

Determination of the magnetic penetration depth with measurements of the vortex-penetration field for type-II superconductors

G. P. Mikitik¹ and Yu. V. Sharlai^{1,2}

¹*B. Verkin Institute for Low Temperature Physics & Engineering of the National Academy of Sciences of Ukraine, Kharkiv 61103, Ukraine*

²*Institute of Low Temperature and Structure Research, Polish Academy of Sciences, 50-422 Wrocław, Poland*



(Received 20 May 2024; accepted 25 July 2024; published 9 August 2024)

Using a known distribution of the Meissner currents over the surface of an infinitely long superconducting slab with a rectangular cross section, we find an applied magnetic field at which vortices begin to penetrate into the superconductor. This vortex-penetration field is determined by an interplay of the geometrical and Bean-Livingston barriers. The obtained results enable one to find the lower critical field and the London penetration depth from measurements of the magnetic induction on the surface of the superconducting slab, using, e.g., the micro-Hall probes.

DOI: [10.1103/PhysRevB.110.064507](https://doi.org/10.1103/PhysRevB.110.064507)

I. INTRODUCTION

A temperature dependence of the magnetic penetration depth λ can shed light on the pairing state of electrons in superconductors [1,2]. In particular, using the measured dependence of λ on the temperature T , important information on this state were obtained for high- T_c $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ [1,3] as well as for the Fe-based [4–6] and heavy-fermion [7–10] superconductors. Below we will discuss one of the methods used for the determination of λ (or of the lower critical field $H_{c1} \propto \lambda^{-2}$), viz., the measurements of the magnetic field H_p at which the vortices begin to penetrate into a type-II superconductor [5–8,11–16]. In this method, it is important to correctly take into account the shape of the superconducting sample, which in most experiments is a rectangular parallelepiped. Strictly speaking, the well-known concept of the demagnetization factor is not applicable to such platelet-shaped superconductors since the geometrical barrier [17] appears in samples different from ellipsoids, and the uniform penetration of the vortices into the superconductor no longer occurs. Although the so-called effective demagnetization factor N of a superconductor in the Meissner state is frequently discussed in the literature [18–22], it does not determine the penetration field. This N is defined by the condition that $-VH_a/(1-N)$ is equal to the total magnetic moment of a sample of the volume V in the applied magnetic field $H_a < H_p$.

The vortex-penetration field H_p for a thin strip was estimated in Ref. [17], and it was shown that due to the geometrical barrier, this H_p essentially exceeds the field $(1-N)H_{c1}$, at which the vortex penetration into an ellipsoid-shaped sample occurs. Based on numerical calculations of the electrostatics of vortices in infinite slabs with rectangular

cross sections, Brandt [18] found a formula for H_p that approximately describes this field for an *isotropic* superconductor in a wide interval of the aspect ratios of the slabs. On the other hand, using the method of conformal mappings in the magnetostatics [23], a distribution of currents over the surface of the infinitely long slab with a rectangular cross section was derived in the case when the slab is in the Meissner state [24]. This strict result was obtained under the only assumption that λ is much less than the width $2w$ and the thickness d of the sample. Using this result, a formula for the penetration field H_p was derived in the case of thin strips with $d \ll 2w$, and an interplay between the Bean-Livingston and geometrical barriers was also analyzed [25]. Since the result for the currents [24] is valid for any $d/2w$, in this paper, we present formulas for H_p in the case of an arbitrary aspect ratio $d/2w$ of the sample, taking into account an anisotropy of the superconductor as well. The vortex pinning is assumed to be negligible (otherwise, the penetration field depends also on the pinning, and λ cannot be accurately found).

The paper is structured as follows. In Sec. II we present the distribution of the currents in the infinitely long slab of the rectangular cross section when the slab is in the Meissner state. Using this distribution, in Sec. III the Bean-Livingston and geometrical barrier are analyzed for anisotropic slab of the arbitrary thickness, and a simple algorithm for the calculation of H_p is proposed. Within this approach, the dependences of penetration field on the aspect ratio of the slab and on the anisotropy of the superconductor are considered in Sec. IV. In Sec. V, we discuss how λ or H_{c1} can be extracted from various experimental data on the vortex penetration into the superconducting slab. The obtained results are briefly summarized in Sec. VI, whereas the Appendixes contain some mathematical details of the calculations.

II. SLAB IN THE MEISSNER STATE

Consider a superconducting slab of a rectangular cross section of width $2w$ ($-w \leq x \leq w$) and thickness

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

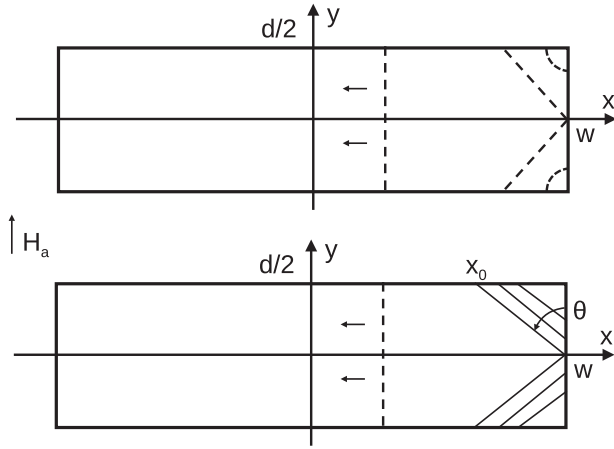


FIG. 1. Two scenarios of the vortex penetration into the infinitely long superconducting slab of the rectangular cross section. The thickness of the slab is d , and its width is $2w$. Top: $p > p_c$, the Bean-Livingston barrier prevails over the geometrical one. Bottom: $p < p_c$, the penetration of vortices is mainly determined by the geometrical barrier. The parameter p is defined by Eq. (22), its critical value p_c depends on the aspect ratio $d/2w$ of the slab and the anisotropy of the superconductor ε , Eq. (23). The dashed lines schematically show mobile vortices in the slab, whereas the solid lines inside the slab designate the immobile vortices that are in the equilibrium. These inclined vortices form the flux-line domes on the upper (lower) plane of the slab and on its right (left) lateral surface.

d ($-d/2 \leq y \leq d/2$), which infinitely extends in the z direction (Fig. 1). The slab is subjected to a perpendicular applied magnetic field $\mathbf{H}_a = (0, H_a, 0)$. It is always assumed below that $d, w \gg \lambda$.

When a superconductor is in the Meissner state, the total magnetic field \mathbf{H} at its surface is tangential to this surface. This $\mathbf{H}(x, y)$ outside the sample, and hence the Meissner sheet currents $J_M = J_z$ flowing in the surface layer of the thickness $\sim \lambda$, can be found by a conformal mapping [23]. For the slab, the Meissner currents were obtained in Ref. [24] (the appropriate mapping was detailed in the Supplemental Material to Ref. [26]). We now present the results of Ref. [24] that will be necessary in subsequent calculations.

