Editors' Suggestion

Shear viscosity of interacting graphene

Kitinan Pongsangangan,[1,2] Pedro Cosme,[1,3] Emanuele Di Salvo,[1] and Lars Fritz[1]
[1]*Institute for Theoretical Physics and Center for Extreme Matter and Emergent Phenomena, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands*
[2]*Institute of Theoretical Physics, Technische Universität Dresden, 01062 Dresden, Germany*
[3]*GoLP - Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisbon, Portugal*

(Received 26 October 2023; revised 4 June 2024; accepted 26 June 2024; published 29 July 2024)

One of the hallmark properties of fluids is their shear viscosity which is, among other things, responsible for parabolic flow profiles through narrow channels. In recent years, there has been a growing number of observations of said flow profiles in electronic transport measurements in a variety of material systems, most notably in graphene. In this paper, we investigate the shear viscosity of interacting graphene from a theoretical point of view. We study both a phenomenological as well as a microscopic model and find excellent agreement between the two. Our main finding is collective modes make a sizable contribution to the viscosity that can equal or even outweigh the electronic contribution that is usually assumed dominant. We comment on how this finding carries over to systems beyond graphene and related Dirac materials.

DOI: 10.1103/PhysRevB.110.045443

I. INTRODUCTION

Hydrodynamics describes the flow properties of classical fluids such as water, air, or plasmas. Its foundations are conserved quantities, such as mass, momentum, and energy, and their slow relaxation towards local equilibrium. This relaxation is driven by interactions between the constituents of the fluid. A direct consequence of said interactions is their shear viscosity. This manifests itself in a friction or drag between adjacent layers of the fluid in the presence of a velocity gradient [1].

In recent years, hydrodynamics gained renewed attention across many disciplines, including condensed-matter physics [2–8]. It is understood as an emergent universal description of dynamical properties of strongly correlated systems in their long-wavelength and low-frequency limit [9]. The framework is generic and applicable to the study of properties of quantum phase transitions [2,10] as well as to quantum spin liquids [11] or magnon dynamics in ferromagnets [12]. Hydrodynamic behavior in more conventional electronic solid-state systems, such as metals, had been elusive for many decades [13]. The main obstacles are usually dominant competing scales due to the underlying lattice of solid-state systems in the form of impurities and lattice vibrations. These limitations have been overcome in recent years, with graphene sitting at the front of this development [14–18].

In the conventional theory of hydrodynamic graphene, there is an underlying assumption: The fluid is composed of electrons and holes that equilibrate locally due to collisions mediated by (long-range) interactions [6,19–28]. However, these interactions not only lead to local equilibration, the main assumption underlying hydrodynamics, they also facilitate the emergence of collective excitations, such as charge-density oscillations called plasmons [29–31]. Those are proper quasiparticles in their own right: In three-dimensional metals, plasmons are inert due to their large excitation gap [32]. In two dimensions, however, they follow a square-root dispersion relation [33–35]. Being gapless, they are important for low-energy equilibrium, for properties such as superconducting instabilities [36,37], and near-equilibrium phenomena [38,39] such as hydrodynamic transport phenomena [40–43].

In recent works, some of us have studied the thermoelectric response of interacting two-dimensional Dirac systems at and away from the Dirac point. We found a small enhancement of the thermal conductivity due to plasmons at the charge-neutrality point. Away from the Dirac point, however, a strong enhancement of the thermal conductivity is found. This can be explained from the undamped gapless nature of plasmons together with their increasing density of states [41–43]. These works suggest that every quantity that depends on energy is sensitive to plasmons. The shear viscosity is directly related to the stress-energy tensor and consequently potentially sees a large plasmon effect.

In this paper, we study the effect of plasmons on the shear viscosity in interacting graphene or, more generally, two-dimensional Dirac theories at varying fillings (our findings are in fact more generically relevant for two-dimensional Fermi liquids). The shear viscosity of the interacting electron-hole fluid in charge-neutral graphene was estimated before in Refs. [22,44]. Now, we go further, including the contribution of the plasmons on equal footing [45]. A corresponding experiment has recently been carried out not only at the charge-neutrality point but also at moderately high dopings corresponding to the Fermi-liquid regime [18]. Thus, the goal of this paper is twofold: (i) to find an effective hydrodynamic description of a viscous fluid composed of electrons, holes, and plasmons, all on equal footing; and (ii) to extend the previous theoretical calculation of the shear viscosity from Ref. [22] away from the Dirac point into the Fermi-liquid regime. We use two complementary approaches: an effective Chapman-Enskog approach and a microscopic description in terms of coupled Boltzmann equations for electrons, holes, and plasmons.

