

Realization of kagome lattice and superconductivity in topological electrides

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Realization of kagome lattice, and hence achieving specific performance, is a challenging issue in condensed matter physics. The corner-sharing triangles of kagome lattice are normally comprised of real atoms occupying certain Wyckoff sites. Here, we propose that the excess electrons in layered electrides Mg_3N and Mg_3O build the two-dimensional kagome lattices. The interstitial electrons further contribute to the nontrivial band topology, i.e., the Dirac points and the Dirac nodal lines in Mg_3N and Mg_3O . Further, the in-plane coupling between the electronic orbitals and the atomic vibrational modes dominate the electron-phonon coupling, and yield superconducting critical temperatures approaching 2.0 and 2.2 K for Mg_3N and Mg_3O , respectively. This work proposes a feasible scheme to construct kagome lattices by interstitial quasiatoms and provides a promising platform to explore the superconductivity and nontrivial band topology in kagome electrides.

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I. INTRODUCTION

Kagome materials have attracted much research interest due to their exotic electronic, magnetic, topological, and superconducting properties. As for the ideal two-dimensional (2D) kagome nets, the emergent Dirac point (DP), flat band, and van Hove singularity (vHS) reveals the deep physics hidden behind the material itself, such as band topology and electronic correlation [1]. Referring to real kagome materials, the geometric frustration of corner-sharing triangles normally varies with element compositions, which produces fruitful phenomena in terms of magnetism [2–5], spin liquid [6,7], spin ice [8–10], quantum spin Hall effect (QSHE) [11–13], anomalous Hall effect (AHE) [2,14,15], quantum anomalous Hall effect (QAHE) [16,17], nontrivial topological states [5,16,18–21], pair density wave (PDW) [22,23], charge density wave (CDW) [3,4,24–28], nematic order [26,27,29,30], and superconductivity [26,27,31–35]. Significantly, the coexistence of multiple orders suggests kagome lattice is a solid ground to investigate the intertwined states [3,25–27,31,32,34,36,37]. Thus, more kagome materials candidates are desired to explore the underlying mechanisms.

It is noticeable that the kagome sublattice in most of the reported kagome materials such as A_xB_y ($A = \text{Mn, Fe, Co, K, Rb, Cs}$; $B = \text{Sn, Ge, Bi}$; $x:y = 1:1, 1:2, 3:1$ and $3:2$)

[18,34,38–40] and $A_xB_yC_z$ ($A = \text{K, Rb, Cs, Sc, Y, La, S}$; $B = \text{V, Nb, Ta, B, Ti}$; $C = \text{Sb, Sn, Rh, Bi, Co}$; $x:y:z = 1:2:3, 1:3:5, 1:6:6$ and $2:2:3$) [2,14,20,21,26,37,41,42] is made up by real atoms occupying specific Wyckoff sites. On the other hand, one notes that electrides, where excess electrons taking up the crystal interstitial sites can exhibit analogous characters to real atoms [43], have broad applications in catalysis [44,45], ion batteries [46,47], optical devices [48], spintronics [49,50], and superconductivity [51–56]. Then, one may ask whether there are any materials with specific properties where kagome lattices are built by the excess electrons rather than the real atoms. Here, we propose Mg_3N and Mg_3O as stable topological electrides with 2D kagome lattices constructed by the interstitial electrons. The electron-phonon coupling (EPC) between the Mg atoms and interstitial quasiatoms (ISQs) make a major contribution to the superconductivity, where the superconducting critical temperature T_c reaches 2.0 and 2.2 K for Mg_3N and Mg_3O , respectively.