Due to the symmetry of the slab, it is sufficient to deal with a quarter of its surface ($x \geq 0, y \geq 0$) and to parametrize the surface with the single variable u changing from 0 to $1/\sqrt{1-m}$ [26]. Here m is a constant parameter, $0 \leq m \leq 1$, the value of which is determined by the aspect ratio of the slab, $d/2w$. In particular, the upper surface of the slab ($0 \leq x \leq w, y = d/2$) is parametrized as follows ($0 \leq u \leq 1$):

$$\frac{x}{w} = \frac{f(u, 1-m)}{f(1, 1-m)}, \quad (1)$$

where

$$f(u, m) \equiv m \int_0^u \frac{\sqrt{1-v^2}}{\sqrt{1-mv^2}} dv. \quad (2)$$

The points $u = 1$ correspond to the upper corner of the slab, $(w, d/2)$. The constant parameter m is found from the equation:

$$\frac{d}{2w} = \frac{f(1, m)}{f(1, 1-m)}. \quad (3)$$

At $d \ll w$, relation (3) gives

$$m \approx \frac{2d}{\pi w}. \quad (4)$$

The upper part of the lateral surface, ($x = w, 0 \leq y \leq d/2$), has the following parametric representation ($1 \leq u \leq 1/\sqrt{1-m}$):

$$\frac{2y}{d} = \frac{f(s(u), m)}{f(1, m)}, \quad (5)$$

where

$$s(u) = \sqrt{\frac{1 - (1-m)u^2}{m}}.$$

The value $u = 1/\sqrt{1-m}$ corresponds to the equatorial point $(w, 0)$ of the slab.

The Meissner sheet currents on the upper and lateral surfaces of the slab (i.e., in the whole interval $0 \leq u \leq 1/\sqrt{1-m}$) are described by the unified formula:

$$J_M(u) = \frac{uH_a}{\sqrt{|1-u^2|}}. \quad (6)$$

Formulas (1)–(6) provide the quantitative description (in the parametric form) of the surface Meissner currents in the slab, including its edge regions. Note that the above formulas describe both the case of a thin strip in the perpendicular magnetic field ($d \ll 2w$, or equivalently, $m \ll 1$) and the case of a plate in the magnetic field parallel to its surface ($d \gg 2w$, or $1-m \ll 1$).

In the limit $|1-u^2| \ll m$, i.e., at $l \equiv w-x \ll d, w$, or at $l \equiv (d/2) - y \ll d, w$, the surface current diverges like $l^{-1/3}$ near the corners of the slab [24,25]. In this limiting case formulas (1)–(3) and (6) lead to the expression

$$J_M \approx H_a \left(\frac{(1-m)d}{6\sqrt{m}f(1, m)l} \right)^{1/3}, \quad (7)$$

which is valid for the slab of an arbitrary thickness. For the thin strips, expression (7) is further simplified since $f(1, m) \approx \pi m/4$ at $m \ll 1$. The divergence of the current in Eq. (7) should be cut off at $l \lesssim \lambda$, and the current density j throughout the corner region [$w - \lambda \leq x \leq w, (d/2) - \lambda \leq y \leq d/2$] is approximately constant, $j_{\text{cm}}(x, y) \sim J_M(x = w - \lambda)/\lambda$,

$$j_{\text{cm}} \sim \frac{H_a}{\lambda} \left(\frac{(1-m)d}{6\sqrt{m}f(1, m)\lambda} \right)^{1/3}. \quad (8)$$

In the case of the thin strip, formula (8) reduces to the appropriate expression of Ref. [25].

III. BEAN-LIVINGSTON AND GEOMETRICAL BARRIERS IN THE SLAB

A. Bean-Livingston barrier

Since the Meissner currents are maximum at the corners of the sample, it is favorable for a vortex to penetrate into the strip through these points. A small circular vortex arc appearing in one of the corners overcomes the Bean-Livingston barrier and begins to expand when the current density in the

corners reaches the value j_0 [25],

$$j_0 \approx \frac{0.92H_{c1}\kappa}{\lambda \ln \kappa}, \quad (9)$$

where $H_{c1} = \Phi_0 \ln \kappa / (4\pi \mu_0 \lambda^2)$ is the lower critical field, Φ_0 is the flux quantum, and κ is the Ginzburg-Landau parameter. This j_0 is of the order of the depairing current density j_{dp} [27], whereas $j_0 \lambda$, the local surface field near the corner, reaches the value of the thermodynamic critical field H_c in the agreement with the results of Refs. [28–31]. Equating this j_0 with the current density j_{cm} defined by Eqs. (8), we find the order of magnitude of the applied field at which the Bean-Livingston barrier disappears for a vortex penetrating through the corner of the sample,

$$\begin{aligned} H_p^{\text{BL}} &= \frac{0.92\kappa H_{c1}}{\ln \kappa} \left(\frac{6\sqrt{m}f(1, m)\lambda}{(1-m)d} \right)^{1/3} \\ &= \frac{0.92\kappa H_{c1}}{\ln \kappa} \left(\frac{3\sqrt{m}f(1, 1-m)\lambda}{(1-m)w} \right)^{1/3}. \end{aligned} \quad (10)$$

At $d \ll w$, formula (10) reduces to the appropriate expression of Ref. [25].

If the superconductor is anisotropic, we will assume that the axis of the anisotropy coincides with the y axis. This means that λ_y differs from the London penetration depth $\lambda_x = \lambda_z \equiv \lambda$ characteristic of the currents in the plane perpendicular to the axis. The anisotropy parameter ε is defined as follows: $\varepsilon \equiv \lambda/\lambda_y$ [27]. (We imply below that $\varepsilon < 1$.) Since the field H_p^{BL} is determined by the current density $j_0 \sim j_{dp} \sim H_c/\lambda$ flowing along the z axis, this field is expected to be practically independent of ε .

B. Geometrical barrier

Consider now the vortex-entry condition caused by the geometric barrier in the slab, assuming that a vortex has already overcome the Bean-Livingston barrier in the corner of the slab. In this case a penetrating vortex can move towards the center of the sample only when its two inclined rectilinear segments meet at the right equatorial point ($x = w, y = 0$), see Fig. 1. Consider the vortex segment, which ends at the point x_0 of the upper plane of the slab and at the point y_0 of its lateral surface. The balance between the line tension of the vortex and the forces generated by the surface currents leads to the following equations in x_0, y_0 [25]:

$$\Phi_0 J(x_0, d/2) = \frac{\delta E_l}{\delta x_0}, \quad (11)$$

$$\Phi_0 J(w, y_0) = \frac{\delta E_l}{\delta y_0}, \quad (12)$$

where Φ_0 is the flux quantum, the sheet currents $J(x, y)$ are determined by formulas (1)–(6), $E_l = Le_l$ is the line energy of a vortex segment of the length L . In an anisotropic superconductor, one has [27]:

$$e_l = \frac{\Phi_0^2 \varepsilon(\theta)}{4\pi \mu_0 \lambda^2} \ln \left(\frac{\kappa}{\varepsilon(\theta)} \right), \quad (13)$$

where $\varepsilon(\theta) = \sqrt{\cos^2 \theta + \varepsilon^2 \sin^2 \theta}$, ε is the parameter of the anisotropy ($\varepsilon \leq 1$), and $\theta < \pi/2$ is the tilt angle of the vortex relative to the y axis, the axis of the anisotropy. There is also

a geometrical relationship between x_0, y_0 , and θ , which is evident from Fig. 1:

$$w - x_0 = \left(\frac{d}{2} - y_0 \right) \tan \theta. \quad (14)$$

Equations (11)–(14) completely determine the three quantities x_0, y_0 , and θ , and these equations in the explicit form are presented in Appendix A. If we set $y_0 = 0$, the equations give x_0 , the appropriate angle θ_0 , and the penetration field H_p^{GB} caused by the geometrical barrier.

