Erratum: de Haas–van Alphen effect and quantum oscillations as a function of temperature in correlated insulators [Phys. Rev. B 109, 235111 (2024)]

Vladimir A. Zyuzin

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In the original paper the calculation of the free energy was not performed correctly. Below we present the corrected derivation and discuss its consequence. In short, one of the four main results of the original paper must be changed; the other three results remain intact.

In terms of the notations and definitions of the original paper, the free energy in the presence of the magnetic field and at $T = 0$, in which case $e^{\frac{-\epsilon_{n;+}}{T}} = 0$ and $e^{\frac{-\epsilon_{n;-}}{T}} \gg 1$ since the $-\epsilon_{n;-} > 0$ for any n, is

$$F = \frac{\omega_{\rm B}}{2}\nu\sum_n\left(\omega_{\rm B}n + \frac{\omega_{\rm B}}{2} - \mu\right) - \frac{\omega_{\rm B}}{2}\nu\sum_n\sqrt{\left(\omega_{\rm B}n + \frac{\omega_{\rm B}}{2} - \mu\right)^2 + 4\theta^2} + \frac{\theta^2}{2U}. \tag{1}$$

The sum over the Landau levels in the second term will be made again with the help of Poisson summation formula.

$$F = F_0 + F_\Sigma, \tag{2}$$

$$F_0 = \frac{1}{2}\nu\omega_{\rm B}^2\int_{-f}^{\Lambda-f} z dz - \frac{1}{2}\nu\omega_{\rm B}^2\int_{-f}^{\Lambda-f}\sqrt{z^2+b^2}dz + \frac{\theta^2}{2U}, \tag{3}$$

$$F_\Sigma = -\frac{1}{2}\nu\omega_{\rm B}^2\sum_{p\neq 0}\int_{-f}^{\Lambda-f} e^{i2\pi p(z+f)}\sqrt{z^2+b^2}dz, \tag{4}$$

where $b = \frac{2\theta_0}{\omega_{\rm B}}$. In the limit $8\pi\frac{\theta_0}{\omega_{\rm B}} > 1$, we calculate

$$F_0 = \frac{1}{4}\nu\big[(\Lambda-f)^2\omega_{\rm B}^2 - f^2\omega_{\rm B}^2\big] - \frac{1}{4}\nu\left\{(\Lambda-f)^2\omega_{\rm B}^2 + f^2\omega_{\rm B}^2 + 4\theta^2 + 4\theta^2\ln\left[\frac{\omega_{\rm B}^2(\Lambda-f)f}{\theta^2}\right]\right\} + \frac{\theta^2}{2U} \tag{5}$$

$$= \frac{1}{4}\nu\big[(\Lambda-f)^2\omega_{\rm B}^2 - f^2\omega_{\rm B}^2\big] - \frac{1}{4}\nu\left[(\Lambda-f)^2\omega_{\rm B}^2 + f^2\omega_{\rm B}^2 + 4\theta^2 - 16\theta^2\cos(2\pi f)e^{-4\pi\frac{\theta}{\omega_{\rm B}}}\sqrt{\frac{\omega_{\rm B}}{2\pi\theta}}\right] - \frac{\theta^2}{2U} + \frac{\theta^2}{2U} \tag{6}$$

$$= -\frac{\nu}{2}\left(\mu - \frac{\omega_{\rm B}}{2}\right)^2 - \nu\theta^2 + 4\nu\theta^2\cos(2\pi f)e^{-4\pi\frac{\theta}{\omega_{\rm B}}}\sqrt{\frac{\omega_{\rm B}}{2\pi\theta}} \tag{7}$$

$$\approx -\frac{\nu}{2}\left(\mu - \frac{\omega_{\rm B}}{2}\right)^2 - \nu\theta_0^2, \tag{8}$$

where, recall $\Lambda\omega_{\rm B} = 2\mu$ and $f\omega_{\rm B} = \mu - \frac{\omega_{\rm B}}{2}$, and where we have used self-consistent equation in obtaining $-\frac{\theta^2}{2U}$ term in the first equality sign. We have applied results of $8\pi\frac{\theta_0}{\omega_{\rm B}} > 1$ limit, but other limits will also lead to the result in the last line [Eq. (8)]. This calculation of the free energy is in agreement with the BCS theory, i.e., the energy gain is equal to $-\nu\theta_0^2$. We note that the response from the noninteracting term (when hybridization is treated as a constant rather than self-consistently) is diamagnetic, although it is not immediately clear from Eq. (8).

In the original paper we have accidentally omitted the oscillating part of the gap equation when going from Eqs. (5) to (6). In other words, the last term in the second square brackets in Eq. (6) has been forgotten. As a result, last line of Eq. (8) contained $-\nu\theta^2$ rather than the correct $-\nu\theta_0^2$. As a consequence of this error a number of sentences and equations in the original paper need to be revised.