In this work, the binary compounds Mg_3N and Mg_3O were identified as stable electrides by the phonon dispersion and *ab initio* molecular dynamics (AIMD) simulations (Fig. S1 in the Supplemental Material (SM) [57]). Remarkably, the excess electrons take up the crystal interstitial space and form 2D kagome lattices in the Mg layers (Fig. 2). Moreover, these ISQs dominate the bands near the Fermi level E_F and contribute to the topological states of Mg_3N (DPs) and Mg_3O [Dirac nodal lines (DNLs)] (Fig. 3). The in-plane EPC between the electronic orbitals and the atomic vibrational modes is much stronger than that of the out of plane (Fig. 4). The T_c calculated upon the Allen-Dynes-modified McMillan formula [58,59] is 2.0 and 2.2 K for Mg_3N and Mg_3O (Table I), respectively. These findings pave an alternative route to realize

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TABLE I. The calculated lattice constants a and c (Å), the electronic DOS at the Fermi level $N(E_F)$ (states/eV), the strength of EPC λ , $\omega_{\log}$ (K) and T_c (K) for Mg_3N and Mg_3O .

Electride	a	c	$N(E_F)$	λ	$\omega_{\log}$	T_c
Mg_3N	5.388	5.729	3.176	0.460	234.78	2.0
Mg_3O	5.386	5.402	2.959	0.467	241.71	2.2

kagome lattices in electrides and will facilitate future research on the potential intertwined orders in topological kagome electrides.

II. COMPUTATIONAL METHODS

The first-principles calculations were performed by the Vienna *ab initio* simulation package (VASP) based on the density functional theory (DFT) [60,61]. The Perdew-Burke-Ernzerhof (PBE) function with projector-augmented wave (PAW) pseudopotential within the generalized gradient approximation (GGA) was adopted to treat the exchange correlation potentials [62,63]. The cutoff energy of the plane wave and the force tolerance were set to 500 eV and -0.01 eV/Å, respectively. The convergence accuracy of the self-consistent iteration was selected as 1×10^{-6} eV/atom, and the k -point grid was set to $9 \times 9 \times 9$. The plane wave cutoff energy and k mesh were respectively increased to 800 eV and $19 \times 19 \times 19$ for precise description of the electron localized function (ELF) and partial charge density (PCD). Phonon spectra were obtained using the $2 \times 2 \times 2$ supercell by the PHONOPY code [64] within density functional perturbation theory (DFPT) approach [65]. The AIMD simulations were performed in the framework of the Nose-Hoover thermostat ensemble. The topological states were demonstrated by the WANNIERTOOLS (WT) package [66], where the tight-binding method was employed with the maximally localized Wannier functions (MLWFs) [67,68]. Since the elements involved in both compounds are very light, the spin-orbit coupling (SOC) effect was ignored. The SOC effect was also excluded in the phonon-related computations, because the SOC is less important in describing the vibration and superconductivity [33,69].

As for the superconducting properties, the QUANTUM ESPRESSO (QE) package [70] was adopted to perform the DFT calculations, in which the PAW pseudopotentials under the PBE scenario [62] were utilized. In detail, the cutoff energy for the wave functions and charge density were set as 100 and 800 Ry, respectively. Phonon dispersion was calculated by the DFPT method [65], in which a denser $18 \times 18 \times 18$ k -point grid and a $6 \times 6 \times 6$ q -point mesh were applied for the EPC calculations. According to the Migdal-Eliashberg equations [71,72], the magnitude of the EPC λ_{qv} , which describes the contribution to λ from individual phonon modes, can be evaluated by

$$\lambda_{qv} = \frac{\gamma_{qv}}{\pi \hbar N(E_F) \omega_{qv}^2}, \quad (1)$$

where γ_{qv} is the phonon linewidth, ω_{qv} stands for the frequency of a lattice vibration with crystal momentum q at the branch v , and $N(E_F)$ represents the density of states at the

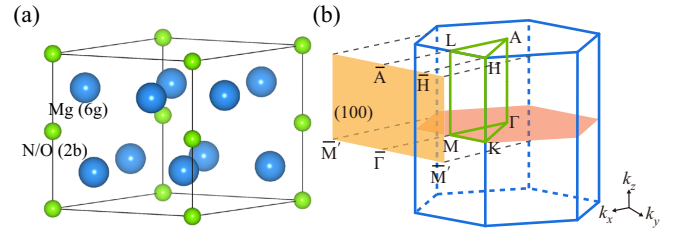


FIG. 1. (a) Crystal structure of Mg_3N and Mg_3O , where Mg and N (O) occupy the 6g and 2b Wyckoff sites, respectively. (b) The first Brillouin zone with high symmetry points and the projected (001) surface.