Below we will neglect the angular dependence of the logarithmic factor in formula (13) for e_l [i.e., we set $\varepsilon(\theta) = 1$ in the logarithmic factor]. Under this assumption and at $y_0 = 0$, the explicit form of Eqs. (11) and (12) looks as follows:

$$\frac{\varepsilon^2 \sin \theta_0}{\varepsilon(\theta_0)} = \frac{H_a}{H_{c1}} \frac{u_0}{\sqrt{1 - u_0^2}}, \quad (15)$$

$$\frac{\cos \theta_0}{\varepsilon(\theta_0)} = \frac{H_a}{H_{c1} \sqrt{m}}, \quad (16)$$

where the parameter u_0 corresponds to the point x_0 according to Eq. (1), and $H_{c1} = e_l(\theta = 0)/\Phi_0$ is the lower critical field of the superconducting material for the magnetic field parallel to the y axis. With the use of formula (1), the geometrical relationship (14) at $y_0 = 0$ takes on the form:

$$\frac{d}{2w} \tan \theta_0 = \frac{f(1, 1-m) - f(u_0, 1-m)}{f(1, 1-m)}. \quad (17)$$

To solve Eqs. (15)–(17), we note that the ratio of formulas (15) and (16) enables us to express $\tan \theta_0$ in terms of u_0 . Inserting this expression into formula (17) and using relation (3), we arrive at the equation determining u_0 as a function of the parameters m and ε ,

$$\frac{\sqrt{m}u_0}{\varepsilon^2 \sqrt{1 - u_0^2}} = \frac{f(1, 1-m) - f(u_0, 1-m)}{f(1, m)}. \quad (18)$$

This simple equation [see formula (2)] has a unique solution since its left-hand side increases with increasing u_0 , while the right-hand side decreases. Knowing u_0 , we find θ_0 from formula (17), whereas Eq. (16) gives the penetration field caused by the geometrical barrier,

$$H_p^{\text{GB}} = H_{c1} \sqrt{m} \frac{\cos \theta_0}{\varepsilon(\theta_0)} = \frac{H_{c1} \sqrt{m}}{\sqrt{1 + \varepsilon^2 \tan^2 \theta_0}}. \quad (19)$$

Consider the two special cases. In the case of a thick anisotropic slab, for which $d/2w \gtrsim 1$ (i.e., $m \sim 1/2$) and $\varepsilon^2 \ll 1$, we assume that the parameter $u_0 \ll 1$. Then, we find $\tan \theta_0 \approx 2w/d$ from Eq. (17), whereas the ratio of Eqs. (15), (16) gives $u_0 \approx \varepsilon^2 \tan \theta_0 / \sqrt{m}$. Thus, our assumption, $u_0 \ll 1$, is really fulfilled if $\varepsilon^2 \ll 1$. In this case, formula (19) is practically independent of ε and reduces to

$$H_p^{\text{GB}} \approx H_{c1} \sqrt{m}. \quad (20)$$

In the case of a thin isotropic strip when $\varepsilon = 1$ and $m \ll 1$, the appropriate angle θ_0 is almost independent of

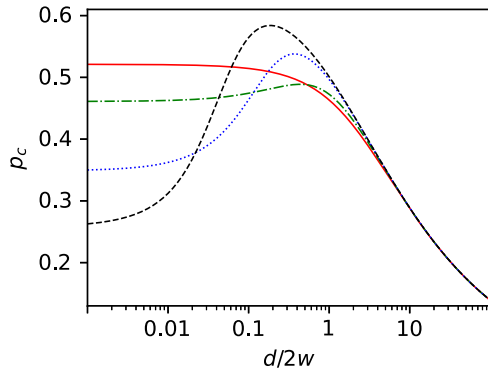


FIG. 2. Dependence of the critical value p_c , Eq. (23), of the parameter p defined by Eq. (22) on the aspect ratio $d/2w$ of the superconducting slab of the thickness d and of the width $2w$ for various values of the anisotropy parameter ε : $\varepsilon = 1$ (red solid line), $\varepsilon = 0.5$ (green dot-dash line), $\varepsilon = 0.15$ (blue dotted line), and $\varepsilon = 0.05$ (black dashed line).

m ($\theta_0 \approx 36.5^\circ$, $\cos \theta_0 \approx 0.8$) [25], and

$$H_p^{\text{GB}} \approx 0.8\sqrt{m}H_{c1} \approx 0.9\sqrt{\frac{d}{2w}}H_{c1}, \quad (21)$$

where we have used formula (4).

C. Interplay of the barriers: Two scenarios of the vortex penetration

A comparison of formulas (10) and (19) shows that the ratio of these penetration fields can be written as p/p_c where the parameter p is defined as follows:

$$p \equiv \frac{\kappa}{\ln \kappa} \left(\frac{\lambda}{d} \right)^{1/3}, \quad (22)$$

and p_c is its critical value which generally depends on ε and m ,

$$p_c \equiv \frac{\cos \theta_0}{0.92\varepsilon(\theta_0)} \left(\frac{m(1-m)}{6f(1,m)} \right)^{1/3}. \quad (23)$$

Although H_p^{BL} , Eq. (10), and hence p_c are determined up to a numerical factor of the order of unity, formula (23) reveals the dependences of p_c on $d/2w$ and ε . For thin isotropic strips ($m \ll 1$, $\varepsilon = 1$), this p_c is practically independent of m ($p_c \approx 0.52$) [25]. For anisotropic slabs with $d/2w \lesssim 1$, p_c depends on the aspect ratio $d/2w$ and ε , but these dependences are relatively weak, and the parameter p_c remains of order of 0.5, Fig. 2. At $d/2w \gg 1$, p_c decreases and is described by the universal function $p_c(d/2w)$ independent of ε ,

$$p_c(d/2w) \approx \frac{1}{0.92} \left(\frac{16w}{3\pi^2 d} \right)^{1/3} \approx 0.7 \left(\frac{2w}{d} \right)^{1/3}.$$

Since the parameter p can be greater or less than its critical value p_c , two scenarios of the vortex penetration into the sample are possible [25]. If $p > p_c$, one has $H_p^{\text{BL}} > H_p^{\text{GB}}$, and the true penetration field H_p coincides with H_p^{BL} , Eq. (10). In this case, small vortex segments appearing at the corners of the strip at $H_a = H_p^{\text{BL}}$ immediately expand, merge at the equatorial point ($x = w$, $y = 0$), and the created vortex moves

towards the center of the sample, Fig. 1. If the parameter p is less than the critical value p_c , one has $H_p^{\text{BL}} < H_p^{\text{GB}}$, and the vortex penetration is a two-stage process. The current density in the vicinity of the corners reaches the depairing value at $H_a = H_p^{\text{BL}}$. At this field a penetrating vortex enters the sample through the corner, but it cannot reach the equatorial point, and so this vortex line will hang between the corner and the equatorial point ($x = w$, $y = 0$). With increasing H_a , two domes filled by these inclined vortex lines will expand in the lateral surface of the strip. The penetration field H_p is determined by the condition that the boundaries of these domes meet at the equatorial point, and this field H_p can be estimated by Eq. (19). However, formula (19) has been derived, considering a single inclined vortex. Since the vortices ending on the lateral surface of the slab modify the current distribution in the sample, the H_p has to be calculated self-consistently, taking into account the currents generated by the domes of the inclined vortices. Nevertheless, as was shown in Ref. [25], for the thin isotropic strip, the maximal decrease of H_p associated with these domes does not exceed 20% as compared to H_p given by Eq. (21) [at $p \ll p_c$, the numerical coefficients 0.8 and 0.9 in Eq. (21) are replaced by 0.63 and 0.71, respectively]. Below, we will neglect the currents generated by the domes since on the one hand, this neglect essentially simplifies the problem, and on the other hand, this approach provides a sufficiently accurate calculation of H_p .

