Letter

Generalized Hertz action and quantum criticality of two-dimensional Fermi systems

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We reassess the structure of the effective action and quantum critical singularities of two-dimensional Fermi systems characterized by the ordering wave vector $\vec{Q} = \vec{0}$. By employing infrared cutoffs on all the massless degrees of freedom, we derive a generalized form of the Hertz action, which does not suffer from problems of singular effective interactions. We demonstrate that the Wilsonian momentum-shell renormalization group (RG) theory capturing the infrared scaling should be formulated keeping $\vec{Q}$ as a flowing, scale-dependent quantity. At the quantum critical point, scaling controlled by the dynamical exponent $z = 3$ is overshadowed by a broad scaling regime characterized by a lower value of $z \approx 2$. This, in particular, offers an explanation of the results of quantum Monte Carlo simulations pertinent to the electronic nematic quantum critical point.

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Introduction. Quantum criticality in Fermi systems constitutes a highly relevant and largely open problem for condensed matter theory. Its significance stems from the growing experimental evidence demonstrating non-Fermi-liquid behavior of thermodynamic as well as transport properties at the onset of different ordered states in a diversity of compounds; the high-T_c cuprate superconductors being the most prominent examples [1]. A persistent question concerns the structure of the low-energy effective action to correctly capture the critical singularities at quantum criticality in Fermi systems. The traditional approach, developed by Hertz [2] and later extended by Millis [3], borrowed the spirit of the Wilsonian theory of classical critical phenomena [4,5]. It proposed to integrate out the original degrees of freedom, resulting in an exact representation of the problem in terms of an effective order parameter action. In the subsequent step, this action was expanded in powers of the ordering field, truncating at quartic order; the two-point function was replaced by its low momentum and frequency asymptotic form, and the (supposedly irrelevant) momentum and frequency structure of the bosonic self-interaction was disregarded. This leads to a relatively simple Hertz action [2,3,6]

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0}\left[m^2 + Z\vec{q}^2 + A\frac{|q_0|}{|\vec{q}|}\right]\phi_{\vec{q},q_0} + u\int_x \phi(x)^4, \tag{1}$$

describing the propagation of a damped collective bosonic mode ϕ, where the interaction with fermions is described by the so-called Landau damping term $\sim|q_0|/|\vec{q}|$, q_0 being the frequency and $\vec{q}$ the momentum of the order parameter field. Here $\{m^2, Z, A, u\}$ are constants, $q := (q_0, \vec{q})$, $\int_q := 1/(2\pi)^3\int dq_0 \int d^2q$, and $\int_x := \int d\tau \int d^2x$ encompasses integration over space and the imaginary time τ. The form $\sim|q_0|/|\vec{q}|$ is valid for instabilities occurring at ordering wave vector $\vec{Q} = \vec{0}$.

In contrast to classical statistical physics systems, this procedure involves integrating out gapless particle-hole excitations across the Fermi surface, the consequence of which becomes revealed by inspection of the nature of the momentum and frequency expansion of the bosonic interaction vertices (for example, the fermionic box diagram), which turns out to be singular at $T = 0$ [7–10]. For this reason, quantum critical Fermi systems (at least in dimensionality $d = 2$ and temperature $T = 0$) cannot be adequately described by a purely bosonic action characterized by local interactions.

The above issues motivated development of a diversity of approaches that retain the fermionic degrees of freedom, which are coupled to order parameter fluctuations [9,11–27]. These theoretical routes come with their own questions. One of these concerns the transition between the microscopic and effective low-energy action. This is transparent, for instance, in the analysis concerning generation of the damped dynamics of the bosonic mode. As emphasized in previous literature (see, in particular, Ref. [15]), appearance of the $\sim|q_0|/|\vec{q}|$ term requires that the fermions be integrated out down to the Fermi level. It is not conceivable to generate the standard Landau damping term by a Wilsonian-type renormalization group (RG) flow until the cutoff on fermions is completely removed and therefore the fermionic degrees of freedom become once and for all integrated out of the theory. As a consequence, accounting for the Landau damping (in its standard form) within such approaches requires fully dressing the boson propagator with self-energy before calculating any loops that involve internal boson lines in the coupled Bose-Fermi theory.