Fermi level. Besides, the phonon linewidth γ_{qv} can be written as

$$\gamma_{qv} = \frac{2\pi\omega_{qv}}{\Omega_{\text{BZ}}} \sum_{k,n,m} |g_{kn,k+qm}^v|^2 \delta(\epsilon_{kn} - \epsilon_F) \delta(\epsilon_{k+qm} - \epsilon_F), \quad (2)$$

where Ω_{BZ} is the volume of the Brillouin zone (BZ), n and m are the Kohn-Sham orbitals, ϵ_{kn} and ϵ_{k+qm} are the Kohn-Sham energies, and ϵ_F is the Fermi energy. The EPC matrix element $g_{kn,k+qm}^v$ can be self-consistently acquired with the linear response theory. Then, the cumulative frequency dependent EPC function $\lambda(\omega)$ can be obtained by

$$\lambda(\omega) = 2 \int_0^\omega \frac{\alpha^2 F(\omega')}{\omega'} d\omega', \quad (3)$$

where the Eliashberg electron-phonon spectral function $\alpha^2 F(\omega)$ is

$$\alpha^2 F(\omega) = \frac{1}{2\pi N(E_F)} \sum_{qv} \frac{\gamma_{qv}}{\omega_{qv}} \delta(\omega - \omega_{qv}). \quad (4)$$

As a result, the T_c can be derived as

$$T_c = \frac{\omega_{\log}}{1.2} \exp \left[-\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right], \quad (5)$$

in which μ^* is the effective screened Coulomb repulsion constant with a typical value of 0.1 [73,74], and $\omega_{\log}$ is the logarithmic average frequency in terms of

$$\omega_{\log} = \exp \left[\frac{2}{\lambda} \int_0^\infty \frac{d\omega}{\omega} \alpha^2 F(\omega) \log \omega \right]. \quad (6)$$

III. RESULTS AND DISCUSSION

Mg_3N and Mg_3O have the anti- TiCl_3 structure with space group $P6_3/mcm$ (No. 193) [Fig. 1(a)], which is similar to Cs_3O and Ba_3N [75], where alkaline-earth metal and non-metal atoms occupy the 6g: $(x, x, \frac{3}{4})$ and 2b: $(0, 0, \frac{1}{2})$ Wyckoff sites, respectively. As tabulated in Table I, the calculated lattice constants are $a = b = 5.388$ Å (5.386 Å) and $c = 5.729$ Å (5.402 Å) for Mg_3N (Mg_3O). Based on the optimized structures, the computed phonon spectra along the selected high symmetry lines [see Fig. 1(b)] are displayed in Figs. S1(a) and (c) for Mg_3N and Mg_3O , respectively. The absence of imaginary frequency mode indicates that both Mg_3N and Mg_3O are dynamical stable. As shown in Figs. S1(b) and (d), the AIMD simulations evaluated the thermodynamic stability of Mg_3N and Mg_3O at room temperature (300 K). Then, we compute

