Two-dimensional Fermi gas*

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A calculation is given of the equilibrium properties of a two-dimensional Fermi gas at 0°K. The ladder diagrams for the self-energy are summed, corresponding to a low-density expansion. We obtain the Fermi energy, the quasiparticle lifetime, the effective mass, the ground-state energy, and the discontinuity at the Fermi surface.

INTRODUCTION

In the past few years there have been a number of experiments with ^{3}He adsorbed on liquid ^{4}He,[1] and on exfoliated graphite (Grafoil).[2] With such substrates the two-dimensional signatures of these thin films have been clearly observed. A model which has not been used to investigate the properties of ^{3}He films is that of a two-dimensional imperfect Fermi gas. In this model, one neglects the substrate interaction and the attractive part of the effective ^{3}He-^{3}He interaction. At 0 °K I have been able to obtain the following expressions for the quasiparticles's Fermi energy, effective mass, and lifetime. One also obtains the ground-state energy of the ^{3}He system and the discontinuity at the Fermi surface:

$$u=\frac{\hbar^2k_F^2}{2m}\left(1-\frac{1}{\ln k_Fa}+\frac{0.45\pm0.05}{(\ln k_Fa)^2}\right),$$

$$\frac{m^*}{m}=1+\frac{0.86\pm0.06}{(\ln k_Fa)^2},$$

$$\frac{1}{\tau}=\frac{\hbar^2k_F^2}{2m}\frac{5}{2\pi(\ln k_Fa)^2}\,\mathrm{sgn}(k_F-|\vec{p}|)\times\left(\frac{k_F-|\vec{p}|}{k_F}\right)^2\left(-\ln\left|\frac{k_F-|\vec{p}|}{k_F}\right|\right),$$

$$\frac{E}{N}=\frac{\hbar^2k_F^2}{4m}\left(1-\frac{1}{\ln k_Fa}+\frac{0.28\pm0.05}{(\ln k_Fa)^2}\right),$$

$$Z_{k_F}=1-\frac{0.26\pm0.05}{(\ln k_Fa)^2}.$$

In the above, a is the scattering length. Briefly, the calculation proceeds by an approximation to the self-energy. For low areal density, it is shown that the ladder graphs are the appropriate subset. The ensuing integral equation is solved and an explicit answer is obtained for the self-energy to second order in $(\ln k_Fa)^{-1}$. These results are generalized from a hard core to any short-range repulsive interaction by introducing a two-dimensional effective-range expansion of the scattering amplitude.

GRAPH ANALYSIS

One of the obvious problems which plagues any straightforward calculation for a dense Fermi liquid is the lack of physical evidence pointing to any subset of diagrams that should be summed. Only in the low-density regime can a systematic expansion be made.[3] For a thin ^{3}He film we can use the same diagrammatic sum and obtain physically useful results.

To determine which graphs should be included, we investigate contributions to the self-energy. We first employ a potential $U(\vec{r})$ which has a well-behaved Fourier transform $V(\vec{k})$. Figures 1(a) and 1(b), representing the two lowest-order processes contributing to the self-energy are

$$\Sigma_a^1(\vec{p})\propto k_F^2V(0)$$

and

$$\Sigma_b^1(\vec{p})=-\int\frac{d^2q}{(2\pi)^2}\,n_{\vec{q}}\,V(\vec{p}-\vec{q}),$$

where

$$n_{\vec{q}}=\Theta(k_F-|\vec{q}|).$$

In second order there are six distinct diagrams which can be grouped into two classes. Figures 1(c)–1(f) represent processes where two holes are present. Because each of these diagrams has two hole lines, their contributions to the self-energy must be quadratic in the areal density, n. Alternatively, they must be proportional to $k_F^4V^2(0)\approx V_0^2(k_Fa)^4$, where $V(0)=\pi V_0a^2$. These diagrams are "events" in which the internal propagator of the lowest-order diagrams is modified and where the many-body medium makes its presence felt via the additional hole line.