IV. PENETRATION FIELD

Using the equations of the preceding section, consider dependences of the vortex penetration field on the aspect ratio of the slab and on the anisotropy parameter ε . If the parameter p exceeds its critical value p_c , the penetration field is estimated by formula (10), which is independent of ε . To understand the dependence of this field on the aspect ratio of the slab, let us assume that its width $2w$ is constant, whereas the thickness d increases. Then, formula (10) shows that H_p^{BL} is proportional to $d^{1/6}$ at small $d/2w$ and reaches its maximal value

$$H_{p,\text{max}}^{\text{BL}} = 0.92 \frac{\kappa H_{c1}}{\ln \kappa} \left(\frac{3\pi\lambda}{4w} \right)^{1/3} \quad (24)$$

in the limit $d \rightarrow \infty$. Note that this limiting value is noticeably less than the Bean-Livingston penetration field $H_c = \sqrt{2}H_{c1}\kappa/\ln \kappa$ characteristic of flat surfaces [28], since the vortices now penetrate into the samples through its corners. However, if this maximal value is less than H_{c1} , the curve $H_p^{\text{BL}}(d/2w)$ necessarily crosses the curve $H_p^{\text{GB}}(d/2w)$ at some d_{cr} (see figures below), and one has $p = p_c$ at the crossing point. When $d > d_{\text{cr}}$, $H_p^{\text{GB}}(d/2w)$ becomes larger than $H_p^{\text{BL}}(d/2w)$, and the penetration field is determined by the geometrical barrier.

Consider now the situation when $p < p_c$, and the penetration field is determined by the geometrical barrier, described by Eqs. (17)–(19). In Figs. 3 and 4, we show the dependences of θ_0 and $(w - x_0)/w$ on the aspect ratio $d/2w$ of the slab for various values of the anisotropy parameter ε . Note that the angle θ_0 remains practically constant for the strips with $d/2w \lesssim 0.1$. In the isotropic case, $\varepsilon = 1$, the quantity $(w - x_0)/w$, defining the position of the inclined vortex on the

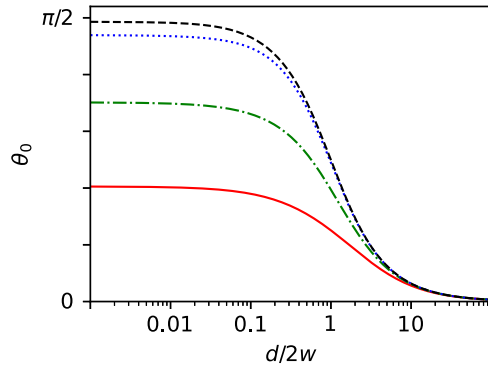


FIG. 3. Dependence of the angle θ_0 of the inclined vortex on the aspect ratio $d/2w$ of the superconducting slab for various values of the anisotropy parameters $\varepsilon = 1, 0.5, 0.15, 0.05$. The correspondence of ε to the color and style of the appropriate line is the same as in Fig. 2. The curves are calculated, solving Eqs. (15)–(19) in the neglect of the angular dependence of the logarithmic factor in Eq. (13).

upper plane of the slab, becomes comparable with unity only at $d/2w > 1$. However, the less ε , the greater $(w - x_0)/w$, and $(w - x_0)/w \sim 1$ even for thin strips if ε is small.

The dependence H_p^{GB}/H_{c1} on the aspect ratio $d/2w$ of the isotropic slab ($\varepsilon = 1$) is presented in Fig. 5. For comparison, in Fig. 5 we also show this dependence obtained from numerical calculations of the electrodynamics of vortices [18],

$$\frac{H_p^{\text{GB}}}{H_{c1}} \approx \tanh\left(\sqrt{0.36 \frac{d}{2w}}\right). \quad (25)$$

The dashed line in this figure corresponds to formula (21) obtained for thin strips ($d \ll w$) [25]. It is seen that this formula well describes the function $H_p^{\text{GB}}(d/2w)$ at $d/2w \lesssim 0.1$. Interestingly, Brandt's formula (25) in the case of the thin

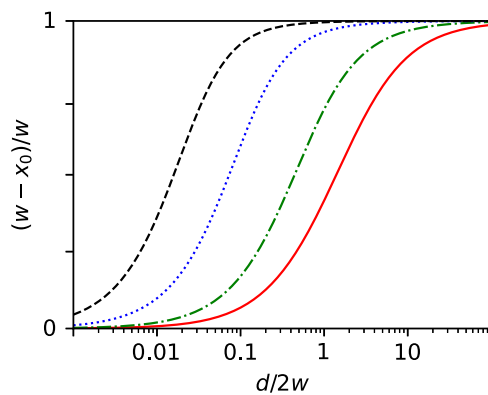


FIG. 4. The position x_0 of the inclined vortex on the upper plane of the slab versus the aspect ratio $d/2w$ of the superconducting slab for various values of the anisotropy parameter $\varepsilon = 1, 0.5, 0.15, 0.05$. The correspondence between the values of ε and the lines is the same as in Figs. 2 and 3. The curves are calculated, solving Eqs. (15)–(19) in the neglect of the angular dependence of the logarithmic factor in Eq. (13).

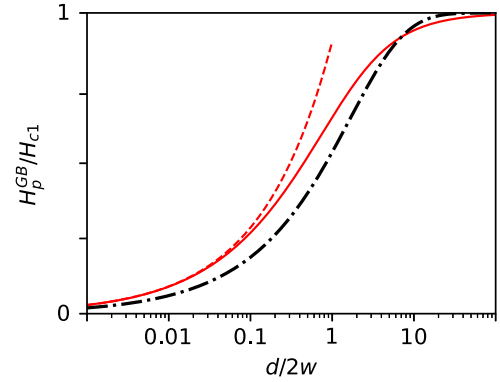


FIG. 5. Dependence of the penetration field H_p^{GB} , caused by the geometrical barrier, on the aspect ratio $d/2w$ of the isotropic superconducting slab ($\varepsilon = 1$). The red curve is obtained, solving Eqs. (15)–(19) in the neglect of the angular dependence of the logarithmic factor in Eq. (13). The black dot-and-dash line corresponds to formula (25), whereas the red dashed line is described by Eq. (21).

strips gives the expression,

$$\frac{H_p^{\text{GB}}}{H_{c1}} \approx 0.6 \sqrt{\frac{d}{2w}},$$

which differs from Eq. (21) only by the numerical factor of the order of unity. The difference between these thin-strip results is further reduced if we take into account that the vortex domes of the inclined vortices lead to the decrease of the coefficient before $\sqrt{d/2w}$ in Eq. (21) (see the end of Sec. III C). Thus, our approach based on the solution of simple equation (18) and Brandt's formula (25) leads to close values of H_p in the isotropic case. However, our approach also describes the anisotropic case; in addition, it permits one to analyze the situations when the complete penetration of vortices does not occur (see the next section).

In Fig. 6, we show the dependences of H_p^{GB} on $d/2w$ for various values of the anisotropy parameter ε . For thick slabs

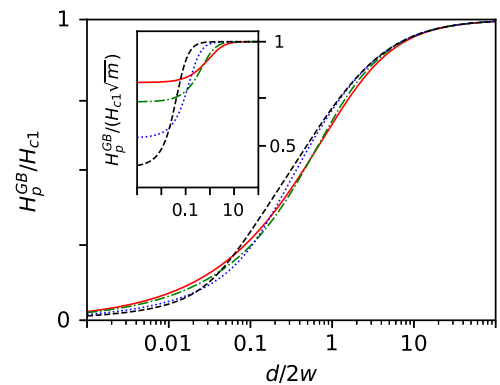


FIG. 6. Dependences of the penetration field H_p^{GB} , caused by the geometrical barrier, on the aspect ratio $d/2w$ of the anisotropic superconducting slab for different values of the anisotropy parameter $\varepsilon = 1, 0.5, 0.15, \text{ and } 0.05$. The notations of the lines are the same as in Figs. 2–4. Inset shows $H_p^{\text{GB}}/(\sqrt{m}H_{c1})$ versus $d/2w$. This ratio $H_p^{\text{GB}}/(\sqrt{m}H_{c1})$ tends to constant values both at $d/2w \rightarrow 0$ and at $d/2w \rightarrow \infty$; see Eqs. (19), (20), and Fig. 3.