In the present Letter, we systematically readdress the theory of quantum criticality in Fermi systems featuring $\vec{Q} = \vec{0}$

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instabilities and develop a Wilsonian RG approach, where both the bosonic and fermionic propagators become equipped with momentum cutoffs and, upon lowering these, generate the RG flow, leading to a generalization of the Hertz action. Our goal is to develop an approximate approach fully encompassing the Hertz-Millis framework, but at the same time refraining from completely integrating femions out, such that the singular effective bosonic interactions never appear. We demonstrate that the RG flow of the bosonic properties involves a completely different, not previously recognized contribution, which encodes a structure richer than the conventional Hertz-Millis theory.

Generalized Hertz action. Our approach relies on the nonperturbative RG framework in the Wetterich formulation [28–30]. This methodology has, in recent years, led to several important insights concerning key problems of condensed matter theory, critical systems, in particular. Examples include identification of strong-coupling fixed points for the Kardar-Parisi-Zhang problem in $d > 1$ [31], resolution of the problem of dimensional reduction and its breaking for the random field Ising model [32], discovery of nonperturbative multicritical RG fixed points for the $O(N)$ models in $d = 3$ [33], and invalidation [34] of the predictions of perturbative approaches concerning nonanalyticity of the critical exponents as function of d and N. In our approach, we integrate the coupled fermionic ($\{\bar{\psi}, \psi\}$) and bosonic (ϕ) fluctuating fields out of the partition function via a renormalization group flow. The central object is the scale-dependent effective action $\Gamma^{\Lambda}[\bar{\psi}, \psi, \phi]$, which continuously interpolates between the bare effective action $S[\bar{\psi}, \psi, \phi]$ and the full effective action (free energy) when the infrared cutoff Λ is lowered from the UV scale towards zero. Below, we suppress the arguments of Γ for readability. The evolution of Γ upon varying Λ is governed by the exact Wetterich flow equation

$$\dot{\Gamma} = \beta_b + \beta_f \ , \tag{2}$$

where

$$\beta_b = \tfrac{1}{2}\mathrm{Tr}\{\dot{\mathcal{R}}_b(\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1}[1 - \tilde{\Gamma}^{(2)}_{\phi\psi}(\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}\tilde{\Gamma}^{(2)}_{\psi\phi}(\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1}]^{-1}\} \ , \tag{3}$$

$$\beta_f = \tfrac{1}{2}\mathrm{Tr}\{\dot{\mathcal{R}}_f(\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}[1 - \tilde{\Gamma}^{(2)}_{\psi\phi}(\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1}\tilde{\Gamma}^{(2)}_{\phi\psi}(\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}]^{-1}\} \ . \tag{4}$$

The quantity $\tilde{\Gamma} := \Gamma + \Delta S$ denotes the action Γ supplemented with the regulator term $\Delta S = \frac{1}{2}\Phi(\mathcal{R}\Phi^T)$, which is quadratic in the fields $\Phi = (\bar{\psi}, \psi, \phi)$ and contains bosonic ($\mathcal{R}_b$) and fermonic ($\mathcal{R}_f$) components. The quantity $\tilde{\Gamma}^{(2)}$ denotes the second (functional) field derivative of $\tilde{\Gamma}$ with the relevant fields specified by the subscript in each case. By $\dot{X}$, we mean $\partial_{\Lambda}X$. Finally, the trace (Tr) sums over the field components, momenta, and frequencies. Our notation is equivalent to that introduced in Ref. [35] (for details and derivations, see also Ref. [36]) with the exception that ϕ is a real scalar in our case. Differentiating the flow equation [Eq. (2)] with respect to fields gives rise to a hierarchy of flow equations for the one-particle irreducible vertex functions. We concentrate on the RG flow equation for the bosonic two-point function, obtained by taking the second functional derivative of Eq. (2) with respect to ϕ. The resulting equation [29,30,35,36] involves terms represented via one-loop Feynman diagrams depicted in Fig. 1.