The remaining diagrams, 1(g) and 1(h) are the second-order Born scattering terms. Since diagram 1(h), the exchange scattering, is similar to the direct interaction, we just consider diagram 1(g). In units where $m=1$ and $\hbar=1$,

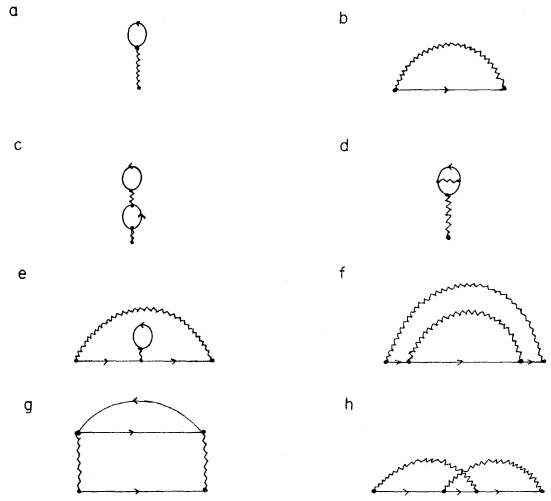

FIG. 1. (a) and (b) are the lowest-order self-energy terms. The other diagrams are the second-order corrections.

$$\Sigma_g^2(p) = \int\int \frac{d^2q dq_0 d^2k dk_0}{(2\pi)^6} V(\vec{q})V(-\vec{q}) \times G_0(p-q)G_0(q+k)G_0(k) \propto V_0^2 a^2 (k_F a)^2 \ln k_F a .$$

V_0 is the strength or the potential, and a is a parameter of the order of the spatial extent of the potential. For low energy and a repulsive square well, it is clear that Σ_c^2 is smaller than Σ_g^2 by a factor of the density. Of all the possible second-order self-energy insertions that can be made for a low-density system, the most important is the Born scattering one. For higher-order processes, we can amend this term by adding two different types of corrections. The first type will involve another hole line and must be of higher order in the density. The second type will just add another interaction between the two particles. We see that for a dilute system, the graphs to retain are the ladder diagrams with only one hole line. These represent the repeated interaction of the same two particles. All other "events" involve at least three particle correlations and are less significant for a low-density calculation. If the potential is large or a hard core, any finite-order Born approximation is inadequate, and the whole series must be summed. In three dimensions the essential effect of summing an infinite number of repeated scatterings is to change the Born scattering length into the full scattering length. In two dimensions, when the infinite series is summed, the scattering length will appear only in the scattering amplitude. It will be shown that the low-energy amplitude goes as $1/\ln k_F a$. This is indicative of the change in analytic properties of the self-energy when an infinite class of terms is kept.

THE APPROXIMATION

We can write the self-energy in terms of an effective interaction

$$\Sigma(p) = -2i\int \frac{d^2k\, dk_0}{(2\pi)^3} G_0(k)\Gamma(p,k;p,k) + i\int \frac{d^2k\, dk_0}{(2\pi)^3} G_0(k)\Gamma(k,p;p,k) . \quad (1)$$

Following Fedder and Walecka,[4] the effective interaction satisfies the integral equation

$$\Gamma(p_1,p_2;p_3,p_4) = V(\vec{p}_1-\vec{p}_3) + i\int \frac{d^2q\, dq_0}{(2\pi)^3} \times V(\vec{q})G_0(p_1-q)G_0(p_2+q)\Gamma(p_1-q,p_2+q;p_3,p_4). \quad (2)$$

Using relative wave vectors $\vec{p} = \frac{1}{2}(\vec{p}_1-\vec{p}_2)$, $\vec{p}\,' = \frac{1}{2}(\vec{p}_3-\vec{p}_4)$, and $\vec{G} = \vec{p}_1+\vec{p}_2$, the equation for Γ can be reduced to

$$\Gamma_0(\vec{p},\vec{p}\,',G) = \tilde{f}(\vec{p},\vec{p}\,') + \int \frac{d^2k}{(2\pi)^2} \tilde{f}(\vec{p},\vec{k}) \left(\frac{1}{\epsilon-\vec{k}^2+i\delta} + \frac{1}{\vec{k}^2-\vec{p}\,'^2-i\delta}\right) \tilde{f}^*(\vec{p}\,',\vec{k}) , \quad (3)$$

$$\Gamma(\vec{p},\vec{p}\,',G) = \Gamma_0(\vec{p},\vec{p}\,',G) + \int \frac{d^2k}{(2\pi)^2} \Gamma_0(\vec{p},\vec{k},G) \left(\frac{N(\vec{G},\vec{k})}{\epsilon-\vec{k}^2+i\delta N(\vec{G},\vec{k})} - \frac{1}{\epsilon-\vec{k}^2+i\delta}\right) \Gamma(\vec{k},\vec{p}\,',G) . \quad (4)$$