$d \gg 2w$, this dependence is well described by formula (20). However, it is important that even for $d/2w \lesssim 1$, the dependences $H_p^{\text{GB}}(d/2w)$ calculated for different ε are close to each other. In other words, the anisotropy of a superconducting material has a relatively small effect on H_p^{GB} for not-too-thin samples. Probably, this result explains the successful applications of formula (25) found for isotropic superconductors to the anisotropic materials [12–14].

V. DISCUSSION

Using the micro-Hall probes, the magnetic-induction component B_y is usually measured on the upper (lower) surface of the slab to detect the penetration of vortices into a type-II superconductor, and therefore, to find the penetration field H_p . Apart from the Hall-probe magnetometry, such measurements can be also carried out using ensembles of nitrogen-vacancy centers [32], magneto-optical imaging [33], and the nanoscale SQUID-on-tip [34]. The formulas of this paper permit one to obtain information on λ (or H_{c1}) from the H_p thus measured. For definiteness, we imply below that B_y is measured by the micro-Hall probes.

Consider $B_y(x)$ on the upper plane of the slab when the first scenario of the vortex penetration into the sample occurs (i.e., when $H_p^{\text{BL}} > H_p^{\text{GB}}$), Sec. III C. In this case, a penetrating vortex arrive at the center of the slab at $H_a = H_p^{\text{BL}}$, and the nonzero B_y can be detected only by the sensor placed at this point. At $H_a > H_p^{\text{BL}}$, the vortices accumulate in the center of the slab, and the vortex dome on its upper plane gradually extends. This dome is described by the formula [26]:

$$B_y(u) = \mu_0 H_a \frac{\sqrt{u_d^2 - u^2}}{\sqrt{1 - u^2}}, \quad (26)$$

where $u^2 \leq u_d^2$, and the boundary of the vortex dome u_d is determined by the condition [26] that the current density at $x = w$ is equal to j_0 defined by Eq. (9). This condition yields

$$u_d^2 = 1 - \left(\frac{H_p^{\text{BL}}}{H_a} \right)^2. \quad (27)$$

When $u_d \ll 1$ and $m \ll 1$, formula (1) gives $u \approx x/w$, and expressions (26), (27) reduces to the well-known result for a thin strip [17]. However, Eqs. (26) and (27) are applicable to the slab with an arbitrary aspect ratio $d/2w$, and one can easily find the field $H_a(x_s)$ at which the boundary of the dome arrives at a point x_s , i.e., the field at which a Hall sensor placed at this point begins to show a nonzero signal. Note that the signal sharply grows at $H_a \gtrsim H_a(x_s)$. For example, Eqs. (26) and (27) at $u = 0$ give, $d(B_y/\mu_0)/dH_a = H_a/\sqrt{H_a^2 - (H_p^{\text{BL}})^2}$. This derivative is large at $H_a \approx H_p^{\text{BL}}$. Thus, in the case of the first scenario, the measurement of the penetration field permits one to find H_p^{BL} determined by formula (10), i.e., to estimate the combination of parameters, $\kappa/\lambda^{5/3}$. However, it is necessary to keep in mind that local defects of the corners of the slab are favorable for the vortex penetration through these defects [35], and the measured H_p can be noticeably suppressed as compared to the theoretical value.

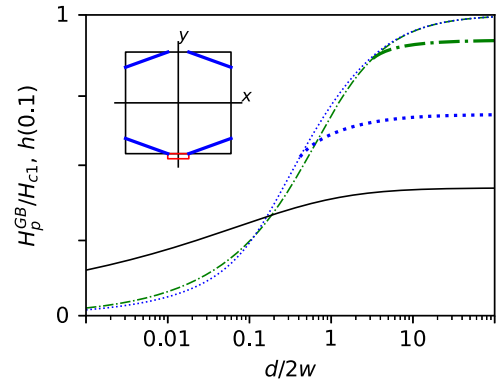


FIG. 7. Dependences of the dimensionless magnetic field $h(x_s/w) = H_a/H_{c1}$, at which the inclined vortex arrives at the point $x_s = 0.1w$, on the aspect ratio $d/2w$ of the anisotropic superconducting slab for the two values of the anisotropy parameter $\varepsilon = 0.5$ (green thick dot-and-dash line) and $\varepsilon = 0.15$ (blue thick dotted line). The similar thin lines show H_p^{GB}/H_{c1} versus $d/2w$, cf. Fig. 6. The dependence $H_p^{\text{BL}}(d/2w)/H_{c1}$, Eq. (10), is shown by the black solid line for the case $H_{p,\text{max}}^{\text{BL}} = 0.424H_{c1}$, Eq. (24). Inset: The slab and the inclined vortices shown schematically at $H_a = H_{c1}h(x_s/w)$. At this field, the vortices reach the Hall sensor (red small rectangle) that covers the lower plane of the slab from $x = -x_s$ to $x = x_s$.

If the second scenario of the vortex penetration occurs (i.e., if $H_p^{\text{BL}} < H_p^{\text{GB}}$), a vortex arrives at the center of the slab at $H_a = H_p^{\text{GB}}$, and for $H_a > H_p^{\text{GB}}$ the physical picture is qualitatively the same as for the first scenario. However, formulas (26) and (27), strictly speaking, should be modified. This modification is due to the currents generated by the inclined vortices that hang near the corners. Within our simplified approach, which neglects these currents, the penetration field H_p^{GB} is determined by formula (19). Thus, the measurement of this field gives H_{c1} if the anisotropy parameter ε is known. It is important to emphasize that, in contrast to H_p^{BL} , the penetration field H_p^{GB} is insensitive to small defects of the surface of the slab, since the geometric barrier is determined by the length of the order of the thickness d of the sample.

However, it is clear that the inclined vortices can generate a nonzero B_y on the surface of the sample if $H_p^{\text{BL}} < H_a < H_p^{\text{GB}}$, i.e., when the second end of the inclined vortex has not yet reached the equatorial point of the slab (Fig. 7, inset). Taking into account the data of Fig. 4, we conclude that this nonzero B_y can be detected on the upper plane of the slab at a sufficiently far distance ($\sim w$) from the corners if $d \gtrsim w$ or $\varepsilon^2 \ll 1$. Thus, in principle, it is possible to find H_{c1} not only from the field H_p^{GB} of the complete vortex penetration into the sample but also from a detection of nonzero B_y generated by the inclined vortices.