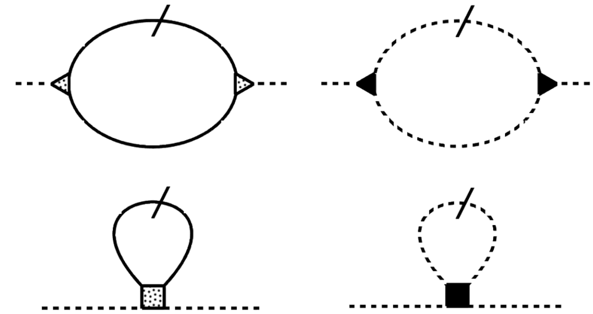

FIG. 1. Terms contributing to the flow of the bosonic two-point function. Dressed (scale-dependent) fermion and boson propagators are depicted as full and dashed lines, respectively. Black triangles and rectangles represent the bosonic vertices, while dotted (grey) triangles and rectangles stand for fermion-boson interactions. The stroked lines represent the single-scale propagators: $\mathcal{S}_f$ for fermion and $\mathcal{S}_b$ for boson propagators (see the main text).

The expressions involve the flowing fermion propagator $\tilde{G} := (\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}$ supplemented with a momentum cutoff $R^{\Lambda}_f(\vec{k})$:

$$\tilde{G}_{k,k',\sigma,\sigma'} = \left(-ik_0 + \xi_{\vec{k}} + R^{\Lambda}_f(\vec{k}) + \Sigma^{\Lambda}(k)\right)^{-1}\delta_{k,k'}\delta_{\sigma,\sigma'} \tag{5}$$

[with $k := (k_0, \vec{k})$]; the flowing regularized boson propagator $\tilde{G}_b := (\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1}$, interaction vertices, as well as the so-called single-scale propagators defined as

$$\mathcal{S}_f := -(\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}\dot{\mathcal{R}}_f(\tilde{\Gamma}^{(2)}_{\psi\psi})^{-1}, \tag{6}$$

$$\mathcal{S}_b := -(\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1}\dot{\mathcal{R}}_b(\tilde{\Gamma}^{(2)}_{\phi\phi})^{-1} \ . \tag{7}$$

In Fig. 1, the single-scale propagators correspond to stroked lines. The bare (microscopic) action contains only contributions quadratic in fields and a Yukawa-type term coupling the bosonic field with two fermionic variables [6]. In addition, the bare boson propagator carries no momentum and frequency dependence. These dependencies are generated by gradually integrating the fermions out via the contribution to the flow given by the first diagram in Fig. 1.

The flow parameter Λ appearing in the Wetterich equation is identified with the bosonic momentum cutoff ($\Lambda_b = \Lambda$). The precise form of the bosonic cutoff will be specified later. We will use the following form of the cutoff function on fermions:

$$R_f(\vec{k}) = \begin{cases} (\xi_{k_F+\Lambda_F} - \xi_{\vec{k}})\theta(\Lambda_F - (|\vec{k}| - k_F)) & \text{for } |\vec{k}| \geqslant k_F \\ (\xi_{k_F-\Lambda_F} - \xi_{\vec{k}})\theta(\Lambda_F - (k_F - |\vec{k}|)) & \text{for } |\vec{k}| < k_F \ . \end{cases} \tag{8}$$

The quantity $\Lambda_F = \Lambda_F(\Lambda)$ is a function of Λ. The effect of adding $R_f(\vec{k})$ to the dispersion $\xi_{\vec{k}}$ amounts to deforming it in a sliver of extension $2\Lambda_F$ around the Fermi level, as depicted in Fig. 2. We expect that our conclusions are completely insensitive to the precise choice of the momentum cutoff function $R_f(\vec{k})$.

The Hertz-like approach corresponds in our framework to sending Λ_F to zero before Λ. This can be realized, e.g., by

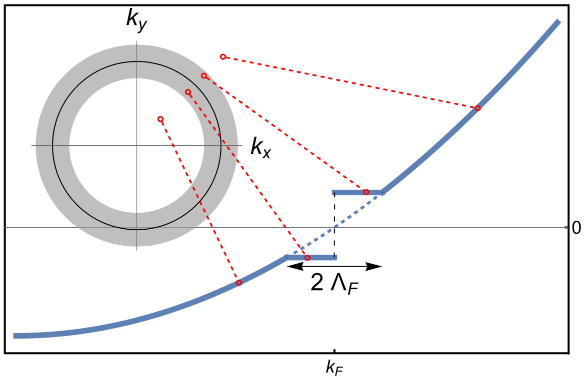

FIG. 2. A schematic plot of the regularized dispersion $\xi_{\vec{k}} + R_f(\vec{k})$. Including the regulator introduces a deformation of the dispersion in a strip of extension $2\Lambda_F(\Lambda)$ around the Fermi level. In the inset, the black line represents the Fermi surface and gray shell designates the area of the deformation.