In the above,

$$\epsilon = G_0 - \tfrac{1}{4}\vec{G}^2 ,$$

$$\tilde{f}(\vec{p},\vec{k}) = \int \frac{d^2q}{(2\pi)^2} V(\vec{q})\psi_{\vec{k}}(\vec{p}-\vec{q}) ,$$

$$N(\vec{G},\vec{k}) = 1 - n_{\vec{G}/2+\vec{k}} - n_{\vec{G}/2-\vec{k}} ,$$

and

$\psi_{\vec{k}}(\vec{p}-\vec{q})$ is the Fourier transform of the two-particle wave function. The quantity $\tilde{f}(\vec{p},\vec{k})$ is related to the scattering amplitude by $\tilde{f}(\vec{p},\vec{k}) = -(8\pi|\vec{k}|)^{1/2} e^{-i\pi/4} f(\vec{p},\vec{k})$ for $|\vec{p}| = |\vec{k}|$. The advantage of using the

modified scattering amplitude in Γ and Γ_0 is that the potential has been formally eliminated from the problem. We can now expand the effective interaction in powers of $1/\ln k a$. For the case of a hard core, we employ the following expression[5] for $\tilde{f}(\vec{p}, \vec{k})$.

$$\tilde{f}(\vec{p}, \vec{k}) = -2\pi |\vec{k}| a \sum_{m=0}^{\infty} c_m e^{i\delta_m} J_m(|\vec{p}|a)[\cos\delta_m J_{m+1}(|\vec{k}|a) - \sin\delta_m Y_{m+1}(|\vec{k}|a)]\cos m\theta. \tag{5}$$

J_m and Y_m are the cylindrical Bessel functions,

$$\cos\delta_m = Y_m(|\vec{k}|a)/[Y_m^2(|\vec{k}|a) + J_m^2(|\vec{k}|a)]^{1/2},$$

$c_m = 1$ for $m = 0$ and 2 otherwise, and δ_m is the mth partial-wave phase shift. We see that for $|\vec{p}|a = |\vec{k}|a \ll 1$, $\tilde{f}(\vec{p}, \vec{k})$ is independent of $|\vec{p}|$. This is a useful result for the off-energy-shell behavior. We note that even though the above was derived for a hard core of radius a, we can extend the result to any repulsive potential which is parameterized by a scattering length a. This is fully equivalent to the replacement of an arbitrary potential with a hard core in the pseudopotential method.[6] A more detailed analysis appears in Appendix B. For $|\vec{k}|a$, $|\vec{p}|a \ll 1$,

$$\tilde{f}(\vec{p}, \vec{k}) = \frac{-2\pi}{\ln|\vec{k}|a} - \frac{2\pi\gamma + i\pi^2}{(\ln|\vec{k}|a)^2},$$

where $\gamma = 0.11593$. Using this expansion for $\tilde{f}(\vec{p}, \vec{k})$, the effective interaction [Eq. (4)] can now be solved explicitly to second order in $1/\ln|\vec{k}|a$.

$$\Gamma(\vec{q}, \vec{q}, G) = \frac{-2\pi}{\ln(|\vec{q}|a)} - \frac{2\pi\gamma}{[\ln(|\vec{q}|a)]^2} + \int \frac{d^2k'}{(2\pi)^2} \frac{4\pi^2}{[\ln(|\vec{k}'|a)]^2} \left(\frac{N(\vec{G}, \vec{k}')}{\epsilon - \vec{k}'^2 + i\delta N(\vec{G}, \vec{k}')} + P\frac{1}{\vec{k}'^2 - \vec{q}^2} \right). \tag{6}$$

Here P denotes the principal part and $\vec{q} = \frac{1}{2}(\vec{p} - \vec{k})$. The self-energy is now

$$\Sigma(\vec{p}, p_0) = \frac{-i}{(2\pi)^3} \int d^2k\, dk_0 G_0(k) \Gamma(\vec{q}, \vec{q}, G). \tag{7}$$