The dome of the inclined vortices appears on the upper plane at $H_a = H_p^{\text{BL}}$, and it expands with increasing H_a , reaching its maximal size at $H_a = H_p^{\text{GB}}$. To calculate the size of the dome correctly, it is necessary to take into account the currents produced by the inclined vortices [25]. Within our simplified approach, in which these currents are disregarded, the boundaries of the dome can be estimated only approximately, calculating x_0 with Eqs. (A1)–(A3) at $H_a = H_p^{\text{BL}}$ and at given $H_a < H_p^{\text{GB}}$. Note that the smaller $(H_a - H_p^{\text{BL}})/H_p^{\text{GB}}$,

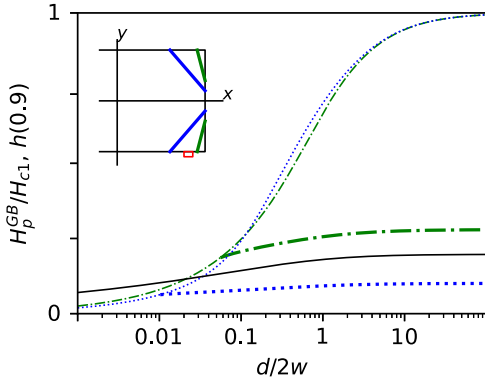


FIG. 8. Dependences of the dimensionless magnetic field $h(x_s/w) = H_a/H_{c1}$, at which the inclined vortex arrives at the point $x_s = 0.9w$, on the aspect ratio $d/2w$ of the anisotropic superconducting slab for the two values of the anisotropy parameter $\varepsilon = 0.5$ (green thick dot-and-dash line) and $\varepsilon = 0.15$ (blue thick dotted line). The similar thin lines show H_p^{GB}/H_{c1} versus $d/2w$, cf. Fig. 6. The dependence $H_p^{BL}(d/2w)/H_{c1}$ is shown by the black solid line for the case $H_{p,max}^{BL} = 0.197H_{c1}$. Inset: The inclined vortices are schematically depicted in the slab at $H_a = H_p^{BL}$. For $\varepsilon = 0.5$, the vortices (green solid lines) have not reached the micro-Hall sensor (red rectangle) yet, whereas for $\varepsilon = 0.15$, the vortices (blue solid lines) have already passed through the sensor.

the better accuracy of this estimate. Thus, if the Hall sensor placed at a point x_s contacts with a small dome of the inclined vortices, Eqs. (A1)–(A3) with the fixed $x_0 = x_s$ permit one to calculate the value of $H_a/H_{c1} \equiv h(x_s/w)$, at which this contact occurs. Comparing the calculated $h(x_s/w)$ and the measured H_a , for which a nonzero B_y is detected by the sensor, one can estimate H_{c1} .

As an example, consider the case when the sensor is placed at the center of the slab, and it covers the region $-x_s \leq x \leq x_s = 0.1w$ on the surface. In Fig. 7, for different values of $d/2w$ and ε , we show the dimensionless magnetic fields $h(x_s/w)$, at which the end of the inclined vortex reaches the point x_s . The $d/2w$ dependences of these fields start on the lines $H_p^{GB}(d/2w)/H_{c1}$ since at low values of the aspect ratio, only the vertical vortex created at $H_a = H_p^{GB}$ can reach this sensor. Thus, for $\varepsilon = 0.15$, Fig. 7 demonstrates that at $d < d_c \approx 0.4w$, the vertical vortices begin to arrive at the sensor when $H_a = H_p^{BL}(d/2w)$. In the interval $d_c < d \lesssim 0.8w$, such vortices reach the sensor at $H_a = H_p^{GB}(d/2w)$, and for $d > 0.8w$, the end of the incline vortex come to the sensor at the magnetic field H_a determined by the blue thick dotted line. Note that due to the large tilt angle θ , the magnitude of B_y produced by the inclined vortex at the sensor is expected to be smaller than at $H_a = H_p^{GB}$ when the vertical vortices completely penetrate into the sample. Thus, the cases of the partial ($H_a < H_p^{GB}$) and complete ($H_a = H_p^{GB}$) vortex penetration can differ in the measured values of dB_z/dH_a .

Consider now the situation when the Hall sensor is near the corner of the slab. In Fig. 8, we consider the case when a very small sensor is at the point $x_s = 0.9w$. In this situation, for the end of the inclined vortex to be at the point x_s , the field H_p^{BL} has to be sufficiently small, and H_a should only slightly exceeds H_p^{BL} . Otherwise, the vortex dome of such vortices

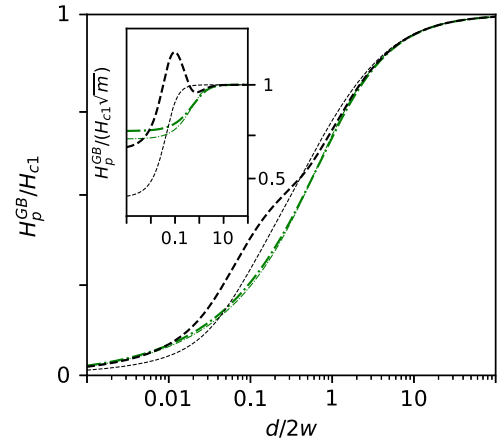


FIG. 9. Dependences of the penetration field H_p^{GB} on the aspect ratio $d/2w$ of the anisotropic superconducting slab for two values of the anisotropy parameter $\varepsilon = 0.5$ (green dot-and-dash lines) and 0.05 (black dashed lines). The thick lines are calculated without taking into account the angular dependence of the logarithmic factor in formula (13), cf. Fig. 6. The similar thin lines are calculated with Eqs. (B1), (B2), (17) and $\kappa = 50$. Inset shows the ratio $H_p^{GB}/(\sqrt{m}H_{c1})$ versus $d/2w$.

can be between the sensor and the center of the slab. It is this case that occurs for $\varepsilon = 0.15$ in Fig. 8. In this case, the blue thick dotted line lies below $H_p^{BL}(d/2w)$, and the inclined vortices cannot come to the sensor. On the other hand, for $\varepsilon = 0.5$, the green thick dot-and-dash line shows the dimensionless fields $h(0.9)$ at which the inclined vortex reaches the sensor.

In deriving Eqs. (17)–(19), we neglected the angular dependence of logarithmic factor in the vortex energy e_l , Eq. (13). In general, this factor depends on the vortex core. To clarify how the core can manifest itself in the dependence $H_p^{GB}(d/2w)$, we derive the appropriate equations in Appendix B, using formula (13) without any simplifications. In Fig. 9, the functions $H_p^{GB}(d/2w)$ obtained with and without taking into account the angular dependence of the logarithmic factor are compared. It is seen that this angular dependence clearly manifests itself only at $d/2w \lesssim 0.3$ for superconductors with strong anisotropy. However, we do not find qualitative changes in the functions $H_p^{GB}(d/2w)$. For thick samples or for $\varepsilon \gtrsim 1/2$, the changes are small.

Throughout this paper, we have considered the infinite slab along the z axis. In Appendix C, we discuss how one can approximately take into account the finite length L of real platelet-shaped samples.

VI. CONCLUSIONS

We analyze the process of the vortex penetration into an anisotropic superconducting slab of a rectangular cross section, with the width $2w$ and the thickness d of the sample being much larger than the London penetration depth λ . The aspect ratio of the slab, $d/2w$, may have an arbitrary value. The axis of the anisotropy of the superconductor and the applied magnetic field H_a are assumed to be directed along the thickness of the sample, and the flux-line pinning is neglected.

It is shown that the vortex-penetration field H_p coincides with the largest field from H_p^{BL} and H_p^{GB} determined by the Bean-Livingston and geometrical barriers, respectively, and the formulas for these H_p^{GB} and H_p^{BL} are derived. The samples, for which the geometrical barrier dominates over the Bean-Livingston one (i.e., when $H_p^{\text{GB}} > H_p^{\text{BL}}$), are most suitable for determining H_{c1} and λ , since the Bean-Livingston barrier is sensitive to the defects in the corners of the slab. In the case $H_p^{\text{GB}} > H_p^{\text{BL}}$, the vortex penetration is a two-stage process. In the interval $H_p^{\text{BL}} < H_a < H_p$, the inclined vortices enter the edge regions of the slab through its corners, and domes of such vortices appear on the upper (lower) planes of the slab. For this case, we describe the simple approach for calculating the field H_a at which the inclined vortex comes to a given point on the upper (lower) plane of the slab.

