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Theory of electronic band structure in intense laser fields*

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The nonlinear interaction of laser radiation with crystalline solids results in a modified band structure which is a natural extension of the usual Bloch picture. A theoretical discussion is presented in which the existence of field-induced gaps is predicted. Applications to model systems such as the Kronig-Penney model are presented. Examination of realistic systems such as the narrow band gap semiconductors suggest that the effect is large and should be readily observable with currently available laser sources.

I. INTRODUCTION

The current availability of high-powered lasers has stimulated renewed interest in the nonlinear interaction of radiation with matter. In this paper, we explore some elementary aspects of this interaction as it relates to crystalline solids. The problem is interesting in a formal sense because it is described by a multiply periodic Hamiltonian. Physically it is interesting because the strong perturbations of the solid caused by the intense field may lead to the alteration of some of the basic characteristics of the solid. One of the main purposes of the paper is to study the evolution of the band structure with increasing field strength. It is found that at particular points in the Brillouin zone, there is a strong modification of the band structure. Radiation-induced gaps are opened up, the sizes of which are related to the field strength.

Interest in temporally periodic quantum-mechanical systems has been evident in recent years,[1] mostly with regards to atomic systems. There has, however, been little attention paid to the more complicated problem of solid-state systems.[2] To summarize what has been learned about such systems would be difficult to do here, so we just concentrate on points relevant to the present paper. First of all, one realizes that energy, *per se*, is not defined for a temporally periodic system. However, one may define a quasienergy whose value is defined modulus $\hbar\omega$, ω being the laser frequency. By specifying a value of this quasienergy, one is actually denoting a ladder of eigenstates whose energy separation between successive states is $\hbar\omega$. Shirley[3] has shown the relation between the index appearing in Floquet solutions to the Schrödinger equation and this quasienergy.

For a finite basis Hamiltonian there is a one-to-one correspondence between low-field eigenstates and these quasistates, each zero-field eigenenergy generating a ladder of its own. As the eigenenergies become Stark shifted, transitions between the various rungs of the ladder manifest themselves as a "splitting" of the spectral lines. This effect, predicted by Mollow,[4] has been observed experimentally recently.[5] These phenomena have been investigated in atomic systems. It is the purpose of the present paper to extend their study of the ac Stark effect to solid-state situations.

In Sec. II, the basic theory for generalizing Bloch's techniques to the radiation-solid system are discussed. We find that the field-induced gaps appear in the band structure whenever n laser photons are able to cause a direct multiphoton interband transition. The two-band approximation is also developed for the radiation-solid system. In Sec. III we apply the theory to the Kronig-Penney model. It is concluded that the Mollow effect,[4] in which the scattered light appears at shifted frequencies, may occur in solid-state situations just as in atomic systems. Section IV concerns itself with an application to the narrow-band-gap semiconductors,[6] where spin-orbit effects must be included. These systems were chosen for study because of the large nonlinearities that occur. Finally, Sec. V contains a brief summary of the paper.

II. BASIC THEORY

Consider the problem of a solid interacting with an electromagnetic wave. Since typical optical wavelengths are much greater than the lattice parameter, we treat the wave as a homogeneous time-varying electric field which is to be characterized by the vector potential $\vec{A}(t)$. We shall neglect many-body effects in this paper and only work in the one-electron approximation. The Hamiltonian is thus

$$H = \tfrac{1}{2}(\vec{p} + \alpha\vec{A})^2 + V(\vec{r}), \tag{1}$$

where we are employing atomic units ($\hbar = e = m = 1$, $\alpha = \frac{1}{137}$). It will be assumed that $\vec{A}(t)$ is periodic in time and $V(\vec{r})$ is periodic in space,

$$\vec{A}(t+\tau) = \vec{A}(t), \tag{2}$$

$$V(\vec{r}+\vec{a}) = V(\vec{r}), \tag{3}$$

$\vec{a}$ being a lattice translation. The object is to solve

the time-dependent Schrödinger equation

$$H(t)\psi(t) = i\frac{\partial\psi(t)}{\partial t}. \tag{4}$$

It is convenient to make a canonical transformation to a new Hamiltonian where the spatial and temporal character of the problem appear in the same term. Thus, letting

$$\psi = \exp\left(-i\alpha\int \vec{p}\cdot\vec{A}\,dt - i\frac{\alpha^2}{2}\int A^2\,dt\right)\phi, \tag{5}$$

we find

$$\left(\frac{p^2}{2} + U(\vec{r},t) - i\frac{\partial}{\partial t}\right)\phi = 0, \tag{6}$$

where

$$U(\vec{r},t) = V\left(\vec{r} + \alpha\int \vec{A}\,dt\right). \tag{7}$$

Here U will be spatially and temporally periodic if $\vec{A}$ is chosen so that on the average it is zero, i.e.,

$$\int_0^\tau \vec{A}(t)\,dt \equiv 0. \tag{8}$$

Thus the problem we have set out to study is, in reality, a special case of the problem of an electron interacting with an arbitrary multiply periodic potential.

