23	2.395	384 ± 11	2.203 ± 0.016	118.9 ± 3.8
Havar	6.55	3.473	626 ± 11	3.160 ± 0.016	95.6 ± 1.9
Havar	3.23	3.473	296 ± 11	3.325 ± 0.016	91.6 ± 3.3
Mu Metal	5.72	2.395	742 ± 11	2.024 ± 0.016	129.7 ± 2.6
Mu Metal	5.72	3.473	568 ± 11	3.189 ± 0.016	99.3 ± 2.3
Permalloy	5.28	2.395	695 ± 11	2.047 ± 0.016	131.6 ± 2.7
Permalloy	5.28	3.473	537 ± 11	3.205 ± 0.016	101.7 ± 2.6

are sufficiently extensive and precise to permit evaluation both of parameters associated with the shell corrections and the mean excitation energy. Since the latter quantity is independent of projectile energy, whereas by contrast the shell corrections are velocity dependent, a fit of stopping-power measurements without inclusion of the velocity-dependent projectile- z^3 effect presumably introduces distortions into the extracted values of both C/Z and of I —in the latter case an error averaged over the energy interval covered by the data.¹³ When Ashley evaluated the effect on experiment-based I and C/Z values of including the projectile- z^3 effect, for stopping-power measurements on elemental targets with $20 \leq Z \leq 30$, he found that the value of I remained essentially unaltered whereas the values of C/Z changed considerably.²⁰ The Ashley technique²⁰ of evaluating projectile- z^3 induced changes in I and C/Z has been modified very little in the present study.

In the Ashley treatment,^{6,19,20} the Bethe-Bloch formula for stopping power, including the z^3 contribution, appears in the form

$$\frac{mc^2\beta^2}{4\pi(ze^2)^2 nZ} \left(-\frac{dE}{dx}\right) = L(\beta, Z) + D(\beta, z, Z), \quad (1)$$

where mc^2 is the electron rest mass energy, ze is the projectile charge, βc is the projectile velocity, n is the target atomic density, L is the stopping number per target electron, and D is the contribution arising from the z^3 effect.^{6,19} The function $D(\beta, z, Z)$ is expressed in terms of a previously defined and graphed function⁶ F by

$$D(\beta, z, Z) = \frac{z}{Z^{1/2}} \frac{F(b/x^{1/2})}{x^{3/2}}, \quad (2)$$

where $x = \beta^2/\alpha^2 Z$, with $\alpha = e^2/\hbar c$, and $b = 1.8$ is a parameter of the theory^{6,19} whose value was obtained from fits of accurate stopping-power measurements.⁴ The function $L(\beta, Z)$, derived and displayed in Ref. 6, is given by

$$L(\beta, Z) = f(\beta) - \ln I - C/Z, \quad (3)$$

where

$$f(\beta) = \ln[2mc^2\beta^2/(1-\beta^2)] - \beta^2. \quad (4)$$

If stopping-power measurements are represented by the function

$$L'(\beta, z, Z) = f(\beta) - \ln I' - C'/Z, \quad (5)$$

then theory and experiment agree²⁰ for

$$L'(\beta, z, Z) = L(\beta, Z) + D(\beta, z, Z). \quad (6)$$

Hence Eqs. (3), (5), and (6) imply that

$$D(\beta, z, Z) = \ln I - \ln I' + C/Z - C'/Z. \quad (7)$$

Since Ashley found no significant changes in mean excitation energies for targets with $20 \leq Z \leq 30$, that result²⁰ was invoked in the present analysis of stopping-power measurements on the alloys Havar, Mu Metal, and Permalloy. That is, if $I = I'$, then

$$C/Z = C'/Z + D(\beta, z, Z), \quad (8)$$

and C'/Z represents shell corrections obtained from fits of stopping-power measurements without inclusion of the projectile- z^3 effect. Moreover, one thus assumes that the aforementioned suspected error in I' , induced by neglect of the projectile- z^3 effect in the fitting of data and averaged over the energy interval of the measurements, is negligibly small. This analysis constitutes a

departure from a previous study of the projectile- z^3 effect, wherein the shell corrections were held fixed in order to observe an induced shift in mean excitation energy.¹³ Justification of the approach rested primarily on the smallness of shell corrections for the low- \bar{Z} hydrocarbons then considered.¹³ Although the crudeness of that approximation was deplored in the case of Havar, the procedure was followed for want of more plentiful and precise experimental data.