The obtained results permit one to estimate the values of H_{c1} and λ from measurements of the penetration field or of the field at which the inclined vortices reach a micro-Hall probe placed on upper (lower) plane of the slab.

ACKNOWLEDGMENT

This research was supported in part by the Program PAS-NASU 2024.

APPENDIX A: EQUATIONS FOR AN INCLINED VORTEX

Consider a vortex, the ends of which are at the point x_0 of the upper plane of the slab and at the point y_0 of its lateral surface. Then, Eqs. (11)–(14) takes on the form [we still set $\epsilon(\theta) = 1$ in the logarithmic factor of formula (13)]:

$$\frac{\epsilon^2 \sin \theta}{\epsilon(\theta)} = \frac{H_a}{H_{c1}} \frac{u_0}{\sqrt{1 - u_0^2}}, \quad (\text{A1})$$

$$\frac{\cos \theta}{\epsilon(\theta)} = \frac{H_a}{H_{c1} \sqrt{m}} \frac{\sqrt{1 - ms_0^2}}{\sqrt{1 - s_0^2}}, \quad (\text{A2})$$

where the parameter u_0 corresponds to the point x_0 , Eq. (1), whereas s_0 corresponds to the point y_0 , Eq. (5). With the use of formula (1) and the ratio of Eqs. (A1) and (A2), which gives an expression for $\tan \theta$ in terms of u_0 and s_0 , the geometrical relationship (14) reduces to

$$\begin{aligned} [f(1, 1 - m) - f(u_0, 1 - m)] \frac{\sqrt{1 - u_0^2} \epsilon^2}{\sqrt{m} u_0} \\ = \frac{\sqrt{1 - s_0^2}}{\sqrt{1 - ms_0^2}} [f(1, m) - f(s_0, m)]. \end{aligned} \quad (\text{A3})$$

At $s_0 = 0$, Eqs. (A1)–(A3) reduce to Eqs. (15), (16), (18). Formulas (A1)–(A3) permit one to find any three of the four

quantities: u_0 (i.e., x_0), s_0 (i.e., y_0), the angle θ , and H_a . For example, at given u_0 , the relation (A3) is equation in s_0 . Knowing s_0 , we find the angle θ and the appropriate H_a from Eqs. (A1) and (A2).

APPENDIX B: PENETRATION FIELD H_p^{GB} FOR SUPERCONDUCTOR WITH ANISOTROPIC VORTEX CORE

With the angular dependence of the logarithmic factor in formula (13), Eqs. (15), (16) take on the form:

$$\frac{\sin \theta_0}{\epsilon(\theta_0)} \frac{[\epsilon^2 \ln(\kappa/e\epsilon(\theta_0)) + [\epsilon(\theta_0)]^2]}{\ln \kappa} = \frac{H_a u_0}{H_{c1} \sqrt{1 - u_0^2}}, \quad (\text{B1})$$

$$\frac{\cos \theta_0}{\epsilon(\theta_0)} \frac{[\ln(\kappa/e\epsilon(\theta_0)) + [\epsilon(\theta_0)]^2]}{\ln \kappa} = \frac{H_a}{H_{c1} \sqrt{m}}, \quad (\text{B2})$$

where κ is the Ginzburg-Landau parameter. Solving the set of equations (B1), (B2), and (17), we can find θ_0 , u_0 , H_p^{GB} .

APPENDIX C: SLAB OF THE FINITE LENGTH

In Ref. [22], the effective demagnetization factor N for a platelet-shaped superconductor with dimensions $d \times 2w \times L$ was considered, and the following formula was proposed:

$$N(r, R) \approx \frac{4}{4 + 3r(1 + R)}, \quad (\text{C1})$$

where $r \equiv d/2w$ and $R \equiv 2w/L$ are the aspect ratios of the cross sections of this platelet. In the limiting case of the infinitely long slab ($L \rightarrow \infty$), Eq. (C1) yields

$$N(r, 0) \approx \frac{4}{4 + 3r}. \quad (\text{C2})$$

Since the currents in the Meissner state of the platelet-shaped superconductor of the volume V and its magnetic moment $-H_a V/(1 - N)$ are proportional to each other, we can express the Meissner currents in a slab of a finite length L via the currents in the infinitely long slab with the same aspect ratio r ,

$$\frac{J_M(r, R)}{J_M(r, 0)} = \frac{1 - N(r, 0)}{1 - N(r, R)} \equiv \frac{1}{F(r, R)}. \quad (\text{C3})$$

On the other hand, the smaller the Meissner currents, the larger the H_p^{BL} and H_p^{GB} determined by these currents. Therefore, to estimate effect of the finite length of the slab on the penetration fields, one can introduce the additional factor $F(r, R)$,

$$F(r, R) = 1 + \frac{4R}{4 + 3r(1 + R)}, \quad (\text{C4})$$

into formulas (10) and (19) for H_p^{BL} and H_p^{GB} , respectively.

[1] W. N. Hardy, D. A. Bonn, D. C. Morgan, Ruixing Liang, and Kuan Zhang, Precision measurements of the temperature

dependence of λ in $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$: Strong evidence for nodes in the gap function, *Phys. Rev. Lett.* **70**, 3999 (1993).