Our main result is shown in Fig. 3 which shows the sizable enhancement of the viscosity due to the plasmons.

II. MODEL

We model the noninteracting electronic structure with a low-energy Dirac Hamiltonian of the type

$$\hat{H}_0 = \int d\mathbf{x}\, \hat{\Psi}^\dagger_{i,\lambda}(\mathbf{x})[-i\hbar v_F \sigma\cdot\nabla_{\mathbf{x}} - \mu + V_{\rm ex}(\mathbf{x})]_{\lambda\lambda'}\hat{\Psi}_{i,\lambda'}(\mathbf{x}). \tag{1}$$

The operators $\hat{\Psi}^\dagger_{i,\lambda}(\mathbf{x})$ and $\hat{\Psi}_{i,\lambda}(\mathbf{x})$ create and annihilate an electron at a position $\mathbf{x}$. The effective Fermi velocity is given by v_F, approximately 10^6 m/s for graphene, while the filling is controlled by a chemical potential μ. The static potential $V_{\rm ex}(\mathbf{x}) = n_0\int d\mathbf{x}' V(\mathbf{x}-\mathbf{x}')$ is added in order to account for the positive charge background of average density n_0 in which the electrons move. The flavor index, denoted i, ranges from $i = 1,\ldots,N = 4$ accounting for spin and valley degrees of freedom (for Dirac systems with a different number of flavors this is an adjustable parameter of the theory). The symbols $\lambda,\lambda' \in \{+,-\}$ denote spinor indices and $\sigma = (\sigma_x,\sigma_y)$ are Pauli matrices (note that double indices are summed over) [46]. Additionally, electrons and holes interact via Coulomb interaction according to

$$\hat{H}_I = 1/2\int d\mathbf{x}d\mathbf{x}'\, \hat{\Psi}^\dagger_{i,\lambda}(\mathbf{x})\hat{\Psi}^\dagger_{i,\lambda'}(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\hat{\Psi}_{i,\lambda'}(\mathbf{x}')\hat{\Psi}_{i,\lambda}(\mathbf{x}). \tag{2}$$

The instantaneous Coulomb interaction between two electrons of charge e locating at $\mathbf{x}$ and $\mathbf{x}'$ is given by the usual $V(\mathbf{x}-\mathbf{x}') = e^2/(4\pi\epsilon|\mathbf{x}-\mathbf{x}'|)$, where ϵ is the average dielectric constant, measuring the average value of the dielectric constant of materials above and below it. The strength of the Coulomb interaction is typically characterized by the ratio of the potential energy to the kinetic energy. This ratio boils down to the fine-structure constant $\alpha = e^2/4\pi\epsilon\hbar v_F$ for Dirac systems. For suspended graphene in vacuum, $\epsilon = 1$ and $\epsilon \approx 7$ for graphene sandwiched between hexagonal boron nitride layers. Thus, for those two cases, $\alpha = 2.2$ and $\alpha \approx 0.3$, respectively [47].

Contrary to Ref. [22], we go beyond perturbation theory in α and explicitly include collective excitations, most notably plasmons. A key complication in this approach is to find an approximation that respects conservation laws and explicitly does not double count degrees of freedom. The basis is the proper use of the random-phase approximation (RPA) which leads to a set of coupled (kinetic) equations of electrons, holes, and plasmons, all on equal footing. The whole procedure is very technical and discussed at length in Refs. [41–43].

III. FLUID

In the hydrodynamic framework of the RPA, the "fluid" is composed of three types of particles, all on equal footing: electrons, holes, and plasmons (see Fig. 1). We introduce an index λ that distinguishes electrons ($\lambda = +$) and holes ($\lambda = -$). The respective energies read $\epsilon_{\mathbf{k},\lambda} = \lambda v_F|\mathbf{k}|$, and the velocity of a quasiparticle is given by $\mathbf{v}_{\mathbf{k},\lambda} = \nabla_{\mathbf{k}}\epsilon_{\mathbf{k},\lambda} = \lambda v_F\hat{\mathbf{k}}$. Note that from this point onwards we set $\hbar = k_B = 1$. Here, $\hbar$ is the reduced Planck constant and k_B denotes the Boltzmann