Shell correction scaling parameters for composite targets were selected from experiment-based²² or interpolated²⁵ values for elements, either as those common to the predominant elements in the composite or simply as those corresponding to the element with atomic number Z closest to the average atomic number \bar{Z} of the composite. An alternate approach would be to calculate a Bragg's rule average,⁹ but the accuracy of known shell correction parameters^{22,25} would hardly warrant so meticulous a procedure. However, calculation of the change in shell corrections for a composite target required that an appropriate average⁹ be taken:

$$\bar{D}(\beta, z, \bar{Z}) = \left(\sum_i n_i D_i(\beta, z, Z_i) \right) / \left(\sum_i n_i \right), \quad (9)$$

where n_i is the atomic concentration of the i th constituent and $D_i(\beta, z, Z_i)$, given by

$$D_i(\beta, z, Z_i) = \frac{z}{Z_i^{1/z}} \frac{F(1.8/x_i^{1/2})}{x_i^{3/2}}, \quad (10)$$

is the contribution to stopping number per target electron for the i th constituent, produced by inclusion of the projectile- z^3 effect. Since all experiment-based shell correction scaling parameters²² were derived from proton and deuteron measurements, the z^3 -effect distortions to be compensated for in these particular shell corrections, C'/Z , were to be evaluated for $z=1$.

All deuteron stopping-power measurements of the present experiment were fitted to extract a value of mean excitation energy without explicit inclusion of the projectile- z^3 effect. (The shell corrections could be modified to reflect the projectile- z^3 contribution, but the resulting stopping power for $z=1$ projectiles should remain unchanged in a subsequent recalculation with explicit projectile- z^3 effect inclusion.) The fitting technique described previously^{11,13,21} was used to extract values of \bar{I}' . (Since $\bar{I}' = \bar{I}$ by assumption, the label \bar{I} will be used hereafter.) The figure of merit utilized in the fitting procedure is defined by

$$\sigma = \left(\frac{1}{N} \sum_{i=1}^N [(S_{\text{exp}_i} - S) / \Delta S_{\text{exp}_i}]^2 \right)^{1/2}, \quad (11)$$

where S is the stopping power obtained from the modified Bethe-Bloch formula, ΔS_{exp} is the uncertainty in S_{exp} as listed in Table I, and N represents

the number of S_{exp} data points for a given projectile-target combination. In this particular experiment, N remained always small, achieving a maximum of 4 in the case of deuterons traversing Havar.

The α -particle measurements were fitted with inclusion of the projectile- z^3 effect for $z=2$ and with corresponding revised shell corrections calculated from Eqs. (9) and (10) for a $z=1$ projectile having the same velocities as the α -particle velocities. The value of \bar{I} thus extracted should lie close to that obtained from the deuteron data. Indeed, the deuteron and α particle mean excitation energies of Havar differed by only 1%, with the α -particle result the greater of the two. Mu Metal results followed the same pattern. Of course, mean excitation energies should be projectile-independent. Hence these results will be further analyzed below.

The aforementioned assumption that I and I' would be approximately the same was applied also to the two low- \bar{Z} targets, Mylar and Teflon, in contrast to the earlier technique of placing the projectile- z^3 effect entirely in the mean excitation energy.¹³ In that former study, the observed shifts in mean excitation energy produced by inclusion of the projectile- z^3 effect were, for protons on methane, carbon dioxide, and air,²⁴ 2.7%, 4.1%, and 4.1%, respectively. Whereas Mylar lies between methane and air in atomic number, Teflon is close in this property to air and carbon dioxide. Thus the basis of the present assumption that \bar{I} and \bar{I}' differ very little when the projectile- z^3 effect has been assigned to the shell corrections is the small shift produced in the $z=1$ case, even when all the projectile- z^3 effect was ascribed to the mean excitation energy.¹³

The compositions of target materials studied in this experiment are displayed in Table II, where the shell correction scaling parameter values actually used are listed in the notation of Ref. 22. (A basic explanation of the notation appears in Ref. 11, also.) Average atomic number \bar{Z} and mass number \bar{A} are shown for each composite material. Shell corrections increased considerably by virtue of the projectile- z^3 effect correction, as shown by the array of C'/Z and \bar{D} values in Table III. The shell corrections are altered only as a sum, so that the detail of shell corrections assigned to individual shells is lost in the adjustment process. The value of \bar{D} ranges from 41% to 88% of C'/Z in the case of deuterons, and from 51% to 64% in the case of α particles.

A convenient parameter with which to test Bragg's rule of the additivity of stopping effects is the mean excitation energy, presumably independent of both projectile identity and velocity. The test requires calculation of an average effective mean excitation energy for a composite material I_B given by²⁶

TABLE II. Composition, average atomic and mass numbers, and shell correction scaling parameters (in the notation of Ref. 22) for each target material.