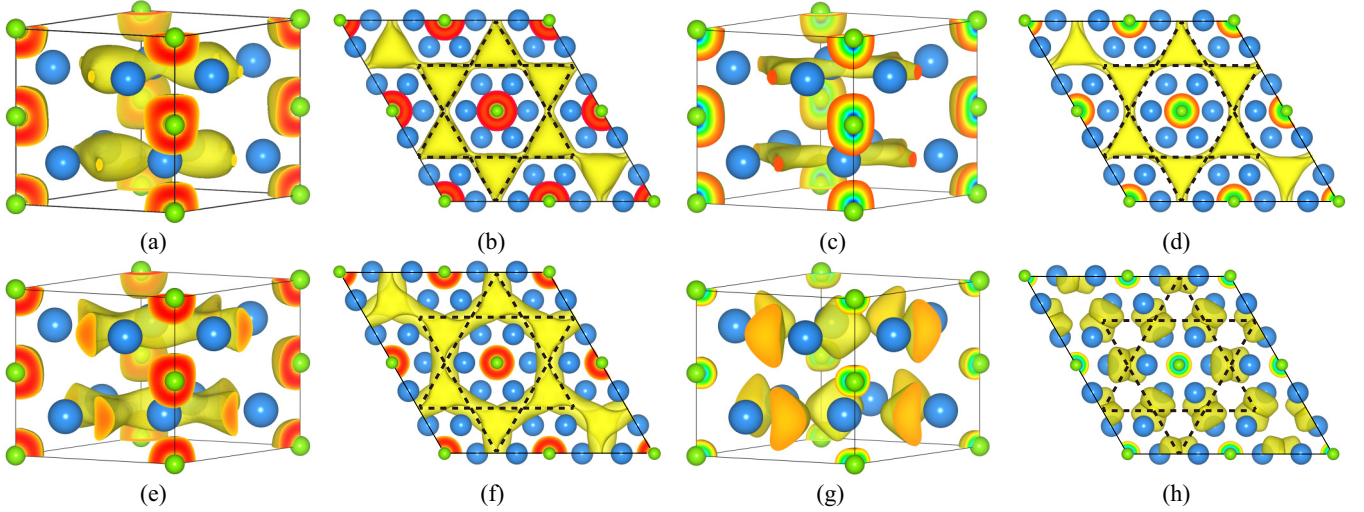


FIG. 2. The calculated ELF and PCD of Mg_3N and Mg_3O . Side view of the ELF for Mg_3N (a) and Mg_3O (e) with a same isosurface $0.67 \text{ e}/\text{\AA}^3$. The corresponding top view for the $2 \times 2 \times 2$ supercell is displayed in (b) and (f), respectively. Side view of the PCD for Mg_3N (c) and Mg_3O (g) with an isovalue $0.009 \text{ e}/\text{\AA}^3$ and $0.013 \text{ e}/\text{\AA}^3$, respectively (see also Fig. S3 [57]). (d) and (h) correspond to the top view of the $2 \times 2 \times 2$ supercell, respectively. The PCD is calculated for the color bands that cross the Fermi level in Figs. 3(a) and 3(h).

the formation energy by $E_f = (E_{\text{bulk}} - 6\mu_{\text{Mg}} - 2\mu_{\text{N/O}})/8$. Here μ_{Mg} and $\mu_{\text{N/O}}$ are the energies of each Mg and N/O atom in their most stable elemental crystal/gas, while E_{bulk} is the total energy of Mg_3N and Mg_3O unit cells. The calculated E_f of Mg_3N and Mg_3O are -0.238 and -1.084 eV per atom, demonstrating the preference formation of Mg_3N and Mg_3O rather than the pure Mg and N/O phases. As shown in Fig S2, Mg_3N and Mg_3O are above the convex hull [57,76,77]. Then, one can infer that these two electrides are metastable and might be experimentally synthesized by designing suitable conditions and methods.

The ELF displayed in Figs. 2(a) and 2(e) clearly reveal that anions localize around the nonmetal atoms and distribute in the crystal interstitial space within the Mg layers of Mg_3N and Mg_3O . The localized electrons around nonmetal atoms can ascribe to the larger electronegativities of N (~ 3.04) and O (~ 3.44) than that of Mg (~ 1.31) [78]. Noticeably, as presented in Figs. 2(b) and 2(f), the interstitial anions provided by excess electrons of Mg atoms build 2D kagome lattices for Mg_3N and Mg_3O . Combining with the PCDs for these bands that cross the Fermi surface [79–81] (Figs. 2(c), 2(d), 2(g), and 2(h), and Fig S3 [57]), one can treat Mg_3N and Mg_3O as electrides [82]. Though there is slight differences in morphology of ELF and PCDs between Mg_3N and Mg_3O , which might be due to variations in crystal and electronic structures that arise from the different element compositions, both Mg_3N and Mg_3O can be regarded as kagome electrides [54,83].

