Letter

Detecting symmetry fractionalization in gapped quantum spin liquids by magnetic impurities

Shuangyuan Lu and Yuan-Ming Lu
Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA

(Received 8 January 2023; accepted 8 August 2024; published 3 September 2024)

We study the Kondo effect of spin-1/2 magnetic impurities in gapped Z_2 spin liquids on two-dimensional lattices. We find that if the impurity is placed at a high-symmetry location, a nontrivial spinon fractionalization class of the impurity site symmetry group will necessarily lead to a non-Kramers doublet in the Kondo screening regime, protected by associated crystalline symmetries. This is in sharp contrast to a featureless screening phase in the usual Kondo effect. We demonstrate this symmetry-protected topological degeneracy by an exactly solvable model and by the large-N theory. Based on this effect, we discuss how thermodynamic measurements in the limit of dilute magnetic impurities can be used to detect symmetry fractionalization in gapped Z_2 spin liquids.

DOI: 10.1103/PhysRevB.110.L100401

Introduction. Quantum spin liquids (QSLs) [1–3] have attracted much interest in the past few decades due to their exotic properties transcending the Landau scheme of symmetry breaking. In particular, the presence of anyons which obey fractional statistics [4] is among the most exciting manifestations of the topological order and long-range entanglement in QSLs [5,6], with potential applications in topological quantum computations [7]. A number of QSL candidate materials with various crystalline symmetries have been discovered experimentally [3,8–10].

Meanwhile, there is a gap between theoretical diagnosis and experimental measurements to identify QSLs. On one hand, various theoretically computable quantities have been proposed to sharply characterize topological orders, such as topological entanglement entropy [11,12] and modular matrices [13,14]. On the other hand, most existing experiments aim at ruling out long-range orders in the low temperature, deterred by the difficulty of directly probing unique features of QSLs. In particular, compared to gapless U(1) spin liquids with clear signatures in inelastic neutron scattering (INS) [15] or thermal transport [3,8], a gapped symmetric QSL is more featureless and harder to detect experimentally. While the long-range entanglement and fractional statistics, as a definitive character of topological orders, is difficult to access experimentally, the fractional symmetry quantum number [16] (formally known as symmetry fractionalization [5,17–20]) of anyons provide extra features to characterize and identify the topological order, which is usually easier to probe experimentally. In the well-known example of fractional quantum Hall effects (FQHEs), indeed the fractional charge is experimentally observed in the nineties, much earlier than the recently confirmed fractional statistics [21]. One question arises naturally: can symmetry fractionalization be experimentally detected as a direct evidence for a QSL state?

For gapped QSLs, which is the focus of this work, there are two major challenges to experimental detection of fractionalization. First of all, the fractionalized excitations such as spinons are charge neutral, therefore insensitive to charge transport probes which played a crucial role in identifying fractionalization in FQHEs [21]. Secondly, a gapped symmetric QSL usually has no features both in the bulk and on the boundary, making it very hard to access experimentally. This is unlike the U(1) spin liquids, whose gapless excitations can be probed by INS in the case of emergent photons [15], or thermal transport in the case of spinon Fermi surfaces [3,8]. Is it possible to experimentally identify a gapped QSL? Previously, INS spectroscopy has been proposed to exhibit features of fractional statistics [22] and spinon symmetry fractionalization [5,23]. Recently, two-dimensional coherent nonlinear spectroscopy has been suggested to reveal both fractionalized spinon excitations [24,25] and their fractional statistics [26] in quantum spin liquids. In this work, we look into magnetic impurities and Kondo effects in gapped QSLs, and show that they can provide distinct thermodynamic signatures of symmetry fractionalization in QSLs, in the Kondo screening regime.

The Kondo effect in QSLs has previously been studied both in theories [27–34] and in experiments [35–37], focusing on gapless QSLs. In this paper, we explore the Kondo effect in gapped Z_2 QSLs. Similar to the distinctions between Kondo effects in metals and in insulators (with a vanishing density of states), the Kondo effect in gapped Z_2 QSLs differs qualitatively from gapless QSLs. In particular, due to the energy gap for spinon excitations, there is a finite threshold of Kondo coupling strength to screen the magnetic impurity [38–43]. Most remarkably, we find that when a half-integer-spin impurity is placed at a high-symmetry location in the crystal hosting a gapped Z_2 QSL, the Kondo screening phase will feature a non-Kramers doublet localized at the impurity site, protected by fractionalized crystalline symmetries in the Z_2 QSL. This symmetry-protected degeneracy lead to distinct signatures in the thermodynamics, such as specific heat, which can serve as "smoking gun" evidence for symmetry fractionalization in a gapped QSL. This phenomena is demonstrated by an exactly solvable model and large-N parton mean-field theory, which agree with each other.