If we expand $V(\vec{r})$ in a Fourier series, we have

$$V = \sum_{\vec{G}} V_{\vec{G}}\, e^{i\vec{G}\cdot\vec{r}}, \tag{9}$$

where $\{\vec{G}\}$ is the set of reciprocal-lattice vectors. While $\vec{A}(t)$ may also be expressed in a general Fourier expansion

$$\vec{A}(t) = \sum_{\substack{n \\ n\neq 0}} \vec{A}_n\, e^{in\omega t}, \tag{10}$$

where $\omega = 2\pi/\tau$, we choose to limit our discussion to the case where it is harmonic in time, i.e.,

$$\vec{A} = -\vec{A}_1 \cos\omega t. \tag{11}$$

Combining Eqs. (7), (9), and (10) leads to the result

$$U(\vec{r},t) = \sum_{\vec{G}n} U_{\vec{G}n}\, e^{i(\vec{G}\cdot\vec{r} - n\omega t)}, \tag{12}$$

where

$$U_{\vec{G}n} = V_{\vec{G}} J_n\left(\frac{\alpha}{\omega}\vec{G}\circ\vec{A}_1\right). \tag{13}$$

Following Shirley,[3] we study the Floquet solutions to Eq. (6). Let

$$\phi(\vec{r},t) = e^{i(\vec{k}\cdot\vec{r} - \epsilon t)}\, u(\vec{r},t), \tag{14}$$

where $u(\vec{r},t)$ is both spatially and temporally periodic. Some remarks as to the significance of such solutions are in order. Imagine allowing an electron to interact with a time varying potential for a very long time. The Floquet states represent solutions of the Schrödinger equation which are, in a sense, steady-state solutions. That is, if we compare the solution with itself at two times translated by a temporal period, they are the same up to a phase factor

$$\phi(\vec{r}, t+\tau) = e^{-i\epsilon\tau}\,\phi(\vec{r},t). \tag{15}$$

In actuality, these states are not eigenstates of the Hamiltonian appropriate to low electric fields but rather represent linear combinations of all those eigenstates. These linear combinations are inert against transitions which might be induced by the temporal variation of $U(\vec{r},t)$, a property not shared by those individual eigenstates.

Inserting Eq. (14) into Eq. (6) and expanding $U(\vec{r},t)$ in a Fourier sum leads to

$$[\tfrac{1}{2}(\vec{k}+\vec{G})^2 - \epsilon - n\omega]u_{n\vec{G}} + \sum_{n'\vec{G}} U_{n-n',\vec{G}-\vec{G}'}u_{n'\vec{G}'} = 0, \tag{16}$$

where

$$u(\vec{r},t) = \sum_{n\vec{G}} u_{n\vec{G}}\, e^{i(\vec{G}\cdot\vec{r} - n\omega t)}. \tag{17}$$

The "eigenvalues" ϵ of Eq. (16) are determined by locating the zeroes of the Hill determinant Δ, where

$$\Delta(\vec{k},\epsilon) = \det\left(\delta_{nn'}\delta_{\vec{G}\vec{G}'} + \frac{U_{n-n',\vec{G}-\vec{G}'}}{\tfrac{1}{2}(\vec{k}+\vec{G})^2 - \epsilon - n\omega}\right). \tag{18}$$

Some general properties of the "eigenvalue" spectrum follow directly from this expression. Firstly, as in the conventional solid-state case, the Hill determinant is invariant under translations of the $\vec{k}$ by a reciprocal-lattice vector

$$\Delta(\vec{k}+\vec{G},\epsilon) = \Delta(\vec{k},\epsilon). \tag{19}$$

Hence, it follows that one need only study the first Brillouin zone. Secondly, the Hill determinant is invariant under translations of ϵ by any integer times ω:

$$\Delta(\vec{k},\epsilon + n\omega) = \Delta(\vec{k},\epsilon). \tag{20}$$

Thus it follows that one need only study a domain of ϵ whose extent is zero to ω. All the "band structure" associated with this problem is contained in a fundamental hyperrectangle in the ϵ-$\vec{k}$ plane. Thus we have a natural extension of Bloch's result for conventional band structure. The Brillouin zone is now a four-dimensional volume.