Integrating over k_0 yields the following expression for the self-energy:

$$\Sigma(\vec{p}, p_0) = \Sigma_a(\vec{p}) + \Sigma_b(\vec{p}, p_0) \tag{8a}$$

$$\Sigma_a(\vec{p}) = \int \frac{d^2k}{(2\pi)^2} n_{\vec{k}} \left(\frac{-2\pi}{\ln|\vec{q}|a} - \frac{2\pi\gamma}{(\ln|\vec{q}|a)^2} \right) \tag{8b}$$

$$\Sigma_b(\vec{p}, p_0) = \int\int \frac{d^2k\, d^2k'}{(2\pi)^4} \frac{4\pi^2}{(\ln|\vec{k}'|a)^2} \left(n_{\vec{k}} P \frac{1}{\vec{k}'^2 - \vec{q}^2} + \frac{n_{\vec{k}}(1 - n_{\vec{G}/2+\vec{k}'})(1 - n_{\vec{G}/2-\vec{k}'})}{p_0 + \vec{q}^2 - \vec{p}^2/2 - \vec{k}'^2 + i\delta} \right.$$
$$\left. + \frac{(1 - n_{\vec{k}}) n_{\vec{G}/2+\vec{k}'} n_{\vec{G}/2-\vec{k}'}}{p_0 + \vec{q}^2 - \vec{p}^2/2 - \vec{k}'^2 - i\delta} \right). \tag{8c}$$

The Green's function for the sum of ladder diagrams is

$$G(\vec{p}, p_0) = \frac{1}{p_0 - \frac{1}{2}\vec{p}^2 - \Sigma(\vec{p}, p_0)}. \tag{9}$$

We can find the energy spectrum and quasiparticle lifetime from the location of the pole, $e(\vec{p}) + i\gamma_p = \frac{1}{2}\vec{p}^2 + \Sigma(\vec{p}, p_0)$. A brief prescription for the calculation of the inverse lifetime appears in Appendix A:

$$\gamma_p = -\frac{k_F^2}{2} \frac{1}{(\ln k_F a)^2} \frac{5}{2\pi} \operatorname{sgn}(k_F - |\vec{p}|) \left(\frac{k_F - |\vec{p}|}{k_F} \right)^2 \ln\left| \frac{k_F - |\vec{p}|}{k_F} \right|. \tag{10}$$

This behavior of the lifetime contrasts with the well know three-dimensional result

$$\gamma_p = \frac{k_F^2}{\pi} (k_F a)^2 \operatorname{sgn}(k_F - |\vec{p}|) \left(\frac{k_F - |\vec{p}|}{k_F} \right)^2.$$

To determine the Fermi energy and the effective mass we expand the real part of the self energy around $|\vec{p}| = k_F$:

$$e(\vec{p}) = \frac{k_F^2}{2} + \operatorname{Re}\Sigma(\vec{k}_F, \tfrac{1}{2}k_F^2) + (|\vec{p}| - k_F) \left(k_F + \frac{\partial \operatorname{Re}\Sigma(|\vec{p}|, p_0)}{\partial |\vec{p}|} \right)_{\substack{|\vec{p}| = k_F \\ p_0 = k_F^2/2}} + \cdots. \tag{11}$$

The constant terms are the Fermi energy, while the linear terms are related to the effective mass by $m^* = \hbar^2 k_F / [\partial e(\vec{p})/\partial |\vec{p}|]_{|\vec{p}|=k_F}$. In Appendix A a short outline of the calculation is given. The Fermi energy is

$$u = \frac{k_F^2}{2}\left(1 - \frac{1}{\ln k_F a} + \frac{0.45}{(\ln k_F a)^2}\right) \tag{12}$$

and the effective mass is

$$\frac{m^*}{m} = 1 + \frac{0.86}{(\ln k_F a)^2} . \tag{13}$$

At this stage a discussion of the physical content of the theory is in order. We must recognize that the mass m is not that of a ^{3}He atom but the effective mass of a single isotopic impurity in a dynamic many-body medium. To date there is no calculation of it. Also, the implicit requirement in the above peturbation calculation that the scattering length be positive precludes any realistic comparison with experiment. It has been known for some time that the low angular momentum phase shifts in mixtures of ^{3}He and ^{4}He, and in pure ^{3}He are positive for certain momenta. The remarkable behavior of bulk ^{3}He at very low temperatures confirms the suspicions of many theorists that these phase shifts imply a superfluid phase. Considering the above arguments little direct connection can be made between the preceeding calculation and experiment. A more realistic calculation of the scattering amplitude coupled with the known binding energy[1] of a ^{3}He atom would give information as to the effective ^{3}He-^{3}He interaction and to its bare mass.