Main results. We first present the major results in Fig. 1 and Tables I–III. Consider a gapped symmetric Z_2 spin liquid on a two-dimensional lattice, whose Hilbert space consists of

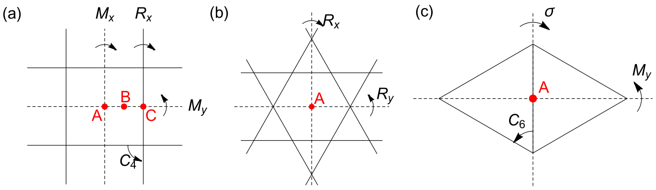

FIG. 1. High-symmetry impurity sites that can be used to detect the symmetry fractionalization of spinons in a gapped symmetric Z_2 spin liquid, on the (a) square, (b) kagome, and (c) triangular lattices in two dimensions.

a spin-1/2 (or a Kramers doublet) on each lattice site. A spin-1/2 magnetic impurity located at a certain high-symmetry position of the lattice can be used to diagnose the symmetry fractionalization class [17–19] of the Z_2 spin liquid phase. Specifically, when such a magnetic impurity is coupled symmetrically to the Z_2 spin liquid, in the Kondo screening regime, there may or may not be a twofold degeneracy (a non-Kramers doublet) protected by the crystalline symmetry of the impurity site, depending on the fractionalization class of spinons in the Z_2 spin liquid.

Figure 1 illustrates the three lattices enumerated in this work, i.e., the square, kagome, and triangular lattices. In the presence of SO(3) spin rotational symmetry, the classifications of symmetric Z_2 spin liquids on these lattices [44,45] are summarized in Tables I–III. A part of the symmetry fractionalization data can be detected by the presence or absence of non-Kramers doublets for Kondo-screened magnetic impurities located at different high-symmetry sites, such as a plaquette center, a nearest-neighbor link center, or on a mirror plane.

In a gapped system with a vanishing density of states, a finite Kondo coupling strength is required to enter the Kondo screening regime [38–43], where the system typically reaches a featureless paramagnetic ground state. The proposed non-Kramers doublet protected by crystalline symmetries at the impurity site of the Z_2 spin liquid is therefore a striking and unusual phenomenon. Below we describe the physical picture behind this observation.

TABLE I. All gapped Z_2 spin liquids of spin-1/2's on the square lattice ($G = p4m \times Z_2^{\mathcal{T}}$), characterized by 2^6 fractionalization classes [44,45], and their realizations in the Schwinger boson [46] and Abrikosov fermion [5] representations. Three of the six independent Z_2 invariants can be detected by magnetic impurities located at sites A, B, and C in Fig. 1(a).

Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	ω^e [46]	ω^ϵ [5]	Impurity site
$(R_x)^2$	ω_{R_x,R_x}	$(-1)^{p_4}$	η_σ	–
$(M_y)^2$	ω_{M_y,M_y}	$(-1)^{p_3+p_4}$	$\eta_\sigma\eta_{xpx}$	–
$(C_4R_x)^2$	$\omega_{C_4R_x,C_4R_x}$	$(-1)^{p_4+p_7}$	$\eta_\sigma\eta_{\sigma C_4}$	–
$M_xM_yM_x^{-1}M_y^{-1}$	$\frac{\omega_{M_x,M_y}}{\omega_{M_y,M_x}}$	$(-1)^{p_1}$	η_{xy}	A
$(M_y\mathcal{T})^2$	$\omega_{M_y\mathcal{T},M_y\mathcal{T}}$	$(-1)^{p_3+p_8+1}$	$-\eta_t\eta_{xpx}$	B
$R_xM_yR_x^{-1}M_y^{-1}$	$\frac{\omega_{R_x,M_y}}{\omega_{M_y,R_x}}$	$(-1)^{p_2}$	η_{xpy}	C

TABLE II. All gapped Z_2 spin liquids of spin-1/2's on the kagome lattice ($G = p6mm \times Z_2^{\mathcal{T}}$), characterized by 2^3 fractionalization classes [44,45], and their realizations in the Schwinger boson [47] and Abrikosov fermion [48] representations. One of the three independent Z_2 invariants can be detected by magnetic impurities located at site A in Fig. 1(b).

Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	ω^e [46]	ω^ϵ [5]	Impurity site
$(R_x)^2$	ω_{R_x,R_x}	$(-1)^{p_2+p_3}$	η_σ	–
$(R_y)^2$	ω_{R_y,R_y}	$(-1)^{p_2}$	$\eta_\sigma\eta_{\sigma C_6}$	–
$R_xR_yR_x^{-1}R_y^{-1}$	$\frac{\omega_{R_x,R_y}}{\omega_{R_y,R_x}}$	$(-1)^{p_1}$	η_{12}	A

A gapped Z_2 spin liquid hosts three types of anyons (or superselection sectors): bosonic spinon e, vison m, and their bound state $\epsilon = e \times m$, known as a fermionic spinon [50]. In a symmetric Z_2 spin liquid on a lattice with an odd number of spin-1/2's in each unit cell, spinons e and ϵ must carry spin-1/2 each, while vison m is spinless [51]. As a result, when a spin-1/2 magnetic impurity is coupled to such a Z_2 spin liquid, to reach a spin-singlet ground state, it can only be screened by a spinon e or ϵ. The same conclusion holds if we replace spin-1/2 by a Kramers doublet with $\mathcal{T}^2 = -1$ in the argument.