Let us proceed by examining the zero-field limit of our solution. Since the Bessel function vanishes for zero argument unless its order is zero, Eq. (18) becomes

$$\Delta(\vec{k},\epsilon) \xrightarrow[\vec{A}_1\to 0]{} \det\left(\delta_{nn'}\delta_{\vec{G}\vec{G}'} + \delta_{nn'}\frac{V_{\vec{G}-\vec{G}'}}{\tfrac{1}{2}(\vec{k}+\vec{G})^2 - \epsilon - n\omega}\right). \tag{21}$$

The property that Δ is periodic in ϵ is still maintained, and thus the zone $0 \leq \epsilon \leq \omega$ still contains all the band-structure information. However, this situation is rather artificial, since the band structure is generated by simply translating the dispersion curve downwards in energy by steps of ω and superimposing the various curves. The fact that there is no change induced in the band structure is the clue that nothing has really happened, there being no coupling between the translated and untranslated curves.

When perturbations due to the electric field are taken into account, however, the translated and untranslated band structure begin to interact. The interaction is particularly important in a place where the curves have crossed in the zero-field limit. The interaction results in the occurrence of an avoided crossing, or to put it another way, the opening of a gap in the band structure. For weak fields, the gap is only sizeable if the number of translations that were involved in generating the composite band structure is small. This means that one is including only one- or two-photon effects and disregarding higher-order effects, for example. For higher field intensities, more translations must be included into the prescription. A rough estimate for the number of translations which result in sizeable gaps is obtained from Eq. (13). The Bessel function tends to be small if its argument is exceeded by its order. Thus

$$N \sim (\alpha/\omega) G A_1 , \tag{22}$$

provides a crude index for this number.

Some analytic expressions for the band gaps may be obtained in the nearly-free-electron model. Assuming V_g to be weak, one may approximate Eq. (18) by

$$\Delta(\vec{k}, \epsilon) \approx 1 - \frac{1}{2} \sum_{\substack{n n' \\ G G'}} \frac{|V_{\vec{G}-\vec{G}'}|^2 J^2_{n-n'}[(\alpha/\omega)\vec{A}_1 \cdot (\vec{G}-\vec{G}')]}{[\frac{1}{2}(\vec{k}+\vec{G})^2 - \epsilon - n\omega][\frac{1}{2}(\vec{k}+\vec{G}')^2 - \epsilon - n'\omega]} . \tag{23}$$

In the vicinity of the intersection of two free-electron bands, both denominators are small for some particular set of values $\vec{G}$, $\vec{G}'$, n, and n'. Thus the eigenvalues are determined through the relation

$$[\tfrac{1}{2}(\vec{k}+\vec{G})^2 - \epsilon - n\omega][\tfrac{1}{2}(\vec{k}+\vec{G}')^2 - \epsilon - n'\omega] - |V_{\vec{G}-\vec{G}'}|^2 J^2_{n-n'}[(\alpha/\omega)\vec{A}_1 \cdot (\vec{G}-\vec{G}')] = 0 . \tag{24}$$

Then the two branches have the form

$$\epsilon = \tfrac{1}{2}[\tfrac{1}{2}(\vec{k}+\vec{G})^2 + \tfrac{1}{2}(\vec{k}+\vec{G}')^2 - n\omega - n'\omega] \pm \left[\{\tfrac{1}{2}[\tfrac{1}{2}(\vec{k}+\vec{G})^2 - \tfrac{1}{2}(\vec{k}+\vec{G}')^2 + (n'-n)\omega]\}^2 + |V_{\vec{G}-\vec{G}'}|^2 J^2_{n-n'}\left(\frac{\alpha\vec{A}_1 \cdot (\vec{G}-\vec{G}')}{\omega}\right)\right]^{1/2} . \tag{25}$$

The actual size of the gap at the intersection is

$$\delta = 2\,|V_{\vec{G}-\vec{G}'}|\,\left|J_{n-n'}[(\alpha/\omega)\vec{A}_1 \cdot (\vec{G}-\vec{G}')]\right| . \tag{26}$$

It should, perhaps, be emphasized that the field-induced gaps discussed here will, in general, lie anywhere inside the Brillouin zone, their position determined by the radiation frequency ω and the weak-field band structure. They occur whenever an n-photon process is capable of bridging two gaps.

III. APPLICATION TO THE KRONIG-PENNEY MODEL

As an elementary example of the application of the theory, we consider the Kronig-Penney model in one dimension. In the field-independent case, as is well known, analytic expressions that lead to the band structure are readily available. For nonzero fields, however, we have not been able to find analogous expressions and have therefore resorted to numerical computations.