To calculate the ground state energy we use the fact that $T = 0\,^\circ$K

$$\frac{E}{N} = \frac{k_F^2}{4}\left(1 - \frac{1}{\ln k_F a} + \frac{0.28}{(\ln k_F a)^2}\right) . \tag{14}$$

For the discontinuity at the Fermi surface we need

$$Z_{k_F} = 1\Big/\left(1 - \frac{\partial\, \mathrm{Re}\Sigma(\vec{p}, p_0)}{\partial p_0}\right)_{\substack{p_0 = e(k_F) \\ |\vec{p}| = k_F}} . \tag{15}$$

In the first appendix an outline of the calculation is presented, we here quote the result

$$Z_{k_F} = 1 - \frac{0.26}{(\ln k_F a)^2} . \tag{16}$$

If we set $p_0 = \frac{1}{2} p^2$ in $\Sigma_b(\vec{p}, p_0)$, the density in momentum space becomes a step function. This is not correct. Although the calculation of the momentum distribution has not been done, one should get the same behavior as found by Belyakov[7] for the three-dimensional case. This view is supported by the fact that a step function of height Z_{k_F} would give an areal density nZ_{k_F}, which is less than the actual value. Also, the finite lifetime of the quasiparticles must alter the momentum distribution.

Finally we use our expression for the effective mass to calculate the low-temperature specific heat, which is $\frac{2}{3}$ the three-dimensional value,

$$C_V = 2\pi^2 N k_b^2 T m^* / 3\hbar^2 k_F^2 .$$

The result obtained is not expected to hold for all temperatures. Experiments at very low temperatures indicate a spin ordering which varies with temperature and density. The spin ordering temperature is in the $m\,^\circ$K range in three dimensions and is expected to be even lower in two. Above that critical temperature this effect should be small. For ^{3}He adsorbed on ^{4}He one must take into account the contribution of surface capillary waves (ripplons). Density fluctuations can be disregarded since the phonon specific heat goes as T^2.

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APPENDIX A

To calculate the lifetime we use the expression $\gamma_p = \mathrm{Im}\Sigma_b(\vec{p}, p_0)$. Taking the imaginary part of 8c we get

$$\gamma_p = \frac{1}{4\pi}\int\int d^2k\, d^2l\, \frac{1}{(\ln|\vec{q}|\, a)^2}\, \delta\left(\left|\frac{\vec{p}-\vec{k}}{2}\right|^2 - \vec{l}^{\,2}\right)\left[(1-n_{\vec{k}})\, n_{(\vec{p}+\vec{k})/2+\vec{l}}\, n_{(\vec{p}+\vec{k})/2-\vec{l}} - n_{\vec{k}}(1-n_{(\vec{p}+\vec{k})/2+\vec{l}})\,(1-n_{(\vec{p}+\vec{k})/2-\vec{l}})\right] .$$

We look for the limiting value as p approaches k_F. Resurrecting Planck's constant and the mass we get Eq. (10).

Knowledge of $\mathrm{Re}\Sigma(\vec{p}, p_0)$ can give us the Fermi energy and the effective mass.

$$\mathrm{Re}\Sigma_a(\vec{p}) = \frac{1}{(2\pi)^2}\int d^2k\, n_{\vec{k}}\left(\frac{-2\pi}{\ln|\vec{q}|\, a} - \frac{-2\pi\gamma}{(\ln|\vec{q}|\, a)^2}\right)$$

$$\mathrm{Re}\Sigma_b(\vec{p}, p_0) = \frac{4\pi^2}{(\ln k_F a)^2}\, P\int\int \frac{d^2k\, d^2l}{\vec{q}^{\,2} - \vec{l}^{\,2}}\left[n_{\vec{k}}(1-n_{(\vec{p}+\vec{k})/2+\vec{l}})\,(1-n_{(\vec{p}+\vec{k})/2-\vec{l}}) + (1-n_{\vec{k}})\, n_{(\vec{p}+\vec{k})/2+\vec{l}}\, n_{(\vec{p}+\vec{k})/2-\vec{l}} - n_{\vec{k}}\right] .$$

An expansion in p is made around k_F. The final integrals must be done by computer. Equation 11 is then

$$e(\vec{p})=\frac{\hbar^2 k_F^2}{2m}\left(1-\frac{1}{\ln k_F a}+\frac{0.45\pm0.05}{(\ln k_F a)^2}\right)+(|\vec{p}|-k_F)\frac{k_F}{m}\left(1-\frac{0.86\pm0.06}{(\ln k_F a)^2}\right).$$