In the presence of crystalline and time reversal symmetries, different gapped Z_2 spin liquids are distinguished by their symmetry fractionalization classes, classified by second group cohomology $\mathcal{H}^2(G, \mathcal{A})$, where the symmetry group is $G = SG \times Z_2^{\mathcal{T}}$ (SG being the space group), and $\mathcal{A} = Z_2 \times Z_2$ is the fusion group of Abelian anyons in the Z_2 spin liquid [17]. Thanks to the SO(3) spin rotational symmetry, the vison fractionalization class is uniquely fixed on the three lattices [52,53], leading to the classification shown in Tables I–III [44,45], characterized by projective representations of G carried by spinons. Consequently, the singlet bound state formed by the impurity spin and the screening cloud of spinon can carry a projective representation of the impurity site symmetry group. This leads to the non-Kramers degeneracy at the impurity site, protected by both crystalline and time reversal symmetries.

There are two types of impurity sites of particular interest to this work: (1) the impurity site lies at the intersection of two mirror planes, such as sites A and C in Fig. 1; and (2) the impurity site lies on a mirror plane, such as site B in

TABLE III. All gapped Z_2 spin liquids of spin-1/2's on the triangular lattice ($G = p6mm \times Z_2^{\mathcal{T}}$), characterized by 2^3 fractionalization classes [44,45], and their realizations in the Schwinger boson [47] and Abrikosov fermion [49] representations. One of the three independent Z_2 invariants can be detected by magnetic impurities located at site A in Fig. 1(c).

Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	ω^e [46]	ω^ϵ [5]	Impurity site
σ^2	$\omega_{\sigma,\sigma}$	$(-1)^{p_2}$	η_σ	–
$(M_y)^2$	ω_{M_y,M_y}	$(-1)^{p_2+p_3}$	$\eta_{\sigma C_6}$	–
$\sigma M_y\sigma^{-1}M_y^{-1}$	$\frac{\omega_{\sigma,M_y}}{\omega_{M_y,\sigma}}$	$(-1)^{p_1}$	η_{12}	A

Fig. 1. In case (1), since the two mirror symmetries M_x and M_y commute in the Hilbert space of the impurity spin, if they anticommute on the spinon screening the impurity spin (i.e., $\omega_{M_x,M_y}/\omega_{M_y,M_x} = -1$), the bound state of impurity spin-1/2 and spinon will carry a projective representation of the site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$, leading to a twofold degeneracy protected by the mirror symmetries. On the other hand, if the two mirror actions commute on the spinon (i.e., $\omega_{M_x,M_y}/\omega_{M_y,M_x} = +1$), the bound state will carry a linear representation of the site symmetry and hence no degeneracy in the screening regime. In case (2), the impurity spin-1/2 on a mirror (M_x) plane carries a projective representation $(M_x\mathcal{T})^2 = -1$. If the spinons screening the impurity carry a linear representation with $\omega_{M_x\mathcal{T},M_x\mathcal{T}} = +1$, their bound state forms a projective representation of the site symmetry group $G_s = Z_2^{M_x} \times Z_2^{\mathcal{T}}$, leading to a twofold non-Kramers degeneracy protected by both mirror M_x and time reversal symmetries. In contrast, if the spinon carries a projective representation with $\omega_{M_x\mathcal{T},M_x\mathcal{T}} = -1$, the bound state would instead form a linear representation $(M_x\mathcal{T})^2 = +1$ with no degeneracy in the screened phase. This shows how the response of a Z_2 spin liquid to impurity spin-1/2's can diagnose a part of the spinon fractionalization data in the spin liquid phase, as summarized in Tables I–III. While this manuscript focuses on Kondo impurities in Z_2 spin liquids, the theory framework can be applied to the impurity problem in a general symmetry-enriched topological order [17–19], which we discussed in Sec. A.3 of the Supplemental Material [54] (see also Ref. [55] therein).

Methods. We use two methods to demonstrate the symmetry-protected non-Kramers degeneracy induced by a spin-1/2 impurity in Z_2 spin liquids, in the Kondo screening regime. We focus on case (1) of impurity site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$, where the impurity spin-1/2 lies at the intersection of two mirror planes $M_{x,y}$. First, we construct an exactly solvable model by modifying the toric code, to show the exact degeneracy protected by two mirror symmetries. Next, we use the large-N approach to solve the Kondo problem in a symmetric Z_2 spin liquid, and to compute the temperature dependence of thermodynamic quantities.