- [2] R. Prozorov and R. W. Giannetta, Magnetic penetration depth in unconventional superconductors, *Supercond. Sci. Technol.* **19**, R41 (2006).
- [3] A. Carrington, R. W. Giannetta, J. T. Kim, and J. Giapintzakis, Absence of nonlinear Meissner effect in $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$, *Phys. Rev. B* **59**, R14173 (1999).
- [4] R. Prozorov and V. G. Kogan, London penetration depth in iron-based superconductors, *Rep. Prog. Phys.* **74**, 124505 (2011).
- [5] C. Putzke, P. Walmsley, J. D. Fletcher, L. Malone, D. Vignolles, C. Proust, S. Badoux, P. See, H. E. Beere, D. A. Ritchie, S. Kasahara, Y. Mizukami, T. Shibauchi, Y. Matsuda, and A. Carrington, Anomalous critical fields in quantum critical superconductors, *Nat. Commun.* **5**, 5679 (2014).
- [6] B. Shen, M. Leroux, Y. L. Wang, X. Luo, V. K. Vlasko-Vlasov, A. E. Koshelev, Z. L. Xiao, U. Welp, W. K. Kwok, M. P. Smylie, A. Snezhko, and V. Metlushko, Critical fields and vortex pinning in overdoped $\text{Ba}_{0.2}\text{K}_{0.8}\text{Fe}_2\text{As}_2$, *Phys. Rev. B* **91**, 174512 (2015).
- [7] T. Yamashita, T. Takenaka, Y. Tokiwa, J. A. Wilcox, Y. Mizukami, D. Terazawa, Y. Kasahara, S. Kittaka, T. Sakakibara, M. Konczykowski, S. Seiro, H. S. Jeevan, C. Geibel, C. Putzke, T. Onishi, H. Ikeda, A. Carrington, T. Shibauchi, and Y. Matsuda, Fully gapped superconductivity with no sign change in the prototypical heavy-fermion CeCu_2Si_2 , *Sci. Adv.* **3**, e1601667 (2017).
- [8] T. Takenaka, Y. Mizukami, J. A. Wilcox, M. Konczykowski, S. Seiro, C. Geibel, Y. Tokiwa, Y. Kasahara, C. Putzke, Y. Matsuda, A. Carrington, and T. Shibauchi, Full-gap superconductivity robust against disorder in heavy-fermion CeCu_2Si_2 , *Phys. Rev. Lett.* **119**, 077001 (2017).
- [9] G. Pang, M. Smidman, J. Zhang, L. Jiao, Z. Weng, E. M. Nica, Y. Chen, W. Jiang, Y. Zhang, W. Xie, H. S. Jeevan, H. Lee, P. Gegenwart, F. Steglich, Q. Si, and H. Yuan, Fully gapped d -wave superconductivity in CeCu_2Si_2 , *Proc. Natl. Acad. Sci. USA* **115**, 5343 (2018).
- [10] K. Ishihara, M. Roppongi, M. Kobayashi, K. Imamura, Y. Mizukami, H. Sakai, P. Opletal, Y. Tokiwa, Y. Haga, K. Hashimoto, and T. Shibauchi, Chiral superconductivity in UTe_2 probed by anisotropic low-energy excitations, *Nature Commun.* **14**, 2966 (2023).
- [11] L. Lyard, T. Klein, J. Marcus, R. Brusetti, C. Marcenat, M. Konczykowski, V. Mosser, K. H. Kim, B. W. Kang, H. S. Lee, and S. I. Lee, Geometrical barriers and lower critical field in MgB_2 single crystals, *Phys. Rev. B* **70**, 180504(R) (2004).
- [12] Z. Pribulova, T. Klein, J. Kacmarcik, C. Marcenat, M. Konczykowski, S. L. Bud'ko, M. Tillman, and P. C. Canfield, Upper and lower critical magnetic fields of superconducting $\text{NdFeAsO}_{1-x}\text{F}_x$ single crystals studied by Hall-probe magnetization and specific heat, *Phys. Rev. B* **79**, 020508(R) (2009).
- [13] R. Okazaki, M. Konczykowski, C. J. van der Beek, T. Kato, K. Hashimoto, M. Shimozawa, H. Shishido, M. Yamashita, M. Ishikado, H. Kito, A. Iyo, H. Eisaki, S. Shamoto, T. Shibauchi, and Y. Matsuda, Lower critical fields of superconducting PrFeAsO_{1-y} single crystals, *Phys. Rev. B* **79**, 064520 (2009).
- [14] T. Klein, D. Braithwaite, A. Demuer, W. Knafo, G. Lapertot, C. Marcenat, P. Rodière, I. Sheikin, P. Strobel, A. Sulpice, and P. Toulemonde, Thermodynamic phase diagram of $\text{Fe}(\text{Se}_{0.5}\text{Te}_{0.5})$ single crystals in fields up to 28 tesla, *Phys. Rev. B* **82**, 184506 (2010).
- [15] Z. Pribulová, Z. Medvecká, J. Kačmarčík, V. Komanický, T. Klein, P. Rodière, F. Levy-Bertrand, B. Michon, C. Marcenat, P. Husančková, V. Cambel, J. Šoltýs, G. Karapetrov, S. Borisenko, D. Evtushinsky, H. Berger, and P. Samuely, Magnetic and thermodynamic properties of Cu_xTiSe_2 single crystals, *Phys. Rev. B* **95**, 174512 (2017).
- [16] J. Juraszek, R. Wawryk, Z. Henkie, M. Konczykowski, and T. Cichorek, Symmetry of order parameters in multiband superconductors $\text{LaRu}_4\text{As}_{12}$ and $\text{PrOs}_4\text{Sb}_{12}$ probed by local magnetization measurements, *Phys. Rev. Lett.* **124**, 027001 (2020).
- [17] E. Zeldov, A. I. Larkin, V. B. Geshkenbein, M. Konczykowski, D. Majer, B. Khaykovich, V. M. Vinokur, and H. Shtrikman, Geometrical barriers in high-temperature superconductors, *Phys. Rev. Lett.* **73**, 1428 (1994).
- [18] E. H. Brandt, Irreversible magnetization of pin-free type-II superconductors, *Phys. Rev. B* **60**, 11939 (1999).
- [19] E. H. Brandt, Geometric edge barrier in the Shubnikov phase of type II superconductors, *Low Temp. Phys.* **27**, 723 (2001).
- [20] R. Prozorov, R. W. Giannetta, A. Carrington, F. M. Araujo-Moreira, Meissner-London state in superconductors of rectangular cross section in a perpendicular magnetic field, *Phys. Rev. B* **62**, 115 (2000).
- [21] E. Pardo, D.-X. Chen, and A. Sanchez, Demagnetizing factors for completely shielded rectangular prisms, *J. App. Phys.* **96**, 5365 (2004).
- [22] R. Prozorov and V. G. Kogan, Effective demagnetizing factors of diamagnetic samples of various shapes, *Phys. Rev. Appl.* **10**, 014030 (2018).
- [23] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Course in Theoretical Physics (Pergamon, London, 1959), Vol. 8.
- [24] E. H. Brandt and G. P. Mikitik, Meissner-London currents in superconductors with rectangular cross section, *Phys. Rev. Lett.* **85**, 4164 (2000).
- [25] E. H. Brandt, G. P. Mikitik, and E. Zeldov, Two regimes of vortex penetration into platelet-shaped type-II superconductors, *J. Exp. Theor. Phys.* **117**, 439 (2013).
- [26] G. P. Mikitik, Critical current in thin flat superconductors with Bean-Livingston and geometrical barriers, *Phys. Rev. B* **104**, 094526 (2021).
- [27] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Vortices in high-temperature superconductors, *Rev. Mod. Phys.* **66**, 1125 (1994).
- [28] P. G. de Gennes, *Superconductivity of metals and alloys* (W. A. Benjamin, New York, 1966).
- [29] A. V. Samokhvalov, A surface barrier for a vortex loop in type-II superconductors, *JETP* **81**, 601 (1995).
- [30] V. P. Galaiko, Formation of vortex nuclei in superconductors of the second kind, *Sov. Phys. JETP* **23**, 878 (1966).
- [31] Y. A. Genenko, Magnetic self-field entry into a current-carrying type-II superconductor, *Phys. Rev. B* **49**, 6950 (1994).
- [32] K. R. Joshi, N. M. Nusran, M. A. Tanatar, Kyuil Cho, W. R. Meier, S. L. Bud'ko, P. C. Canfield, and R. Prozorov, Measuring the lower critical field of superconductors using nitrogen-vacancy centers in diamond optical magnetometry, *Phys. Rev. Appl.* **11**, 014035 (2019).

- [33] Ch. Jooss, J. Albrecht, H. Kuhn, S. Leonhardt, and H. Kronmüller, Magneto-optical studies of current distributions in high- T_c superconductors, *Rep. Prog. Phys.* **65**, 651 (2002).
- [34] L. Embon, Y. Anahory, Ž. L. Jelić, E. O. Lachman, Y. Myasoedov, M. E. Huber, G. P. Mikitik, A. V. Silhanek, M. V. Milošević, A. Gurevich, and E. Zeldov, Imaging of super-fast dynamics and flow instabilities of superconducting vortices, *Nat. Commun.* **8**, 85 (2017).
- [35] A. Yu. Aladyshkin, A. S. Mel'nikov, I. A. Shereshevsky, and I. D. Tokman, What is the best gate for vortex entry into type-II superconductor? *Physica C: Supercond.* **361**, 67 (2001).