taking

$$\Lambda_F = (\Lambda - \Lambda_0)\theta(\Lambda - \Lambda_0), \tag{9}$$

with $\Lambda_0 > 0$, such that Λ_F becomes zero at positive Λ. In what follows, we will perform a detailed comparison between the pictures emergent for $\Lambda_0 = 0$ and $\Lambda_0 > 0$ (Hertz-Millis case).

The flow equation represented by the terms depicted in Fig. 1 is exact, but can be solved only approximately. Its present truncation is devised such that it encompasses the Hertz-Millis theory if Λ_F is scaled to zero first [e.g., when one takes $\Lambda_0 > 0$ in Eq. (9)], but does not require this in any way. The gradual generation of the dynamics of the boson propagator can be followed upon reducing Λ towards zero. The key present approximation amounts to disregarding the Fermi self-energy $[\Sigma^\Lambda(k) = 0]$ and the flow of the Yukawa coupling g as well as other fermionic interactions generated by the flow. This allows us to write the contribution to the flow of the boson propagator represented by the first diagram in Fig. 1 as

$$\mathcal{X}(q, \Lambda_F) = -2g^2 \int_k \partial_\Lambda R_f(\vec{k}) \tilde{G}_0(k)^2 (\tilde{G}_0(k+q) + \tilde{G}_0(k-q)), \tag{10}$$

where the scale-dependent (regularized) fermion propagator is given by $\tilde{G}_0(k)^{-1} = [-ik_0 + \xi_{\vec{k}} + R_f(\vec{k})]$. To simplify the calculations and highlight the theoritical insight clearly, we employ the standard quadratic dispersion, $\xi_{\vec{k}} = (\vec{k}^2 - k_F^2)/2m_f$. We then evaluate the integrals in Eq. (10) and subsequently integrate over the cutoff scale, which results in the momentum and frequency structure of the boson propagator (generated from integrating the fermions from the UV cutoff scale Λ_u down to the scale Λ). Computing

$$B(\vec{q}, q_0, \Lambda_F(\Lambda)) := \int_{\Lambda_u}^{\Lambda} d\Lambda' \mathcal{X}\,, \tag{11}$$

we obtain

$$B(\vec{q}, q_0, \Lambda_F) = B_< \theta(-|\vec{q}| + \Lambda_F) + B_> \theta(|\vec{q}| - \Lambda_F)\,, \tag{12}$$

where

$$B_< = -\mathcal{N}_< \frac{|\vec{q}|\Lambda_F}{q_0^2 + 4v_F^2\Lambda_F^2}\,, \tag{13}$$

$$B_> \approx -\mathcal{N}_< \frac{\vec{q}^2}{q_0^2 + 4v_F^2\vec{q}^2} + \mathcal{N}_> \frac{q_0}{|\vec{q}|}\left[\arctan\frac{2v_F|\vec{q}|}{q_0} - \arctan\frac{2v_F\Lambda_F}{q_0}\right], \tag{14}$$

and $\mathcal{N}_< = \mathcal{N}_> v_F^3 = 4g^2 k_F v_F/\pi^2$ [37]. The above expressions are essential for the present Letter. Equation (13) follows from exactly evaluating the integrals of Eq. (10) for $|\vec{q}| < \Lambda_F$ and subsequently integrating over the cutoff scale according to Eq. (11). Equation (14) results from evaluating Eq. (10) for $|\vec{q}| \gg \Lambda_F$ retaining the terms, which generate the standard Landau damping $\sim |q_0|/|\vec{q}|$ if we first take $\Lambda_F \to 0$ and subsequently consider $\frac{|\vec{q}|}{q_0} \to \pm\infty$; the dropped terms are regular in q in the limit $\Lambda_F \to 0$ and we make no assumptions concerning the relative magnitude of $|q_0|$ and $v_F|\vec{q}|$. Concerning the structure of $B(\vec{q}, q_0, \Lambda_F)$, we emphasize that (i) for $\Lambda_F \to 0$ it recovers, via $B_>$, the standard Landau damping term of the Hertz action and (ii) it takes minimum at $(q_0, |\vec{q}|) = (0, \Lambda_F)$, which indicates that the ordering wave vector depends on the cutoff scale and falls at $|\vec{Q}_\Lambda| = Q_\Lambda = \Lambda_F$, thus scaling to zero under RG. Note, in particular, that artificially putting $Q_\Lambda = 0$ suppresses the flow of the mass generated from fermionic bubble. This explains (and evades) the unwelcome features of the mass flow under the Wilsonian RG, discussed in Ref. [15]. Observe that the mass flow is generated from $B_<$ [evaluated at $(q_0, |\vec{q}|) = (0, \Lambda_F)$]. In the present generalization of the Hertz-Millis approach, we will parametrize the flowing inverse boson propagator as