First, we present an exactly solvable model illustrated in Fig. 2. The bulk Z_2 spin liquid is described by

$$\hat{H}_{\text{bulk}} = -\sum_s A_s - \sum_p B_p - \sum_s \Delta\left[\frac{(A_s+1)}{2}P_s(S=0) + \frac{(1-A_s)}{2}P_s(S=1/2)\right], \tag{1}$$

where A_s and B_p are the star and plaquette operators in Kitaev's toric code [50]. In addition to one qubit on each link, there is a three-dimensional Hilbert space of spin $0 \oplus \frac{1}{2}$ on each site or vertex (see Fig. 2). In the limit $\Delta \gg 1$, each e and ϵ particle will carry spin-1/2 of the site or vertex Hilbert space, while m particles are spinless. The fractionalization class associated with the impurity site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$ is given by [17]

$$\frac{\omega^e_{M_x,M_y}}{\omega^e_{M_y,M_x}} = \frac{\omega^m_{M_x,M_y}}{\omega^m_{M_y,M_x}} = +1, \quad \frac{\omega^\epsilon_{M_x,M_y}}{\omega^\epsilon_{M_y,M_x}} = -1. \tag{2}$$

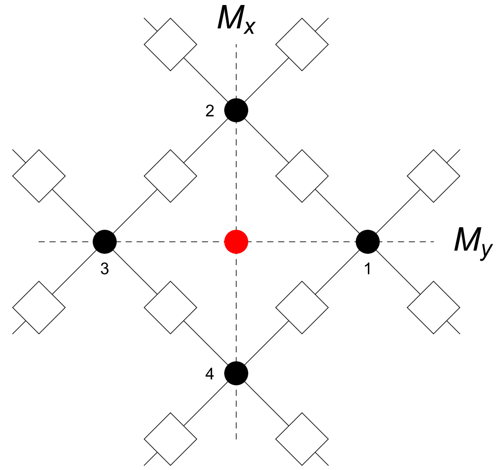

FIG. 2. An exactly solvable model for a spin-1/2 impurity in a Z_2 spin liquid with SU(2) symmetry. The squares on link centers are qubits in the toric code [50]. Each black dot on a star/vertex represents a Hilbert space of spin $0 \oplus 1/2$. The red dot denotes the spin-1/2 impurity with a site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$ [54].

Next, we introduce a spin-1/2 impurity located at the center of the plaquette (1234) in Fig. 2, which is coupled to the bulk spin liquid as follows:

$$H_{\text{imp}} = J\sum_{i=1}^{4} \vec{S}_i \cdot \vec{S}_{\text{imp}} + E_c(A_1 + A_2 + A_3 + A_4 - 3)^2 + \Delta_\epsilon B_p[1234]. \tag{3}$$

In addition to the usual Kondo coupling J, we also introduce (1) a Coulomb repulsion E_c for spinons, which makes sure the impurity is screened by one spinon, and (2) an energy Δ_ϵ coupled to the plaquette operator on the plaquette [1234], to control which type of spinons (e vs ϵ) will screen the impurity spin. Assuming $E_c \gg 1, J$, the Kondo screening regime happens when $J > 4/3$, leading to four degenerate states in the low-energy manifold where a single spinon is located at one neighboring site (out of 1,2,3,4). When $\Delta_\epsilon < 1$, the bosonic spinons cost less energy and will screen the impurity, and the fourfold degenerate can be completely lifted with a unique paramagnetic ground state without breaking any symmetry [54]. This is consistent with the trivial fractionalization class of e particles in (2). When $\Delta_\epsilon > 1$, however, the fermionic spinons cost less energy and are responsible for the Kondo screening. As detailed in the Supplemental Material [54], the four-dimensional low-energy space can be split into two doublets, each of which forms an irreducible projective representation of the impurity site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$. As a result, a twofold degeneracy protected by two mirror symmetries $M_{x,y}$ will emerge in the Kondo screening regime, as indicated by the nontrivial fractionalization class (2) of ϵ particles. Therefore, we have demonstrated the correspondence between nontrivial fractionalization class of spinons screening the impurity, and protected twofold degeneracy in the Kondo screening regime.

Next, we use a large-N mean-field theory to solve the Kondo problem in symmetric Z_2 spin liquids. Both the bulk and impurity spins are represented by fermionic partons with

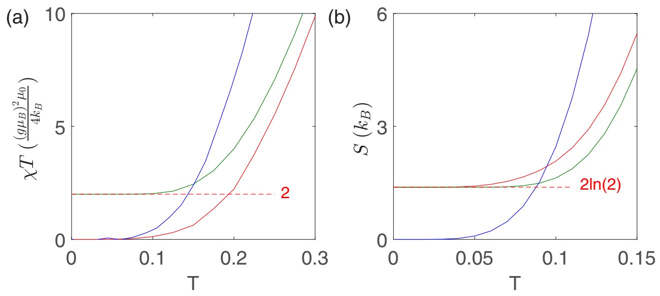

FIG. 3. The temperature dependence of (a) uniform magnetic susceptibility $\chi(T)$ and (b) entropy $S(T)$ contributed by Kondo impurities in different regimes: the unscreened regime of free moments at the impurity sites (green), the Kondo screening regime in Z_2 spin liquids with a trivial (blue) vs nontrivial (red) spinon fractionalization class. The calculations are performed for two distant impurities with site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$ on a 20×20 lattice [54].