We take V_G to be a constant independent of G in the Kronig-Penney model. The following parameters are introduced: $k = g\xi$, $\epsilon = \frac{1}{2}\eta g^2$, $\omega = \frac{1}{2} z g^2$, $x = \alpha A_1 g/\omega$, $y = 2V_0/g^2$, where $g = 2\pi/a$. In practice we found it simpler to directly diagonalize the matrix associated with Eq. (16) rather than to hunt for the zeroes of Eq. (18). Since a truncation of the infinite determinant was required, strict periodicity was not maintained. Our main concern in this project was the temporal periodicity, so we included nine values for the index n ($-4 \leq n \leq +4$) but only three G values ($\pm g$ and 0). It was found that in the domain $0 \leq \eta \leq 1$ and the neighboring domains, the band structure replicated itself fairly well. As the number of n-states increased, the periodicity became more firmly established, consistent with our expectation from Eq. (20).

In Fig. 1 we present the results of a model calculation for the fundamental half-rectangle (there being symmetry under $\xi \to -\xi$). Here we have the "dimensionless" field strength $x = 0$, lattice potential $y = 0.3$, and frequency $z = 0.3$. In Fig. 2 we present the same curve for $x = 0.3$. Note that the band structure is largely unaffected aside from the

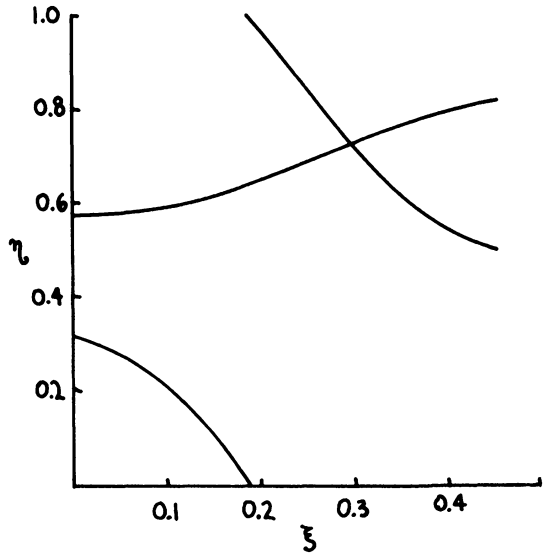

FIG. 1. Band structure for a Kronig-Penney model. The abcissa is the scaled wave vector and the ordinate is the energy in units of the laser frequency. Here the electromagnetic field strength x, is zero.

opening of a gap at the intersection of the two bands. When $x=0.5$, as in Fig. 3, the gap has widened and the band structure is beginning to deform, while in Fig. 4, where $x=1.0$, the gap is rather substantial and the structure is strongly perturbed. The values of the field strengths employed in this calculation were taken to be artificially large for illustrative purposes, so we now turn to the practical question as to how one may experimentally observe the

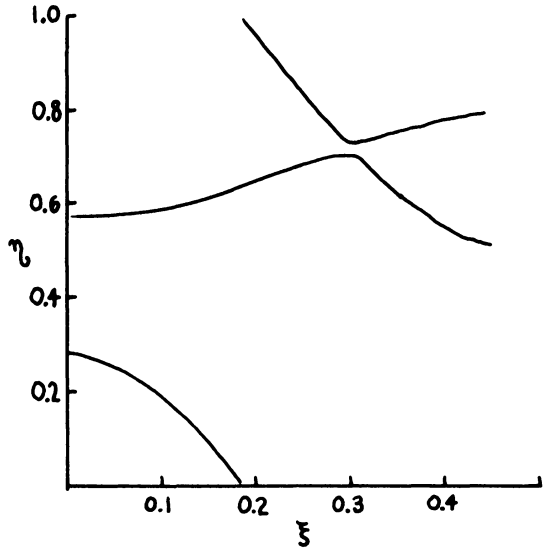

FIG. 3. Same as Fig. 1, but $x=0.5$.

distorted band structure and, in particular, the gaps.