$$\Gamma^{(2)}_\Lambda = Z(|\vec{q}| - Q_\Lambda)^2 + Aq_0^2 + m_\Lambda^2 + B(\vec{q}, q_0, \Lambda_F(\Lambda))\,. \tag{15}$$

The essential modification of the standard Hertz action amounts to replacing the term $\sim |q_0|/|\vec{q}|$ occurring in Eq. (1) with the formula $B(\vec{q}, q_0, \Lambda_F)$ obtained above, such that fermionic fluctuations are included only down to the scale $\Lambda_F(\Lambda)$ (which is sent to zero as $\Lambda \to 0$). We emphasize that the RG flow of the boson propagator will be strongly influenced by the first term in Eq. (12), corresponding to $|\vec{q}|$ small.

The dynamical exponent. We now examine the consequences of the term $B_<$ for the dynamical exponent z of the order parameter field. If we first take $\Lambda_F \to 0$ setting $\Lambda_0 > 0$ in Eq. (9) [thus removing the term $B_<$ from B in Eq. (15)], and subsequently consider the limit $v_F|\vec{q}|/q_0 \to \pm\infty$, we recover the Hertz result $z = z_H = 3$, proceeding along the standard path [6].

The situation radically changes if we instead integrate both bosons and fermions in parallel by considering Eq. (9) with $\Lambda_0 = 0$ ($\Lambda_F = \Lambda$), in which case $B_<$ plays a prominent role. The anticipated value of z resulting from the q_0 dependence of $B_<$ can be deduced by putting $|\vec{q}| = \Lambda$ in $B_<$ and expanding for $q_0 \ll 2v_F\Lambda$. The leading term renormalizes the mass m_Λ^2 in Eq. (15) and the second term is proportional to q_0^2/Λ^2. We find that the $B_<$ term in $\Gamma^{(2)}_\Lambda$ scales as Λ^2 (thus leading to scale invariant propagator) provided $q_0 \sim |\vec{q}|^2$, which corresponds

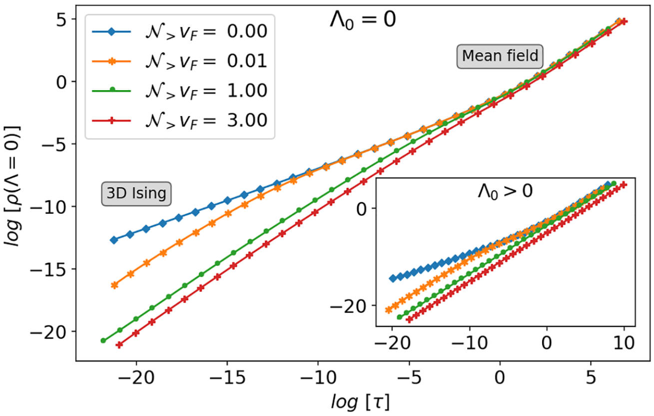

FIG. 3. The renormalized value of $\rho = \phi_0^2/2$ plotted as function of the control parameter τ (bare mass) at $T = 0$ for a sequence of values of the boson-fermion coupling g ($\mathcal{N}_> v_F \propto g^2$). The uppermost curve (blue) exhibits crossover between scaling pertinent to the classical 3D Ising universality class and mean-field scaling outside the true critical region. Upon gradually switching on g, the asymptotic critical scaling corresponding to efffective dimensionality $D = d + z \geqslant 4$ sets in and the system again crosses over to mean-field behavior. The behavior exhibited in the inset corresponds to $\Lambda_0 > 0$ (integrating out fermions first). There is no qualitative difference between these two cases, which demonstrates that including $B_<$ has no impact on the critical singularities of the order parameter.