Sp(2N) symmetry [56]:

$$S^{ab+} = \tfrac{1}{2}\big(c_\uparrow^{a\dagger} c_\downarrow^b + c_\uparrow^{b\dagger} c_\downarrow^a\big), \quad S^{ab,z} = \tfrac{1}{2}\big(c_\uparrow^{a\dagger} c_\uparrow^b - c_\downarrow^{b\dagger} c_\downarrow^a\big), \quad (4)$$

with $1 \leqslant a, b \leqslant N$. They reduce to the familiar SU(2) spin-1/2 case when $N = 1$. The model consists of a Z_2 spin liquid in the bulk described by parton mean-field ansatz of Sp(2N) partons

$$H_{\text{bulk}} = \sum_{a=1}^{N} \sum_{i,j} J_{i,j} \psi_i^{a,\dagger} u_{i,j} \psi_j^a + \text{H.c.}, \quad (5)$$

where we denote $\psi_i^a = (c_{i,\uparrow}^a, c_{i,\downarrow}^{a\dagger})^T$, and the Kondo coupling between a Sp(2N) impurity spin and its neighboring spins:

$$H_{\text{imp}} = \sum_{\langle j,\text{imp}\rangle} \frac{J}{N} \mathbf{S}_j^{ab} \cdot \mathbf{S}_{\text{imp}}^{ba} + \frac{J'}{N^3}\big(\mathbf{S}_j^{ab} \cdot \mathbf{S}_{\text{imp}}^{ba}\big)^2 \quad (6)$$

As detailed in the Supplemental Material [54] (see also Ref. [57] therein), the bulk parton ansatz can be exactly realized in solvable models in analogy to Kitaev's honeycomb model [58], and choosing different link parameters $\{u_{ij}\}$ can lead to either trivial or nontrivial fractionalization classes for fermionic spinons $\{\psi_i^a\}$, with $M_x M_y M_x^{-1} M_y^{-1} = \pm 1$. A self-consistent mean-field calculation, which becomes exact in the large N limit, reveals a Kondo screening phase separated from the unscreened phase by a Kondo temperature $T_K(J)$, for Kondo couplings beyond a finite threshold $J > J_c$ [54]. In the Kondo screening regime with $T < T_K(J)$, while the trivial fractionalization class ($M_x M_y M_x^{-1} M_y^{-1} = +1$) shows a unique paramagnetic ground state, the nontrivial fractionalization class ($M_x M_y M_x^{-1} M_y^{-1} = -1$) exhibits two degenerate ground states which cannot be mixed by any local perturbations, preserving mirror symmetries $M_{x,y}$ [54]. This again demonstrated our conclusion that a nontrivial spinon fractionalization class will lead to symmetry-protected zero modes localized at high-symmetry impurity sites.

Experimental implications. The large-N mean-field theory also allows us to predict distinct experimental signatures of the anomalous Kondo screening phase described above. The temperature dependence of uniform magnetic susceptibility $\chi(T)$ and the entropy $S(T) = \int_0^T \frac{C_v(t)}{t} dt$ are shown in Fig. 3. The impurity contribution is shown in the figure susceptibility $\chi(T)$ and specific heat $C_v(T)$, by subtracting the bulk contribution of Hamiltonian H_{bulk} from the total amount of $H_{\text{bulk}} + H_{\text{imp}}$. Three different regimes can be differentiated from each other by inspecting the susceptibility and entropy (by integrating the specific heat) at low temperatures: (1) In the unscreened regime, the magnetic impurity behaves as a free moment, leading to $\chi(T) \sim 1/T$ and a finite entropy of $k_B \ln 2$ per impurity, colored green in Fig. 3. This scenario also applies to the situations where the magnetic impurity is underscreened or overscreened, leaving a residual local moment near the impurity. (2) In the Kondo screening regime, for Z_2 spin liquid with trivial fractionalization class, Kondo screening leaves a unique paramagnetic ground state below the Kondo temperature, and therefore exponentially decaying thermodynamic responses $\chi(T), C_v(T) \sim e^{-\Delta/k_B T}$, as colored blue in Fig. 3. (3) A Z_2 spin liquid with a nontrivial fractionalization class, on the other hand, features a symmetry-protected non-Kramers doublet (twofold degeneracy) localized at each impurity site in the Kondo screening regime. As a result, while the susceptibility vanishes exponentially at low temperatures $\chi(T) \sim e^{-\Delta/k_B T}$, there is a low-energy entropy of $k_B \ln 2$ per impurity below the Kondo temperature, as colored red in Fig. 3. The sharp differences between the three scenarios provide clear experimental features to detect a gapped Z_2 spin liquid with a nontrivial symmetry fractionalization class for spinons.