For the discontinuity at the Fermi surface we start with the expression

$$Z_{k_F}=1\Big/\left(1-\frac{\partial \mathrm{Re}\Sigma(\vec{k}_F,p_0)}{\partial p_0}\right)_{p_0=k_F^2/2}$$

The part of the self-energy that is frequency dependent is $\Sigma_b(\vec{k}_F,p_0)$. We easily find

$$\left(\frac{\partial \mathrm{Re}\Sigma_b(\vec{k}_F,p_0)}{\partial p_0}\right)_{p_0=k_F^2/2}=-\frac{1}{\pi^2(\ln k_F a)^2}\int\int\frac{d^2s\,d^2k}{(|\vec{s}+\vec{k}_F|^2-\vec{k}^2)^2}(1-n_{2\vec{s}+\vec{k}_F})n_{\vec{s}+\vec{k}_F}n_{\vec{s}-\vec{k}_F}.$$

This integral must be done numerically, and

$$Z_{k_F}=1-\frac{0.26\pm0.05}{(\ln k_F a)^2}.$$

APPENDIX B

We now give a more detailed explanation of why one can include any well behaved short range repulsive potential. Using standard techniques of partial wave analysis we can derive the following expression for $\tilde{f}(\vec{p},\vec{k})$

$$\tilde{f}(\vec{p},\vec{k})=2\pi\sum_{m=0}^{\infty}c_m e^{i\delta_m}\cos m\theta\times\int_0^{\infty}r\,dr\,U(r)F_m(|\vec{k}|r)J_m(|\vec{p}|r),$$

where c_m is the same as before. The function $F_m(|\vec{k}|r)$ is related to the wave function $\psi_{\vec{k}}(\vec{r})$ by

$$\psi_{\vec{k}}(\vec{r})=\sum_{m=0}^{\infty}c_m e^{i\delta_m}F_m(|\vec{k}|r)\cos m\theta.$$

For $|\vec{p}|a\ll 1$ where a is the range of the potential,

$$\tilde{f}(\vec{p},\vec{k})=2\pi c_0 e^{i\delta_0}\int_0^{\infty}U(r)F_0(|\vec{k}|r)r\,dr.$$

All other terms in the expansion go to zero at least as fast as $|\vec{p}|a$. We reach the useful conclusion that $\tilde{f}(\vec{p},\vec{k})$ is independent of $\vec{p}$. Because of this, we can investigate the low energy behavior of $\tilde{f}(\vec{p},\vec{k})$ off the mass shell with $|\vec{p}|=|\vec{k}|$. Once again using two-dimensional scattering theory it is easy to show that the modified scattering amplitude can be written for low energy as

$$\tilde{f}(\vec{p},\vec{k})=\frac{-4}{\cot\delta_0-i}.$$

Outside the region of the potential we must match the logarithmic derivative of the wave function with $\alpha_m(E)$

$$\alpha_m(E)=\frac{1}{F_m(|\vec{k}|a)}\left(\frac{dF_m(|\vec{k}|r)}{dr}\right)_{r=a}$$

$$F_m(|\vec{k}|r)=\tfrac{1}{2}[H_m^*(|\vec{k}|r)+e^{2i\delta_m}H_m(|\vec{k}|r)],$$

$H_m(|\vec{k}|r)$ is the cylindrical Hankel function. One can then obtain

$$\cot\delta_0=\left(\frac{\partial Y_0(|\vec{k}|r)}{\partial r}-\alpha_0 Y_0(|\vec{k}|r)\right)\Big/\left(\frac{\partial J_0(|\vec{k}|r)}{\partial r}-\alpha_0 J_0(|\vec{k}|r)\right)_{r=a}.$$

For $|\vec{k}|a\ll 1$,

$$\cot\delta_0\approx(2/\pi)[\ln(|\vec{k}|a)-\gamma],$$

which implies that

$$\delta_0\approx\pi/2[\ln(|\vec{k}|a)-\gamma]$$

and

$$\tilde{f}(\vec{p},\vec{k})=-2\pi\Big/[\ln(|\vec{k}|a)-\gamma]\left[1-\frac{i\pi}{2[\ln(|\vec{k}|a)-\gamma]}\right].$$

This is the same formula that one obtains for a hard core only now a is not the spatial extent of the potential but some constant that parameterizes the scattering.

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