Material	Constituents and per cent composition by weight	\bar{Z}	\bar{A}	Shell correction scaling parameter				
				B_1	V_M	H_M	V_N	H_N
Havar	Be(0.04), C(0.20), Cr(20.0), Mn(1.60), Fe(17.5), Co(42.5), Ni(13.0), Mb(2.40), W(2.80)	26.62	57.79	1.0	1.0	5.5	0	0
Mu Metal	Cr(2.0), Cu(5.0), Fe(18.0), Ni(75.0)	27.58	58.24	1.0	1.0	5.5	0	0
Permalloy 4750	Fe(49.7-52.7), Ni(50-47), Mn(0.03)	26.94	57.20	1.0	1.0	5.5	0	0
Mylar (C ₁₀ H ₈ O ₄)	C(62.5), H(4.2), O(33.3)	4.54	8.74	0.6	0	0	0	0
Teflon (C ₂ F ₄) _n	C(24.0), F(76.0)	8.00	16.67	1.0	0	0	0	0

$$\ln I_B = \frac{\sum_i n_i Z_i \ln I_i}{\sum_i n_i Z_i}, \quad (12)$$

where Z_i and I_i indicate the atomic number and mean excitation energy of the i th element, respectively, and the sum extends over the number of constituent elements. Values of I_i for all but one of the low- Z_i major constituents employed in the present study have been defended previously¹³ insofar as freedom from projectile- z^3 distortions is concerned. The sole new light element involved herein is fluorine, for which the best choice of I_i appears to be a theoretical result,²⁷ despite the fact that the similarly calculated values of I for several elemental metals are 5-10% higher than those determined in a very accurate experiment.²⁸ Values of I_i for the elements with $20 \leq Z_i \leq 30$, taken from Ref. 29, are the aforementioned mean excitation energies found by Ashley to be essentially free of projectile- z^3 distortions.²⁰ Mean excitation energy values for constituent elements are shown along with the sources of information in Table IV.

Mean excitation energies extracted from the measurements \bar{I} are displayed in Table V with corresponding figures of merit $\sigma(\bar{I})$ and the additivity-based value I_B . The four significant figures given \bar{I} values indicate the precision of the calculation (± 0.2 eV), whereas the accuracy of these quantities is suggested by the interval of \bar{I} values over which $\sigma(\bar{I})$ remains less than unity, $\Delta\bar{I}$. Deviations of \bar{I} from the additivity prediction I_B are expressed by $D_B = (I_B - \bar{I})/I_B$.

The deuteron data clearly indicate that \bar{I} values generally lie close both to respective I_B values and to the corresponding α -particle \bar{I} values. In addition to the current stopping-power measurements, those of Duder *et al.*¹⁴ for protons and deuterons traversing Havar foils were analyzed in similar fashion for comparison. Resulting \bar{I} val-

ues for these accurate proton and deuteron data were 294.2 and 297.6 eV, respectively. The figures of merit for these fits were, in order, 1.24 and 0.72. Moreover, shell corrections C'/Z and the z^3 -effect revision \bar{D} for protons on Havar fall reassuringly close to those encountered by Ashley in his study²⁰ of equal energy protons on iron and copper targets.³⁰ α particles provided only two measurements of stopping power, at energies close to each other and spanned by the deuteron energies utilized. Hence the deuteron data, buttressed by additional, very accurate, measurements¹⁴ with $z=1$ projectiles, are given precedence in the evaluation of a mean excitation energy characteristic of Havar. It appears that an appropriate value of \bar{I} for all these data is 296 eV, a value thoroughly

TABLE III. Original shell corrections C'/Z and projectile- z^3 shifts \bar{D} .

Target	E (MeV)	C'/Z	\bar{D}
Deuterons			
Havar	1.979	0.1056	0.0878
	2.203	0.1235	0.0842
	3.160	0.1618	0.0708
	3.325	0.1668	0.0687
Mu Metal	2.024	0.0991	0.0871
	3.189	0.1559	0.0709
Permalloy	2.047	0.1056	0.0870
	3.205	0.1589	0.0707
α particles			
Havar	4.672	0.1333	0.0818
	4.983	0.1421	0.0795
Mu Metal	4.780	0.1272	0.0813
Mylar	5.056	0.0916	0.0469
Teflon	4.939	0.1154	0.0663

TABLE IV. Values of constituent mean excitation energies used in Bragg's rule calculations.