Theoretically we only discussed the case of isolated impurities in the large-N self-consistent mean-field theory described earlier. In real materials, we expect our predictions in Fig. 3 to hold in the case of *dilute magnetic impurities*, where the average distance r between neighboring impurities is much larger than the bulk correlation length ξ of the gapped spin liquid. In this case, the splitting of the symmetry protected zero modes localized at impurity sites will be small $\sim J e^{-C_0 r/\xi}$, where C_0 is a constant of order one. This will lead to a peak in specific heat at low temperature $T \sim J e^{-C_0 r/\xi}$. The splitting of protected degeneracy can also be caused by crystalline symmetry breaking due to, e.g., Jahn-Teller effect, which can smear out the plateau of residue entropy in Fig. 3(b). Detailed calculations regarding broken crystalline symmetry are shown in Sec. C.5 in the Supplemental Material [54].

Among the candidate materials of quantum spin liquids, Herbertsmithite [59] is most relevant to our proposal. Nuclear magnetic resonance measurements [60] pointed to a spin liquid ground state with a finite gap of around 10 K, a small fraction of the intralayer Cu-Cu exchange interaction $J \simeq 200$ K. Previous resonant x-ray scattering results revealed 15% intersite impurities [61] between the kagome layers of Cu^{2+} ions, where excessive spin-1/2 Cu^{2+} ions occupy the interlayer A sites in Fig. 1(b) ("intersites") of Zn^{2+} ions. Neutron scattering results [62] suggested a coupling strength of about 10 K between the intersite Cu impurities, comparable to the bulk gap in the kagome layer. Therefore, Herbertsmithite is a potential material candidate our theory proposal can apply to.

Concluding remarks. In this work, we show that when magnetic impurities with half-integer spins are coupled to symmetric Z_2 spin liquids in an isotropic magnet with SU(2) symmetry, a spinon will form a singlet bound state with the impurity in the Kondo screening regime. This bound state will feature a symmetry-protected twofold degeneracy, if the spinon fractionalization class of the impurity site symmetry is nontrivial, therefore leading to a non-Kramers doublet localized at the impurity site. We further show that this local degeneracy in the Kondo screening regime can be distinguished from other scenarios by the low-temperature behaviors of magnetic susceptibility and specific heat, hence unveiling a different way to detect symmetry fractionalization in QSLs.

In the future, it will be insightful to apply the angle proposed in this work to examine the candidate materials of QSLs, where magnetic impurities are known to exist at high-symmetry sites, e.g., in Herbertsmithite [60]. Theoretically, this work also provides a different idea to probe fractionalization in models of strongly correlated electrons, by studying the impurity problem, e.g., using numerical methods.

Acknowledgments. We thank Biao Huang for discussions and related collaborations at an early stage of this work. This work is supported by the National Science Foundation (NSF) under Award No. DMR 1653769 (Y.M.L.), and by the Center for Emergent Materials at The Ohio State University, a NSF MRSEC, through NSF Award No. DMR-2011876 (S.L.).

[1] L. Balents, Spin liquids in frustrated magnets, Nature (London) **464**, 199 (2010).

[2] L. Savary and L. Balents, Quantum spin liquids: a review, Rep. Prog. Phys. **80**, 016502 (2017).

[3] Y. Zhou, K. Kanoda, and T.-K. Ng, Quantum spin liquid states, Rev. Mod. Phys. **89**, 025003 (2017).

[4] *Fractional Statistics and Anyon Superconductivity*, edited by F. Wilczek (World Scientific, Singapore, 1990).

[5] X.-G. Wen, Quantum orders and symmetric spin liquids, Phys. Rev. B **65**, 165113 (2002).

[6] X.-G. Wen, *Quantum Field Theory of Many-body Systems: From the Origin of Sound to an Origin of Light and Electrons* (Oxford University, New York, 2007).

[7] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-Abelian anyons and topological quantum computation, Rev. Mod. Phys. **80**, 1083 (2008).

[8] K. Kanoda and R. Kato, Mott physics in organic conductors with triangular lattices, Annu. Rev. Condens. Matter Phys. **2**, 167 (2011).

[9] J. Wen, S.-L. Yu, S. Li, W. Yu, and J.-X. Li, Experimental identification of quantum spin liquids, npj Quantum Mater. **4**, 12 (2019).

[10] H. Takagi, T. Takayama, G. Jackeli, G. Khaliullin, and S. E. Nagler, Concept and realization of Kitaev quantum spin liquids, Nat. Rev. Phys. **1**, 264 (2019).

[11] A. Kitaev and J. Preskill, Topological entanglement entropy, Phys. Rev. Lett. **96**, 110404 (2006).

[12] M. Levin and X.-G. Wen, Detecting topological order in a ground state wave function, Phys. Rev. Lett. **96**, 110405 (2006).

[13] E. Keski-Vakkuri and X.-G. Wen, The ground state structure and modular transformations of fractional quantum Hall states on a torus, Int. J. Mod. Phys. B **07**, 4227 (1993).