to the dynamical exponent $z = z_< = 2$. We also note that the choice $\Lambda_F \sim \Lambda$ is the only one, which allows us to write $\Gamma_\Lambda^{(2)}$ given by Eq. (15) (keeping either $B_<$ or $B_>$) in a scaling form. From this above heuristic picture, one anticipates a competition between two scaling behaviors governed by $z \approx 2$ and $z \approx 3$. This is checked and confirmed by solving the RG equations as described below.

RG flow. A convenient way to extract the dynamical exponent z from the RG flow, capturing possible crossovers, is to inspect the behavior of the flowing order parameter expectation value, which follows $\phi_0^\Lambda \sim \Lambda^{z/2}$. This is, for $d = 2$, implied from $m_\Lambda^2 \sim \Lambda^2$, $u^\Lambda \sim \Lambda^{4-(d+z)}$ and the relation $m_\Lambda^2 = u^\Lambda(\phi_0^\Lambda)^2$ (see, e.g., Refs. [38,39]). Equivalently, one may invoke the scaling dimension of the ϕ field $[\phi^2] = d + z - 2 + \eta$, which gives z for $d = 2$. Here we neglect the anomalous dimension η.

We evaluate the flow [37] of the boson order parameter ϕ_0^Λ and quartic coupling u^Λ within a simple truncation of the Wetterich equation, where the bosonic propagator is dressed as dictated by Eq. (15). We include [39,40] the renormalization of u via bosonic fluctuations of order $\sim u^2$, which allows for also capturing the 3D Wilson-Fisher fixed point. The latter governs the critical behavior in the absence of the Fermi-Bose coupling g and gives rise to an intermediate scaling regime described by the dynamical exponent $z = 1$, as observed in Ref. [15] and also clearly captured in our approach (see Fig. 3). Within our present framework, the flow equations for ϕ_0^Λ and the quartic coupling u^Λ are derived following the standard procedure described, for example, in Refs. [38,39]. We choose the Litim cutoff [41] on bosons

$$R_b(\vec{q}) = Z[\Lambda^2 - (|\vec{q}| - Q_\Lambda)^2]\theta[\Lambda^2 - (|\vec{q}| - Q_\Lambda)^2] . \quad (16)$$

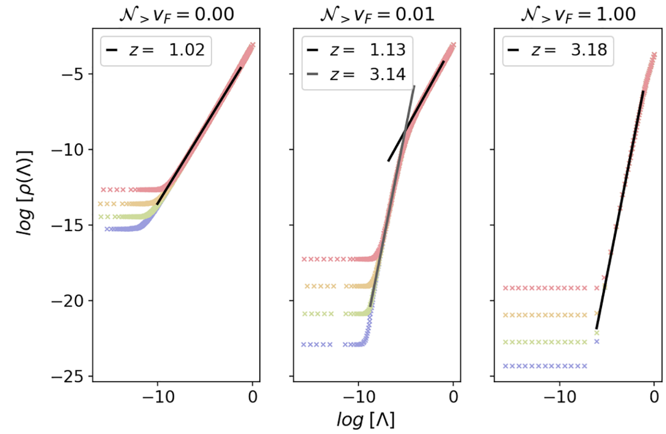

FIG. 4. The RG flows of $\rho^\Lambda = (\phi_0^\Lambda)^2/2$ for a sequence of values of τ progressively tuning the system towards the QCP in a situation, where fermions are integrated first [$\Lambda_0 > 0$ in Eq. (9)]. The value of z can be read by fitting the power law (see the main text). The crossover between $z = 1$ and $z = 3$ is clearly visible both as a function of g ($\mathcal{N}_> v_F \propto g^2$) and the cutoff scale Λ.

We verified that implementing the Wetterich cutoff [38] instead does not change any of our results. Our major conclusion concerning z is best summarized by comparing the RG flows of the order parameter depicted in Figs. 4 and 5.