Actually, the different element compositions indeed have some effects on the electronic properties of Mg_3N and Mg_3O . As shown in Figs. 3(a) and 3(h), several bands cross the Fermi level, indicating the metallic character of Mg_3N and Mg_3O . The characteristic features of kagome bands [54,84] can be revealed by the DPs that are highlighted by dashed circles in Figs. 3(a) and 3(h) for Mg_3N and Mg_3O , respectively. However, these DPs are well above the Fermi level that is not conducive to experimental observation. Then, we focus on the bands below the Fermi level and the topological

elements are marked by solid circles in Figs. 3(a) and 3(h), respectively. To fully illustrate the electronic properties, the irreducible representations (irreps) of the little group at distinct k points in the BZ are obtained [85]. As for Mg_3N , the little group of the high symmetry H point ($\frac{1}{3}, \frac{1}{3}, \frac{1}{2}$) has three generating elements, i.e., S_3^+ , C_{21}'' , and PT . Here, S_3^+ denotes $\{S_3^+|000\}$, C_{21}'' indicates $\{C_{21}''|00\frac{1}{2}\}$, and PT stands for the combined inversion symmetry P and time-reversal symmetry T . Then, the obtained irreps, written in the BCS convention [86,87], of H would be $H5 + H6$ and the effective Hamiltonian model $\mathcal{H}(k)$ for the emergent particle at the H point is constrained by $\Delta(\mathcal{O})\mathcal{H}(k)\Delta^{-1}(\mathcal{O}) = \mathcal{H}(\hat{\mathcal{O}}k)$ [88]. Here $\mathcal{O} \in \{S_3^+, C_{21}'', PT\}$ is one of the three operators and $\Delta(\mathcal{O})$ denotes the corresponding matrix representation at the H point [88]. The obtained model given in Ref. [88] shows that the set of bands whose irreps is $H5 + H6$ forms a fourfold nodal point, namely a Dirac point, as shown in Figs. 3(e) and 3(f). Besides, the intersections of bands Q1+Q3 and Q2+Q4 form the fourfold degenerate Dirac points DP2 locating at the high-symmetry line $A-H$, near the H point, as illustrated in Figs. 3(e) and 3(f).

Referring to Mg_3O , the intersections of the twofold bands LD1 and LD4 form DP along the $M-K$ path. Combining with the inversion symmetry P , the time-reversal symmetry T , and the rotation symmetry C_{3z}^+ [88], there would be six DNLs in the whole BZ [see Fig. 3(m)]. The topology of DNLs can be further evaluated by the Berry phase in terms of $\gamma_n = \oint_C d\mathbf{R} \cdot \mathbf{A}_n(\mathbf{R})$, in which C is a closed path encircling a generic point of the DNL and the vector $\mathbf{A}_n(\mathbf{R})$ stands for the Berry connection [89]. Remarkably, each DNL generates a Berry phase of π , implying that each DNL is nontrivial. Besides, the acquired edge states for the DPs of Mg_3N and the DNLs of Mg_3O , which are projected to the (100) surface [Fig. 1(b)], are well demonstrated in Figs. 3(g) and 3(n), respectively.