Two approaches seem possible. One is to spectroscopically probe the perturbed solid and the other is to monitor the transport properties of the material. In either case, there are several restrictions placed on the lower limit to the field strength that can be used. First of all, we would like the gap to be larger than the thermal spread of the electrons. From Eq. (26) we see that if an nth-order transition is involved, it would imply that

$$\frac{2V_G}{n!}\left(\frac{\alpha A_1 G}{2\omega}\right)^n > k_B T. \tag{27}$$

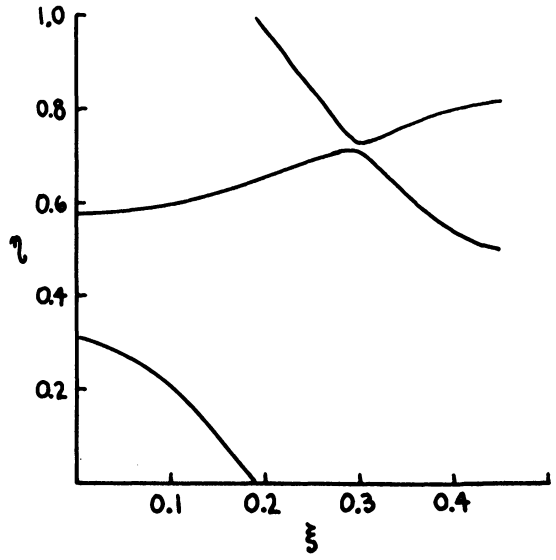

FIG. 2. Same as Fig. 1, but $x=0.3$

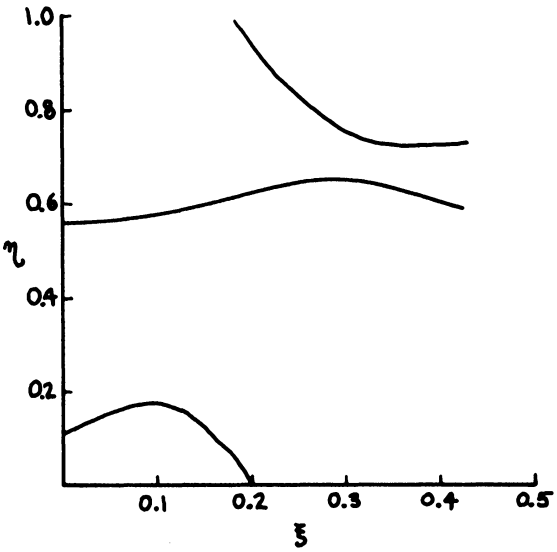

FIG. 4. Same as Fig. 1, but $x=1.0$.

Thus the laser intensity would have to satisfy the inequality

$$I > \frac{c}{2\pi}\left(\frac{m\omega^2}{eG}\right)^2 \left(\frac{n! k_B T}{2V_G}\right)^{2/n}. \tag{28}$$

For typical parameters, $\omega = 1.7\times10^{14}$ rad/sec, $V_G = 1$ eV, $G = 2\pi\times10^8$ cm^{-1}, $n = 1$, $T = 300$ °K, this gives the condition $I > 600$ W/cm^2. This lower limit can be reduced five orders of magnitude by lowering the temperature to 1 °K. Thus, thermal effects do not seem to impose an essential limitation on the experiment.

Next let us consider radiative-lifetime constraints Since phonon emission is generally highly favored over photon emission, we would like the gap to be larger than the decay width for phonon emission, $\hbar/\tau_p$. This leads to the expression

$$I > \frac{c}{2\pi}\left(\frac{m\omega^2}{eG}\right)^2 \left(\frac{\hbar n!}{2V_G\tau_P}\right)^{2/n}. \tag{29}$$

Taking $\tau_p \sim 10^{-13}$ sec and $n = 1$, this leads to the inequality $I > 35$ W/cm^2, again a nonrestrictive condition.

Thus it appears that a direct spectroscopic probe of the band structure seems possible. Needless to say, the resolution of the spectrometer must be fine enough to resolve the gap. A particular prediction of the present model is that scattered light should appear not only at frequency ω but at $\omega \pm \delta$. This is analogous to the situation in atoms where, in fact, this effect has already been observed.

In a spectroscopic study, one is generally sensitive to anomalies in the density of states. Such anomalies would appear in the vicinity of a gap that has been opened.

IV. APPLICATION TO NARROW-BAND-GAP SEMICONDUCTORS

It is well known that there are strong nonlinearities that occur in the narrow-band-gap semiconductors such as InSb. This is mainly due to the very low effective mass of the conduction electrons, which allows for a strong coupling to the electromagnetic field. It therefore seems reasonable to study the perturbed band structure in such a material.