Element	I (eV)	Source
Hydrogen in compounds	16.5	Ref. 10
Beryllium	64.2	Ref. 1
Carbon in compounds	78.5	Ref. 10
Carbon in mixtures	77.5	Ref. 1
Oxygen in compounds	96.0	Ref. 10
Fluorine	137.7	Ref. 27
Chromium	258.0	Ref. 29
Manganese	273.1	Ref. 29
Iron	280.6	Ref. 29
Cobalt	298.8	Ref. 29
Nickel	303.2	Ref. 29
Copper	320.8	Ref. 29
Molybdenum	436.8	Ref. 27
Tungsten	769.6	Ref. 27

consistent with additivity predictions.

In order to compare $z=2$ and $z=1$ measurements graphically, the projectile- z^3 effect influence must be evaluated explicitly. If the quantity S is subscripted according to projectile, then

$$S_d = S_{0d} + \gamma S_{0d}, \quad (13)$$

where S_0 represents the unmodified Bethe-Bloch calculation of stopping power, and γ is the relative increase produced by inclusion of the projectile- z^3 effect. Stopping powers calculated for deuterons in the manner described above correspond to S_d , because the projectile- z^3 effect is implicitly included in original fits of the data by virtue of using unmodified shell corrections. Similarly,

$$S_\alpha = S_{0\alpha} + \delta S_{0\alpha}, \quad (14)$$

where δ is the $z=2$ counterpart of γ . The dependence of γ or δ on z is linear,^{6,13,19} so that for projectiles of the same velocity, $\delta = 2\gamma$. Hence the

TABLE V. Comparison of experiment-based mean excitation energies (\bar{I}) with those expected from Bragg's-rule calculations (I_B).

Material	\bar{I} (eV)	σ (\bar{I})	$\Delta\bar{I}$ (eV)	I_B (eV)	D_B (%)
Deuteron Measurements					
Havar	296.6	0.37	284-310	294.9	-0.6
Mu Metal	292.0	0.03	279-305	298.9	+2.3
Permalloy	274.2	0.18	261-287	291.4	+5.9
α Particle Measurements					
Havar	299.4	0.13	287-311	294.9	-1.5
Mu Metal	294.4	0	281-307	298.9	+1.5
Mylar	81.0	0	75-87	73.9	-9.6
Teflon	120.4	0	112-129	119.6	-0.7

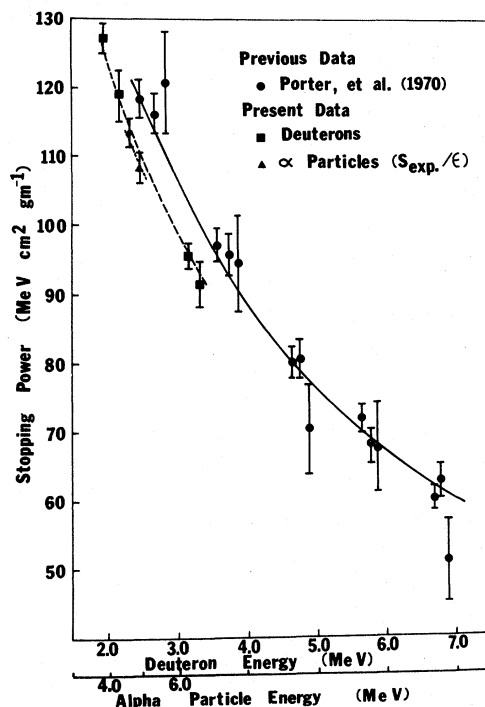


FIG. 1. Stopping-power of Havar for deuterons and α particles. The solid curve represents the best fit to the data of Ref. 11, whereas the dashed and dotted curves represent best fits to the deuteron and α -particle data of the present study, respectively.

ratio of stopping powers in this case is

$$S_\alpha/S_d = 4(1 + \delta)/(1 + \frac{1}{2}\delta), \quad (15)$$

or if the ratio is given the symbol $\epsilon = 4(1 + \delta)/(1 + \frac{1}{2}\delta)$, then an evaluation of ϵ for a given projectile velocity permits a direct comparison of stopping powers. The values of ϵ at α -particle energies of 4.672 and 4.983 MeV are 4.138 and 4.131, respectively. Measurements and best-fit curves are accordingly displayed in Fig. 1, along with the earlier data of Porter *et al.*¹¹ The fact that the present set of stopping-power values lies systematically lower than the previous set invites speculation as to the source of difference. Whereas the present experiment was conducted with a detection system of superior resolution, the previous measurements involved a greater range of deuteron energies. A thorough review of the earlier experiment¹¹ revealed no plausible source of error hitherto overlooked. In view of excellent agreement between present data and the more recent measurements of Duder *et al.*,¹⁴ these data will be accepted as more reliable than the previous set.¹¹ Thus the additivity rule is followed by Havar.