(i) The correct expression for the oscillating part of free energy at $T = 0$ is

$$\delta F_{\rm osc} = -4\nu\theta_0^2\left(\frac{\omega_{\rm B}}{2\pi\theta_0}\right)^{\frac{3}{2}}\cos\left(\frac{2\pi\mu}{\omega_{\rm B}}\right)e^{-\frac{4\pi\theta_0}{\omega_{\rm B}}}, \tag{9}$$

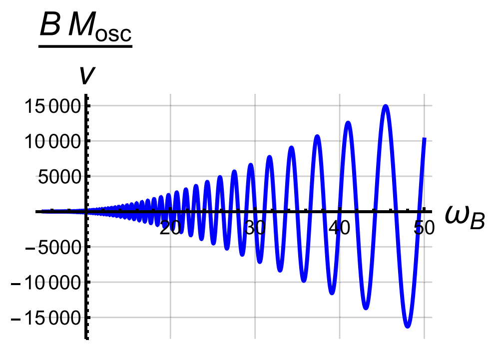

FIG. 1. Plot of the first harmonic of the oscillating part of the torque, i.e., of $M_{\rm osc}B$ where $M_{\rm osc}$ is given for $2\theta_0 \approx 40\,\mathrm{K}$, effective mass is picked to be $m = 0.2m_{\rm e}$, at $T = 0$, and $\mu m/m_{\rm e} = 420\,\mathrm{K}$, and $\nu = \frac{m}{2\pi}$ where m is an effective mass of the d-fermions.

where the leading contribution to the oscillating part of free energy originates from Eq. (4), which is the oscillating part of the noninteracting model (when θ in the Hamiltonian is inserted by hands rather than self-consistently). This confirms the theoretical result of Ref. [1]. Furthermore, Eq. (1) in the original paper must be changed to

$$M_{\rm osc} = -\frac{2\mu\nu e}{\pi mc}\left(\frac{2\pi\theta_0}{\omega_{\rm B}}\right)^{\frac{1}{2}} \sin\left(\frac{2\pi\mu}{\omega_{\rm B}}\right) e^{-4\pi\frac{\theta_0}{\omega_{\rm B}}}. \tag{10}$$

Equation (13) in the original paper must be changed to

$$\delta F_{\rm osc} = -4\nu\theta_0^2\left(\frac{\omega_{\rm B}}{2\pi\theta_0}\right)^{\frac{3}{2}} \cos\left(\frac{2\pi\mu}{\omega_{\rm B}}\right) e^{-\frac{4\pi\theta_0}{\omega_{\rm B}}}. \tag{11}$$

Equation (26) in the original paper must be changed to

$$M_{\rm osc} = -\frac{2\mu\nu e}{\pi mc}\left(\frac{2\pi\theta_0}{\omega_{\rm B}}\right)^{\frac{3}{2}} R(T) \sin\left(\frac{2\pi\mu}{\omega_{\rm B}}\right). \tag{12}$$

In other words, there is no vanishing of the amplitude of oscillations at $\omega_{\rm B} = 2\pi\theta_0$.

(ii) Figure 1 in the original paper must be changed to the one shown as Fig. 1 here. In other words, there is no vanishing of the amplitude of oscillations at $\omega_{\rm B} = 2\pi\theta_0$. Figure 7 in the Appendix in the original paper must now be replaced with Fig. 2.

(iii) As a consequence of the error two claims in the original paper must be dismissed. The first claim is a sentence regarding experiment by Ref. [2] i.e., Ref. [20] in the original paper. It turns out our results are not related to this experiment. While the other claim, which must be dismissed is our comment on the paper [1] written in the Appendix C of the original paper. Our corrected calculation of the free energy matches the one in [1]. In addition to those two changes, all credit must be given to Ref. [1] for calculation of the free energy of the Keldysh-Kopaev model [3] in magnetic field.

(iv) Abstract should thus now read: “We theoretically study a model of excitonic insulators, which show de Haas-van Alphen oscillations as well as periodic dependence of magnetization on the inverse temperature. The insulating behavior arises from the Coulomb interaction-driven hybridization of fermions at the crossing point of their energy bands. We study this hybridization self-consistently and reproduce the known results from [1]. In addition to these known results, we show that the hybridization gap decreases with the magnetic field, which corresponds to the diamagnetism. Furthermore, we show that the amplitude of the de Haas-van Alphen is oscillating with the inverse temperature with a period defined by a combination of the hybridization gap and magnetic field. We analytically determine the position and the height of the first and dominant peak of these oscillations.”

On the other hand, three other main results of the original paper remain correct. This is because the analysis leading to them is independent from the calculation of the free energy discussed above. The title of the original paper is intact since it is based on the correct result.

We repeat our other main results here.