A rather complete survey of the theoretical model used to calculate the zero-field band structure has been given by Kane.[6] The Hamiltonian is taken by us to be

$$H = \tfrac{1}{2}(\vec{p} + \vec{k} + \alpha\vec{A})^2 + V + \tfrac{1}{4}\alpha^2 \vec{\sigma}\cdot\nabla V\times(\vec{p} + \vec{k} + \alpha\vec{A}), \tag{30}$$

where V is the lattice potential and the last term is the spin-orbit interaction. We have generalized Kane's Hamiltonian by including an interaction with the radiation field. Following Kane, we note that $\vec{\sigma}\cdot\nabla V\times\vec{k}$ is much smaller than the other terms and disregard it. Similarly, one may neglect $\vec{\sigma}\cdot\nabla V\times\vec{A}$. Thus the Schrödinger equation to be studied becomes

$$\left(\frac{p^2}{2} + V + \vec{p}\cdot(\vec{k} + \alpha\vec{A}) + \frac{\alpha^2}{4}\vec{\sigma}\cdot\nabla V\times\vec{p} + \tfrac{1}{2}(\vec{k} + \alpha\vec{A})^2 - i\frac{\partial}{\partial t}\right)\psi = 0. \tag{31}$$

For the present discussion, we limit our attention to the vicinity of the bottom of the conduction band. Here it is more convenient not to make the transformation of Eq. (5), and we study Eq. (31) directly. Let

$$\psi = \exp\left(-i\epsilon t - \frac{i}{2}\int(\vec{k} + \alpha\vec{A})^2\,dt\right)\phi, \tag{32}$$

be the Floquet solution to Eq. (31). Then

$$\left(\frac{p^2}{2} + V + (\vec{k} + \alpha\vec{A})\cdot\vec{p} + \frac{\alpha^2}{4}\vec{\sigma}\cdot\nabla V\times\vec{p} - \epsilon - i\frac{\partial}{\partial t}\right)\phi = 0, \tag{33}$$

where ϕ is a periodic spinor. In the limit where the spin-orbit splitting dominates over the band gap, one may decompose the solution to Eq. (33) in the following way: The manifold of significant states contributing to ϕ consist of the three p-orbitals of the valence band and one s-orbital of the conduction band. One valence and one conduction orbital strongly couple and are described by the equations

$$\begin{pmatrix} E_c - \epsilon - i\frac{\partial}{\partial t} & \sqrt{\frac{2}{3}}(k + \alpha A)P \\ \sqrt{\frac{2}{3}}(k + \alpha A)P & E_V - \epsilon - i\frac{\partial}{\partial t}\end{pmatrix}\begin{pmatrix} u \\ v\end{pmatrix} = 0, \tag{34a}$$

where

$$\phi = \begin{pmatrix} u \\ v\end{pmatrix} = \sum_m \begin{pmatrix} u_m \\ v_m\end{pmatrix} e^{-im\omega t}. \tag{34b}$$

Here E_c and E_v are the conduction and valence energies at the band center, P denotes the matrix element

$$P = -i\langle s|p_x|x\rangle, \tag{35}$$

and we have chosen k and A along the 100 directions Solution of Eq. (34) is equivalent to finding the eigenvalues ϵ of the following matrix:

$$\begin{aligned}
&\langle cm'|\mathcal{H}|cm\rangle = (E_c - m\omega)\delta_{m'm},\\
&\langle cm'|\mathcal{H}|vm\rangle = \sqrt{\tfrac{2}{3}}P[k\delta_{m',m} + \tfrac{1}{2}\alpha A(\delta_{m',m+1} + \delta_{m',m-1})],\\
&\langle vm'|\mathcal{H}|vm\rangle = (E_v - m\omega)\delta_{m'm}.
\end{aligned} \tag{36}$$

Thus the structure of the problem is similar to that encountered in Eq. (16). The only difference is that Eq. (16) is valid throughout the entire Brillouin zone, and Eq. (36) is restricted to the bottom of the band and is further restricted to a particular manifold of states.

The eigenvalues of Eq. (36) result in a pair of ladders whose rungs are separated by energy ω.