It should be pointed out that when one takes excess electrons as interstitial quasiatoms [79,80,90,91], i.e., places ISQs at the $4c$: ($\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$) Wyckoff sites to construct

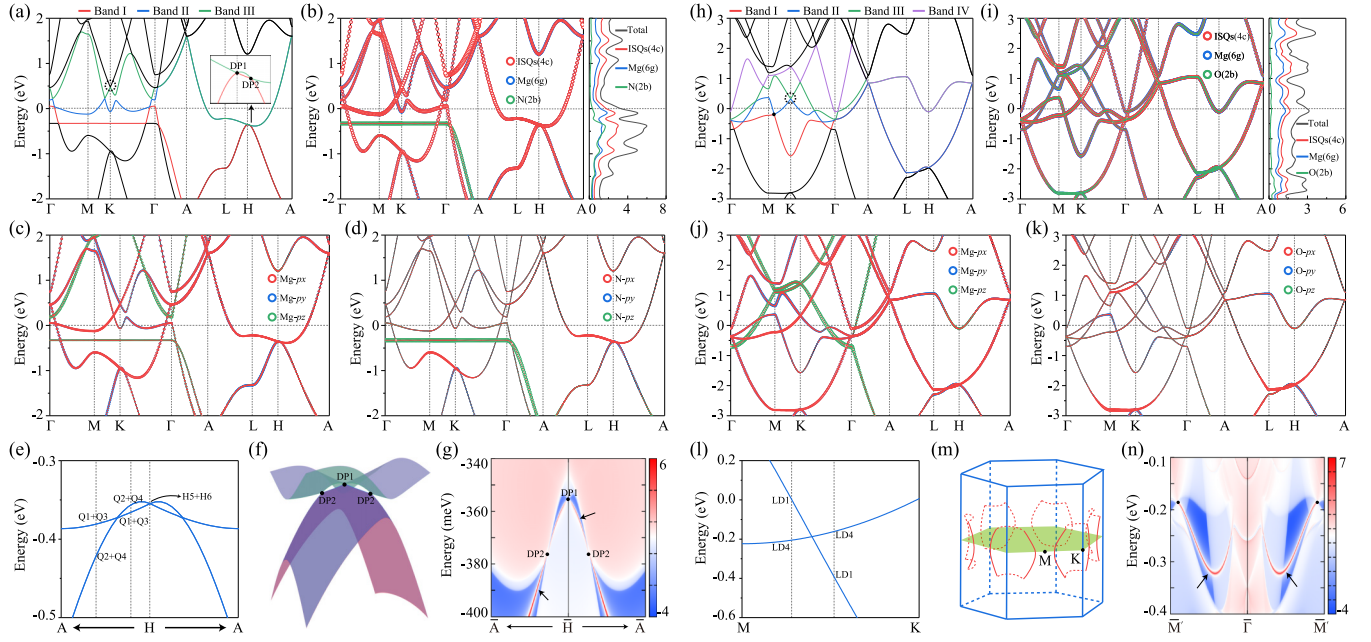


FIG. 3. Electronic structure and topological elements of Mg_3N (left panel) and Mg_3O (right panel). Band structure of Mg_3N (a) and Mg_3O (h). The bands crossing the Fermi level are highlighted by different colors. The inset in (a) is an enlarged view for the DP1 and DP2. The calculated projected band structure and density of states of Mg_3N (b) and Mg_3O (i). The orbital-resolved band structures of Mg_3N (c), (d) and Mg_3O (j), (k). The computed irreducible representations for the bands forming the topological elements of Mg_3N (e) and Mg_3O (l). Distribution of the DPs (f) and the DNLs (m). The topological surface states of Mg_3N (g) and Mg_3O (n), where the band crossings and edge states are indicated by the black solid circles and arrows, respectively.

kagome nets for Mg_3N and Mg_3O [see dashed lines in Figs. 2(b), 2(d), 2(f), and 2(h)], one can obtain projected band structures (PBANDS) and projected density of states (PDOS) including contributions from ISQs. Figures 3(b) and 3(i) suggest Mg and ISQs dominate the PBANDS and PDOS around the E_F , where the in-plane orbits (p_x and p_y) make a greater contribution than that of the out-of-plane orbits (p_z) for Mg, N, and O [Figs. 3(c), 3(d), 3(j), and 3(k)]. Interestingly, the above phenomena happen to coincide with the distribution of ELF and PCDs in Fig. 2, that is, the localized electrons mostly lie in the plane of Mg layers (see also Fig. S3 [57]). Then, one may ask what about the atomic vibrations?