[14] Y. Zhang, T. Grover, A. Turner, M. Oshikawa, and A. Vishwanath, Quasiparticle statistics and braiding from ground-state entanglement, Phys. Rev. B **85**, 235151 (2012).

[15] O. Benton, O. Sikora, and N. Shannon, Seeing the light: Experimental signatures of emergent electromagnetism in a quantum spin ice, Phys. Rev. B **86**, 075154 (2012).

[16] R. B. Laughlin, Nobel lecture: Fractional quantization, Rev. Mod. Phys. **71**, 863 (1999).

[17] A. M. Essin and M. Hermele, Classifying fractionalization: Symmetry classification of gapped $\mathbb{Z}_2$ spin liquids in two dimensions, Phys. Rev. B **87**, 104406 (2013).

[18] N. Tarantino, N. H. Lindner, and L. Fidkowski, Symmetry fractionalization and twist defects, New J. Phys. **18**, 035006 (2016).

[19] M. Barkeshli, P. Bonderson, M. Cheng, and Z. Wang, Symmetry fractionalization, defects, and gauging of topological phases, Phys. Rev. B **100**, 115147 (2019).

[20] X. Chen, Symmetry fractionalization in two dimensional topological phases, Rev. Phys. **2**, 3 (2017).

[21] D. E. Feldman and B. I. Halperin, Fractional charge and fractional statistics in the quantum Hall effects, Rep. Prog. Phys. **84**, 076501 (2021).

[22] S. C. Morampudi, A. M. Turner, F. Pollmann, and F. Wilczek, Statistics of fractionalized excitations through threshold spectroscopy, Phys. Rev. Lett. **118**, 227201 (2017).

[23] A. M. Essin and M. Hermele, Spectroscopic signatures of crystal momentum fractionalization, Phys. Rev. B **90**, 121102(R) (2014).

[24] W. Choi, K. H. Lee, and Y. B. Kim, Theory of two-dimensional nonlinear spectroscopy for the Kitaev spin liquid, Phys. Rev. Lett. **124**, 117205 (2020).

[25] R. M. Nandkishore, W. Choi, and Y. B. Kim, Spectroscopic fingerprints of gapped quantum spin liquids, both conventional and fractonic, Phys. Rev. Res. **3**, 013254 (2021).

[26] M. McGinley, M. Fava, and S. A. Parameswaran, Signatures of fractional statistics in nonlinear pump-probe spectroscopy, Phys. Rev. Lett. **132**, 066702 (2024).

[27] G. Khaliullin and P. Fulde, Magnetic impurity in a system of correlated electrons, Phys. Rev. B **52**, 9514 (1995).

[28] K. Dhochak, R. Shankar, and V. Tripathi, Magnetic impurities in the honeycomb Kitaev model, Phys. Rev. Lett. **105**, 117201 (2010).

[29] R. L. Doretto and M. Vojta, Quantum magnets with weakly confined spinons: Multiple length scales and quantum impurities, Phys. Rev. B **80**, 024411 (2009).

[30] P. Ribeiro and P. A. Lee, Magnetic impurity in a U(1) spin liquid with a spinon fermi surface, Phys. Rev. B **83**, 235119 (2011).

[31] M. Vojta, A. K. Mitchell, and F. Zschocke, Kondo impurities in the Kitaev spin liquid: Numerical renormalization group solution and gauge-flux-driven screening, Phys. Rev. Lett. **117**, 037202 (2016).

[32] S. D. Das, K. Dhochak, and V. Tripathi, Kondo route to spin inhomogeneities in the honeycomb Kitaev model, Phys. Rev. B **94**, 024411 (2016).

[33] A. Kolezhuk, S. Sachdev, R. R. Biswas, and P. Chen, Theory of quantum impurities in spin liquids, Phys. Rev. B **74**, 165114 (2006).

[34] W.-Y. He and P. A. Lee, Magnetic impurity as a local probe of the U(1) quantum spin liquid with spinon fermi surface, Phys. Rev. B **105**, 195156 (2022).

[35] T. D. Yamamoto, H. Taniguchi, and I. Terasaki, Dynamical coupling of dilute magnetic impurities with quantum spin liquid state in the $s = \frac{3}{2}$ dimer compound $Ba_3ZnRu_2O_9$, J. Phys.: Condens. Matter **30**, 355801 (2018).

[36] M. Gomilšek, R. Žitko, M. Klanjšek, M. Pregelj, C. Baines, Y. Li, Q. M. Zhang, and A. Zorko, Kondo screening in a charge-insulating spinon metal, Nat. Phys. **15**, 754 (2019).

[37] Y. Chen, W.-Y. He, W. Ruan, J. Hwang, S. Tang, R. L. Lee, M. Wu, T. Zhu, C. Zhang, H. Ryu, F. Wang, S. G. Louie, Z.-X. Shen, S.-K. Mo, P. A. Lee, and M. F. Crommie, Evidence for a spinon Kondo effect in cobalt atoms on single-layer 1T-$TaSe_2$, Nat. Phys. **18**, 1335 (2022).