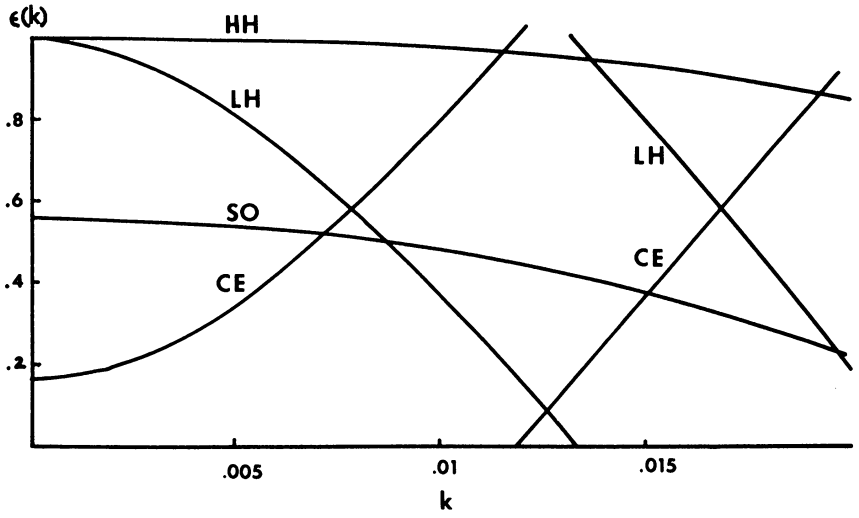

FIG. 5. Band structure for InSb. Here CE denotes the conduction-electron band, LH the light-hole band, HH the heavy-hole band, and SO the split-off band. The abcissa, k, is the wave vector in atomic units, while the ordinate is the band energy in units of the laser energy. Here the electromagnetic field strength is zero.

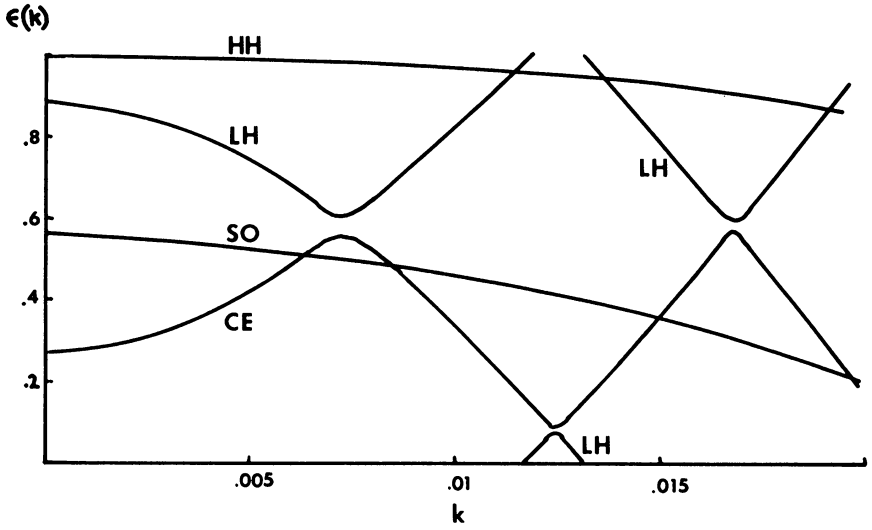

FIG. 6. Same as Fig. 5, but $E = 10^5$ V/cm.

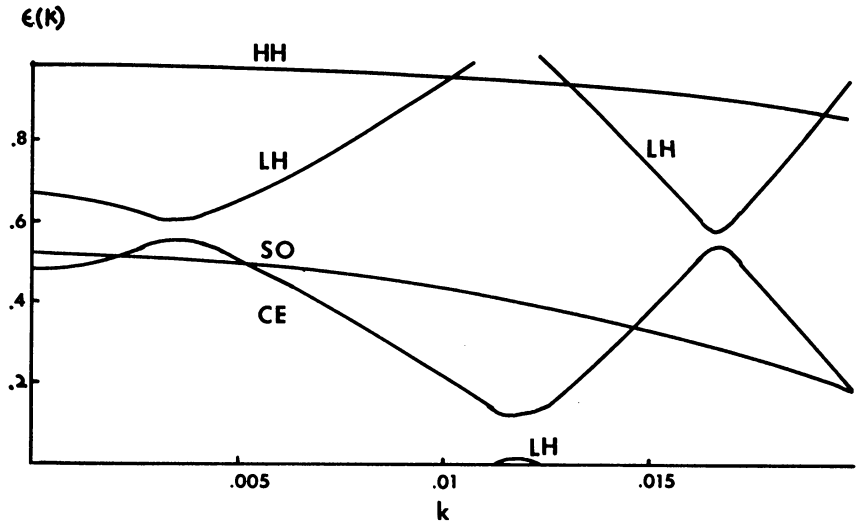

FIG. 7. Same as Fig. 5, but $E = 2\times10^5$ V/cm.