As shown in Figs. 4(a) and 4(c), the distinctive atomic vibration information can be roughly summarized as follows: the vibrations in low-frequency regions are dominantly originated from Mg atoms (0–40 meV for Mg_3N and 0–30 meV for Mg_3O); the medium-frequency vibrations mainly come from the Mg and nonmetal atoms (40–55 meV for Mg_3N and 30–50 meV for Mg_3O); and the high-frequency vibrations are dominated by N and O atoms (60–85 meV for Mg_3N and 50–70 meV for Mg_3O). Furthermore, combining with Figs. 4(b) and 4(d), it is clear that the superconducting parameter λ_{qv} mainly comes from the vibrations of Mg atoms, especially the in-plane vibrations. Moreover, the vibrations trigger some Eliashberg spectra function $\alpha^2F(\omega)$ peaks and subsequently form the accumulated EPC constants $\lambda(\omega)$. Meanwhile, combining with the ISQs distribution in Figs. 2 and 3, the interactions between the kagome lattice electrons (ISQs) and the in-plane vibration modes of Mg atoms may play an essential role in the EPC. Finally, according to the BCS theory, the superconducting parameters $\omega_{\log}$, λ , and T_c

for Mg_3N (Mg_3O) are 234.8 (241.7) K, 0.46 (0.47), and 2.0 (2.2) K, respectively (Table I). Noticeably, the total EPC λ suggests the electron-phonon strength is weak in our predicated 2D kagome electrides [52–55] and the obtained T_{cs} are comparable to the reported superconducting electrides Li_5Si (~ 1.1 K) [54], Li_5P (~ 2.61 K) [52], Li_6P (~ 2.87 K for the $P1$ phase) [52], Li_5C (~ 3.1 K) [53], and Ba_2N (~ 3.4 K) [55].

It is noticeable that the different electronic orbitals at the Fermi level could lead to the emergence of superconducting multigaps [92–95]. As shown in Fig. 3, several orbitals, such as p (p_x , p_y , and p_z) orbitals of Mg atoms, in-plane p (p_x and p_y) orbitals of O (N) atoms, and ISQs, emerge at the E_F , which means that the superconducting multigaps might be observed for Mg_3N and Mg_3O . However, it is quite costly and time consuming to obtain the superconducting multigaps, and here we will focus on the realization of kagome lattice and superconductivity in Mg_3N and Mg_3O .

Since the interplay between the charge density wave (CDW), the structural (electronic) instability, and the magnetism has been reported for kagome materials [4, 74, 93, 96, 97], we proceed with investigations for these properties. The results of the temperature-dependent AIMD simulations (1–300 K) (Fig. S4) [57] indicate these two proposed electrides are stable and will not undergo structural phase transition. Meanwhile, Mg_3N and Mg_3O are confirmed as nonmagnetic materials, which implies no magnetic phase transition. Moreover, there are no soft modes around the high-symmetry points or special routes in the phonon dispersion, suggesting the structural phase transition is absent. Finally, there are no flat band or van Hove singularity around the E_F (Fig. 3), and hence the Fermi nesting is absent in the