[38] D. Withoff and E. Fradkin, Phase transitions in gapless Fermi systems with magnetic impurities, Phys. Rev. Lett. **64**, 1835 (1990).

[39] K. Satori, H. Shiba, O. Sakai, and Y. Shimizu, Numerical renormalization group study of magnetic impurities in superconductors, J. Phys. Soc. Jpn. **61**, 3239 (1992).

[40] T. Saso, $1/N$ and quantum Monte Carlo studies of a magnetic impurity in an insulator with a small gap, J. Phys. Soc. Jpn. **61**, 3439 (1992).

[41] Y. Itoh, The Kondo effect for a vanishing density of states near the Fermi energy, J. Phys. Soc. Jpn. **62**, 2184 (1993).

[42] K. Takegahara, Y. Shimizu, N. Goto, and O. Sakai, Magnetic impurity in nonmetallic systems, Phys. B: Condens. Matter **186-188**, 381 (1993).

[43] K. Chen and C. Jayaprakash, Kondo effect in Fermi systems with a gap: A renormalization-group study, Phys. Rev. B **57**, 5225 (1998).

[44] Y.-M. Lu, Symmetry-protected gapless Z_2 spin liquids, Phys. Rev. B **97**, 094422 (2018).

[45] Y. Qi and M. Cheng, Classification of symmetry fractionalization in gapped $\mathbb{Z}_2$ spin liquids, Phys. Rev. B **97**, 115138 (2018).

[46] X. Yang and F. Wang, Schwinger boson spin-liquid states on square lattice, Phys. Rev. B **94**, 035160 (2016).

[47] F. Wang and A. Vishwanath, Spin-liquid states on the triangular and Kagomé lattices: A projective-symmetry-group analysis of Schwinger boson states, Phys. Rev. B **74**, 174423 (2006).

[48] Y.-M. Lu, Y. Ran, and P. A. Lee, $\mathbb{Z}_2$ spin liquids in the $s = \frac{1}{2}$ Heisenberg model on the kagome lattice: A projective symmetry-group study of Schwinger fermion mean-field states, Phys. Rev. B **83**, 224413 (2011).

[49] Y.-M. Lu, Symmetric Z_2 spin liquids and their neighboring phases on triangular lattice, Phys. Rev. B **93**, 165113 (2016).

[50] A. Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. **303**, 2 (2003).

[51] M. P. Zaletel and A. Vishwanath, Constraints on topological order in Mott insulators, Phys. Rev. Lett. **114**, 077201 (2015).

[52] Y. Qi, M. Cheng, and C. Fang, Symmetry fractionalization of visons in $\mathbb{Z}_2$ spin liquids, arXiv:1509.02927.

[53] Y.-M. Lu, G. Y. Cho, and A. Vishwanath, Unification of bosonic and fermionic theories of spin liquids on the kagome lattice, Phys. Rev. B **96**, 205150 (2017).

[54] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.110.L100401 for details on symmetry fractionalization in Z_2 spin liquids, the exactly solvable model and the large-N model.

[55] Y. Qi and L. Fu, Detecting crystal symmetry fractionalization from the ground state: Application to $\mathbb{Z}_2$ spin liquids on the kagome lattice, Phys. Rev. B **91**, 100401(R) (2015).

[56] Y. Ran and X.-G. Wen, Continuous quantum phase transitions beyond Landau's paradigm in a large-N spin model, arXiv:cond-mat/0609620.

[57] J. B. Marston and I. Affleck, Large-n limit of the Hubbard-Heisenberg model, Phys. Rev. B **39**, 11538 (1989).

[58] A. Kitaev, Anyons in an exactly solved model and beyond, Ann. Phys. **321**, 2 (2006).

[59] M. R. Norman, *Colloquium*: Herbertsmithite and the search for the quantum spin liquid, Rev. Mod. Phys. **88**, 041002 (2016).

[60] M. Fu, T. Imai, T.-H. Han, and Y. S. Lee, Evidence for a gapped spin-liquid ground state in a kagome Heisenberg antiferromagnet, Science **350**, 655 (2015).

[61] D. E. Freedman, T. H. Han, A. Prodi, P. Müller, Q.-Z. Huang, Y.-S. Chen, S. M. Webb, Y. S. Lee, T. M. McQueen, and D. G. Nocera, Site specific x-ray anomalous dispersion of the geometrically frustrated Kagomé magnet, Herbertsmithite, $ZnCu_3(OH)_6Cl_2$, J. Am. Chem. Soc. **132**, 16185 (2010).

[62] T.-H. Han, M. R. Norman, J.-J. Wen, J. A. Rodriguez-Rivera, J. S. Helton, C. Broholm, and Y. S. Lee, Correlated impurities and intrinsic spin-liquid physics in the kagome material Herbertsmithite, Phys. Rev. B **94**, 060409(R) (2016).