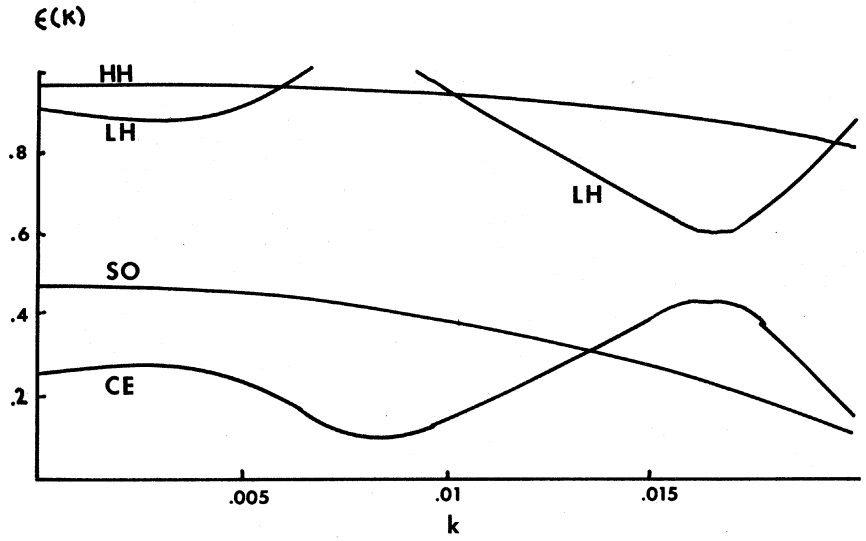

FIG. 8. Same as Fig. 5, but $E=3\times10^5$ V/cm.

These are the distorted conduction-electron and light-hole bands. In addition, we have as solutions to Eq. (33) the heavy-hole band and the split-off band. By making the appropriate minimal substitution, the Hamiltonian for the heavy-hole band becomes

$$H_{HH}=E_v-(1/2m_{HH})(\vec{k}+\alpha\vec{A})^2\,. \tag{37}$$

Since the solution to the Schrödinger equation is

$$\psi=\exp\left(-i\int[E_v-(1/2m_{HH})(k+\alpha A)^2]\,dt\right)\psi_0\,, \tag{38}$$

one may identify ϵ as the nonperiodic part of ψ,

$$\epsilon_{HH}=E_v-\frac{k^2}{2m_{HH}}-\frac{\alpha^2A_0^2}{4m_{HH}}\,. \tag{39}$$

Similarly, we can study the split-off band by generalizing the expression given by Kane

$$H_{SO}=E_v-\Delta-\frac{p^2}{3}\frac{(k+\alpha A)^2}{E_c-E_v+\Delta}\,, \tag{40}$$

where

$$\Delta=\frac{-3i\alpha^2}{4}\langle x|\,(\nabla V\times\vec{P})_y\,|Z\rangle$$

is a measure of the spin-orbit force. One finds the following expression:

$$\epsilon_{SO}=E_v-\Delta-\frac{P^2}{3}\frac{k^2+\alpha^2A_0^2/2}{E_c-E_v+\Delta}\,. \tag{41}$$

In the present model, we are neglecting A-dependent couplings between the split-off (SO), heavy-hole (HH), and other bands. The only bands that are coupled by the radiation field are the light-hole (LH) and conduction electron (CE) bands.

In Figs. 5–8, we explore the systematics of the evolution of the hyperband structure with increasing field strength for fixed ω. The parameters appropriate to InSb, interacting with a CO_2 laser ($\omega=0.0039$ atomic units $=1.7\times10^{14}$ rad/sec) are $m_{CE}=0.016=m_{LH}$, $m_{HH}=0.4$, $E_c-E_v=0.0085$, $P=0.64$, and $\Delta=0.033$. Figure 5 gives ϵ as a function of k for zero-field strength. We see that the CE and LH bands cross at three values of k (0.008, 0.013, 0.017). In Fig. 6 where the field is 10^5 V/cm, one sees that gaps have opened up at these crossings. Our model does not allow for gaps to open at the crossings with the HH and SO bands. They are expected to be much smaller than the LH-CE gaps and so are neglected in the present analysis. In Fig. 7 we see the gaps widen when the field is 2×10^3 V/cm. Also note that for nonzero field, the LH and HH degeneracy at $k=0$ has been lifted. Finally, in Fig. 8 where $E=3\times10^5$ V/cm, the band structure has been significantly altered with huge gaps now dominating the picture. The tendency is for the bands to flatten out as the field is increased.

V. SUMMARY

In this paper, we have considered the interaction of a coherent radiation field with a solid. We found that the usual Bloch band picture can be suitably generalized and that new features appear in the band structure. Gaps are opened up within the Brillouin zone at locations where direct multiphoton transitions can occur. We have calculated the field-dependent magnitude of these gaps for the Kronig-Penney model and for narrow-band-gap semiconductors. It is concluded that these features are observable in both spectroscopy and transport experiments.

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