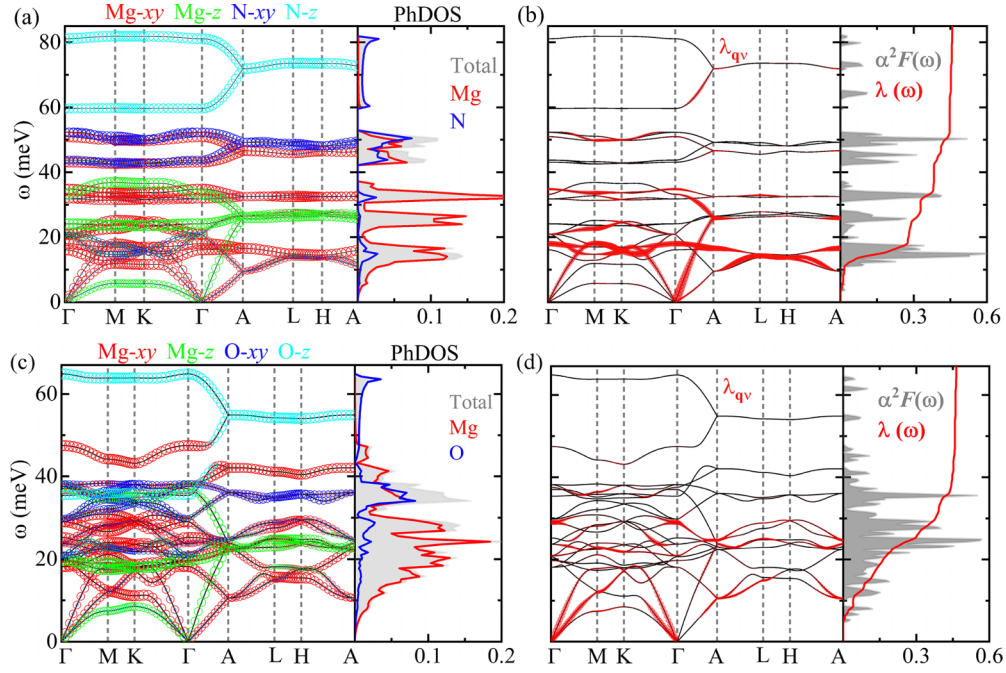


FIG. 4. Phonon and superconducting features of Mg_3N and Mg_3O . Phonon dispersion weighted by different atomic vibrational modes of Mg_3N (a) and Mg_3O (c). The right panels of (a) and (c) are the total (gray shaded zone) and vibrational mode-resolved (colored lines) phonon density of states (PhDOS). (b) and (d) demonstrate the phonon dispersion weighted by the magnitude of the EPC, the Eliashberg spectral function $\alpha^2 F(\omega)$, and the integrated strength of EPC $\lambda(\omega)$.

Fermi surface (Fig. S5) [57]. Thus, there is no magnetism, no structural instability, or electronic instability in Mg_3N and Mg_3O , which indicates the CDW would be absent.

In the end, we evaluate the influence of pressure on the lattice displacement and superconductivity. For simplicity, we take Mg_3N as an example and the pressure is set to 5 GPa. The in-plane (Fig. S6), out-of-plane (Fig. S7), and hydrostatic pressure (Fig. S8) is respectively applied on Mg_3N [57]. The corresponding superconducting parameters ω_{log} , λ , and T_c are estimated to be 240, 205, 174 K, 0.44, 0.63, 0.66 and 1.7, 5.4, 5.3 K, respectively. The results suggest that the out-of-plane (along the c axis) pressure can effectively enhance the EPC strength λ and T_c . However, the in-plane (along the ab axes) pressure suppress them. Meanwhile, with the introduction of pressure, the values of ω_{log} are decreasing. As depicted in Figs. S8 and S9, the in-plane vibrations of Mg and N atoms are more susceptible to the out-of-plane pressure, while the in-plane vibrations respond more slowly to the in-plane pressure. Moreover, as shown in Figs. S8–S10, the in-plane vibration of N atoms around 15 meV plays an essential role in the EPC under pressure, rather than the in-plane vibration of Mg atoms. The influence of different pressures on the superconductivity will be illustrated in the next work.

IV. CONCLUSION

In summary, kagome lattice and superconductivity were realized in the stable topological electrides Mg_3N and Mg_3O . Remarkably, the excess electrons donated by Mg atoms dis-

tribute within the Mg layers and construct the 2D kagome lattices. These ISQs contribute to the nontrivial topological elements and take part in the interplay between the in-plane electronic orbitals and atomic vibrations that act as an important role in the EPC. The superconductivity in both systems are primarily contributed by the electronic states and phonon vibrations from the Mg atoms and the ISQs. This work proposes two stable superconducting electrides and supplies a potential playground to investigate the band topology and superconductivity in kagome electrides.

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