(a) Dependence of the nonoscillating part of the gap on the magnetic field is given by $\theta_0 = \sqrt{\mu^2 - \frac{\omega_{\rm B}^2}{4} e^{-\frac{1}{4U\nu}}}$. The gap decreases with the magnetic field resulting in a diamagnetic contribution to the magnetic susceptibility.

(b) Analytical expression for the temperature dependence of the dHvA oscillations amplitude and realization that the dependence is not monotonous (what we called as the quantum oscillations as a function of temperature). This result comes from our analytic analysis of $R(T)$, defined in Eq. (17) in original paper. This behavior is illustrated in Eq. (19) and Figs. 3, 4, 5 in the original paper. We verified that the same $R(T)$ will appear in the expression for $M_{\rm osc}(T)$.

(c) Analytic expressions for the position of the first peak in the temperature dependence of the dHvA oscillations amplitude and its height. The position of the first peak is shown to be proportional to the gap θ_0. For example, see Eq. (22) in the original paper for the limit of large magnetic fields as compared to the gap and an estimate given in Eq. (21) in the original paper for the general limit. The height of the first peak is estimated in Eq. (25) in the original paper.

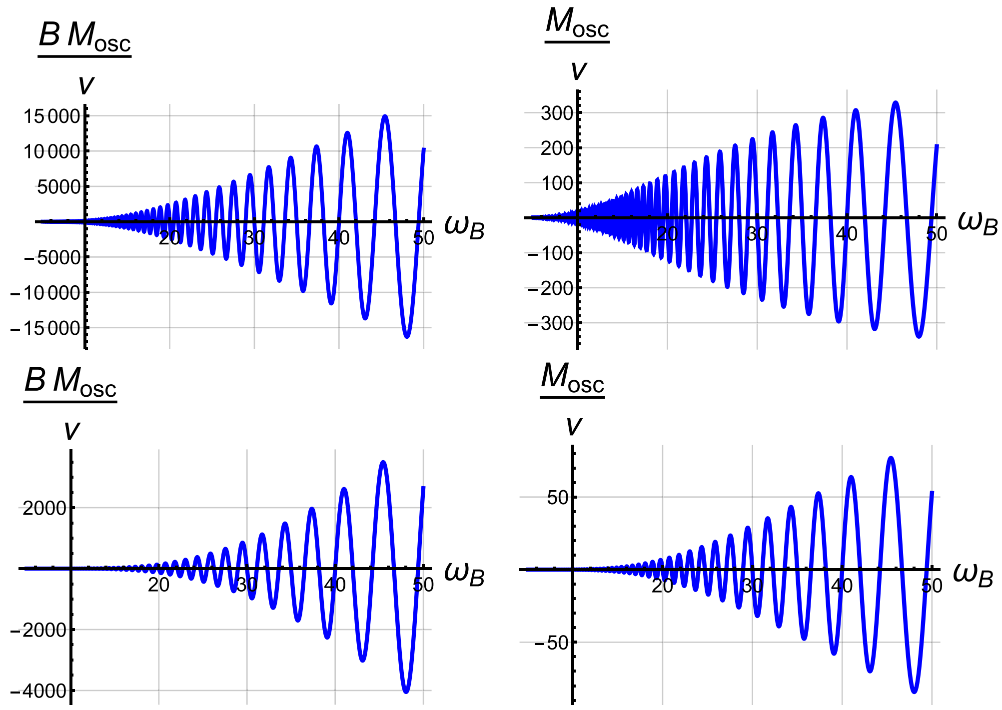

FIG. 2. (Left) Plot of the first harmonic of the oscillating part of the torque, i.e., of $M_{\mathrm{osc}}B$ where M_{osc} is given for $2\theta_0 \approx 40\,\mathrm{K}$. (Top) Effective mass is $m = 0.2m_e$; (bottom) $m = 0.4m_e$. (Right) magnetization for the same parameters. All plots are for $T = 0$ and $\mu m/m_e = 420\,\mathrm{K}$.

We note that results similar to the (b) part above have been obtained before in the literature but only numerically [4]. Analytic analysis have appeared in our original paper. The title of the original paper highlights the results of the (b) part.

[1] A. Allocca and N. Cooper, Quantum oscillations in interaction-driven insulators, SciPost Phys. **12**, 123 (2022).

[2] H. Aoki, N. Kimura, and T. Terashima, Fermi surface properties, metamagnetic transition and quantum phase transition of $CeRu_2Si_2$ and its alloys probed by the dHvA effect, J. Phys. Soc. Jpn. **83**, 072001 (2014).

[3] L. V. Keldysh and Yu. V. Kopaev, Possible instability of the semimetallic state toward Coulomb interaction, Sov. Phys. - Solid State. **6**, 2219 (1965).

[4] J. Knolle and N. R. Cooper, Quantum oscillations without a Fermi surface and the anomalous de Haas-van Alphen effect, Phys. Rev. Lett. **115**, 146401 (2015).