Mu Metal stopping powers for both projectiles indicate consistency with Bragg's rule. The two Permalloy 4750 measurements with deuterons

mildly suggest deviation from additivity, but these limited data hardly constitute conclusive evidence. Values of stopping power calculated from Bragg's rule lie only $1\frac{1}{3}$ standard deviations from the two measurements. Moreover, it was noted above that this target was the only one for which no thickness uniformity precautions were taken. Recent accurate tests of Bragg's rule with 0.5–2.25-MeV ${}^4\text{He}^+$ ions as projectiles and binary metal systems as targets yielded results consistent with the rule, and led to the speculation that additivity obtains for all metal alloys.³¹ Present Mu Metal and Permalloy 4750 measurements are essentially consistent with the additivity assumption.

Mylar has been investigated on several past occasions, not only because this material serves as a window on gas target cells and gas counters, but in addition possesses an electron density within a few per cent of dry protein.³²

Studies with heavy ions^{32,33} were not directly concerned with additivity. Experiments with α particles,^{34,35} although of limited experimental accuracy, indicated that measured stopping powers of several organic materials, including Mylar, fell below Bragg's rule calculations. These results imply that the experiment-based mean excitation energy exceeded I_B in each case. In one experiment³⁴ the difference between experimental and theoretical stopping power for Mylar was found to be 2.8%, which corresponds to a D_B of -10.5%, a result in close agreement with the present finding. Reports of additional measurements of Teflon stopping power were not found in the literature for comparison. The additivity test of the present measurement relies heavily on a theoretical value of mean excitation energy for fluorine,²⁷ whose credibility rests to a great extent on experimental confirmation of the general trend of stopping cross section behavior as a function of target atomic number.³⁶ The experiment alluded to featured measurements of stopping cross sections of gaseous fluorocarbons for 0.3–2.0-MeV α particles.³⁶

The results of the Mylar study agree with those of another experiment³⁷ on low- \bar{Z} composite materials, wherein 10–30-MeV protons bombarded targets of Lucite ($\text{C}_5\text{H}_8\text{O}_2$), sapphire (Al_2O_3), and quartz (SiO_2); these measurements yielded mean excitation energies greater than I_B by 7.2%, 9.7%, and 7.1%, respectively. Mylar differs only slightly from Lucite in chemical composition, and when the constituent I values of Ref. 37 are used to calculate I_B , the present value of \bar{I} for Mylar exceeds

I_B by 5.5%.

Additivity investigations need not feature the mean excitation energy as the tested parameter. In the past decade, numerous studies have been carried out in order to test directly the additivity of stopping cross sections in hydrocarbons and other low- \bar{Z} compounds. Experiments with 30–350-keV protons and helium ions on solid targets,³⁸ and with 0.3–2.0-MeV α particles on gaseous targets,^{39,40} have revealed, in addition to mere departures from Bragg's rule, systematics of the observed deviations according to types of chemical bonds in the targets.^{39,40} Indeed, recent studies of the stopping cross sections of metallic oxides have managed to elucidate physical-state effects of constituents in additivity tests.^{31,41} Future checks of obedience to the additivity rule will benefit from such detailed investigations. Current tests of Bragg's rule applicability in compounds remain quite uncertain. Deviations from the rule are often encountered in low- \bar{Z} compounds, as expected.¹⁰ However, when stopping-power data for targets of small atomic number and for projectiles with energies in the interval of applicability of the Bethe-Bloch formula are analyzed by including the projectile- z^3 effect, caution must be exercised¹³ since the theory^{6,19,20} assumes that the statistical model of the atom pertains. Further precise stopping-power measurements for such target and projectile combinations could furnish probes to search for limitations of the projectile- z^3 effect formalism.

VI. CONCLUSION

Alloys of medium atomic number obey Bragg's rule of additivity of stopping effects. The experiment-based mean excitation energy of Permalloy 4750 lies 5.9% below the additivity-based value, but the number and precision of present measurements do not warrant a conclusion that the alloy defies Bragg's rule. The experiment-based mean excitation energy of Mylar exceeds the additivity prediction by nearly 10%. Whereas Teflon appears to follow Bragg's rule closely, the validity of the test is questionable because of the uncertainty associated with the values of constituent mean excitation energies.

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