Before discussing the scale dependencies arising in the RG flow, we inspect the $T = 0$ phase diagram—see Fig. 3. We observe hardly any difference between the situations corresponding to $\Lambda_0 = 0$ and $\Lambda_0 > 0$ that would be visible in the scaling of the order parameter as a function of the nonthermal control parameter τ. A similar situation may be anticipated for $T > 0$ in the behavior of the critical temperature $T_c(\tau)$. This is because (at least in the Hertz-Millis framework) one has $T_c \sim \tau^\psi$ with $\psi = z/(d + z - 2)$, which for $d = 2$ yields $\psi = 1$ irrespective of the value of z. As far as the behavior of T_c is concerned, the distinction between $z = 2$ and $z = 3$

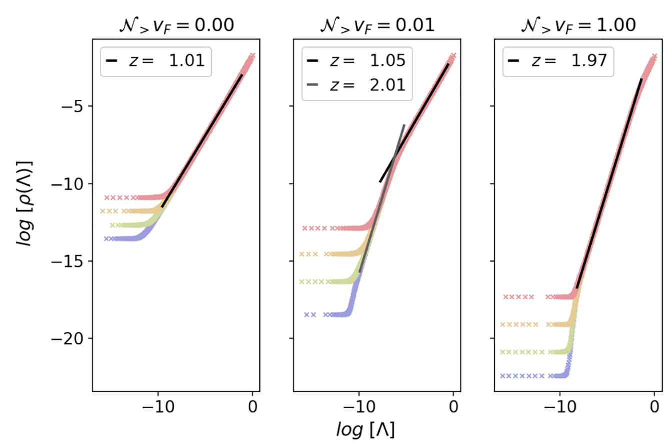

FIG. 5. The RG flows of $\rho^\Lambda = (\phi_0^\Lambda)^2/2$ for a sequence of values of τ progressively tuning the system towards the QCP in the case when $\Lambda_F = \Lambda$. The value of z can be read by fitting the power law (see the main text). The value $z \approx 2$ following from $B_<$ is clearly visible for nonzero values of the boson fermion coupling g ($\mathcal{N}_> v_F \propto g^2$).

is only revealed at the level of logarithmic corrections [3]. In a realistic (experimental or simulation) situation, the value of z may, for example, be read from the scaling behavior of the correlation function above the quantum critical point (see, e.g., Ref. [42]).

While presence of the term $B_<$ has no impact on the phase diagram (see Fig. 3), switching on g leads to scaling of the order parameter characterized by $z \approx 3$ in absence of $B_<$ and $z \approx 2$ when $B_<$ is present (compare Figs. 4 and 5). At a vanishingly low cutoff scale, deviation from the $z = 2$ scaling towards higher values of z is clearly visible in the right plot in Fig. 5. Despite high numerical accuracy, for the considered parameter values, we were, however, not able to obtain a scaling regime, which could be trustfully interpreted as capturing the behavior corresponding to $z = 3$. This emergent picture seems to be consistent with, and offer a potential explanation to, results of quantum Monte Carlo simulations of certain models of the electronic nematic [42] as well as the itinerant ferromagnetic [43] quantum critical points which provide evidence for $z = 2$-type scaling behavior, rather than the conventionally anticipated behavior corresponding to $z = 3$.

Conclusion and perspective. Within the nonperturbative RG framework, we have derived a generalization of the Hertz action pertinent to fermionic quantum critical systems in $d = 2$ characterized by the ordering wave vector $\vec{Q} = \vec{0}$. In our approach, both the fermionic and bosonic degrees of freedom are regularized and integrated out of the partition function in parallel, which uncovers another term in the corresponding order-parameter action and gives rise to a broad scaling regime characterized by the dynamical exponent $z \approx 2$. Our results indicate that a consistent formulation of the momentum-shell Wilsonian RG approach to this problem necessarily requires treating the ordering wave vector as a flowing quantity, which scales to zero exclusively in the infrared limit. The present Letter addresses only properties pertinent to the order parameter degrees of freedom and the structure of the bosonic effective action. An extension accounting for the feedback of bosonic fluctuations on fermionic properties (i.e., the flowing fermion self-energy) can be naturally achieved within our framework, but requires a fully numerical treatment and self-consistent evaluation of the interplay of the two flowing propagators.

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