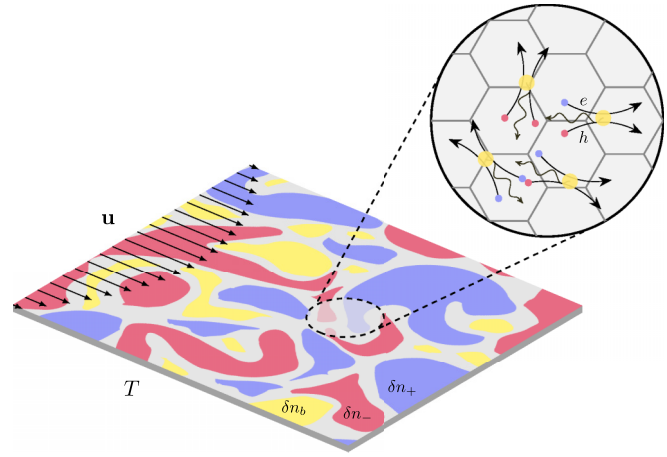

FIG. 1. Beyond weak coupling, the fluid of hydrodynamic graphene is composed of three types of particles, electrons, holes, and plasmons, allowing for the local variation of the respective number densities δn_+, δn_-, and δn_b. These particles contribute equally to viscosity, which leads to a parabolic flow profile of the joint velocity $\mathbf{u}$ through a narrow channel. The three components are coupled via the long-range collisions [cf. Eqs. (4)], here schematically represented in the inset.

constant. We denote the corresponding distribution functions $f_\lambda(\mathbf{k},\mathbf{x},t)$ that reduce to the Fermi function in thermal equilibrium.

For the plasmons, we review the most salient features here (more details can be found in Refs. [41–43] and cited papers). The plasmon dispersion relation is given by

$$\omega_{\mathbf{q}} = \sqrt{\frac{N}{2}\alpha v_F T\log[2+2\cosh(\mu/T)]q} \equiv \sqrt{\mathcal{N}q}, \tag{3}$$

where T is the temperature and q is the modulus of the two-dimensional momentum. The velocity of the plasmons is given by $\mathbf{w}_{\mathbf{q}} = \nabla_{\mathbf{q}}\omega(\mathbf{q})$. The requirement for a Boltzmann equation to be valid is the existence of well-defined quasiparticle excitation. A plasmon of high momentum is overly damped into a particle-hole continuum, so only the plasmon of small momentum, or long wavelength, are well defined. Consequently, there is a momentum cutoff for plasmons beyond which their spectral function is broadened significantly. The momentum cutoff is found to be $q_c = \mathcal{N}$ [41]. The distribution function of the plasmons is henceforth called $b(\mathbf{q},\mathbf{x},t)$.

Although the fluid is composed of three distinct species with differing quasiparticle velocities, all the components are locked in a uniform flow of velocity $\mathbf{u}$. This locking is a consequence of entropic constraints (see the Supplemental Material [48]). Therefore, the viscous forces from each sector—be they fermions or plasmons—arise from the same velocity field.

IV. EFFECTIVE BOLTZMANN EQUATIONS

Formally, the Boltzmann equations (BEs) can be derived from the Keldysh formalism. Here, however, we content ourselves with using them and refer the reader to Refs. [41–43].

We find three coupled equations:

$$\partial_t f_\lambda + \mathbf{v}_{\mathbf{k},\lambda}\cdot\nabla_{\mathbf{x}} f_\lambda - \nabla_{\mathbf{x}}\epsilon_{\mathbf{k},\lambda}\cdot\nabla_{\mathbf{k}} f_\lambda = \mathcal{I}^f_\lambda[f,b],$$
$$\partial_t b + \mathbf{w}_{\mathbf{q}}\cdot\nabla_{\mathbf{x}} b - \nabla_{\mathbf{x}}\omega_{\mathbf{q}}\cdot\nabla_{\mathbf{q}} b = \mathcal{I}^b[f,b]. \quad (4)$$

The left-hand side (LHS) contains the so-called streaming terms accounting for spatial and temporal variations. The right-hand side contains the collision terms, $\mathcal{I}^b[f,b]$ and $\mathcal{I}^f_\lambda[f,b]$, that couple the three equations explicitly. A detailed discussion of the scattering terms has been provided in the Supplemental Material [48]. It is important to note that for the problem at hand we do not require a translational symmetry-breaking collision term for the reason that viscosity manifests itself in a situation in which translational symmetry is broken by boundary conditions.

Performing momentum integrals with respect to the moments of the distribution function allows to derive the conservation laws. In the present paper we are considering the first two moments, i.e., $\int_{\mathbf{k}}\delta f_{\lambda,\mathbf{k}}$ and $\int_{\mathbf{k}}\mathbf{k}\delta f_{\lambda,\mathbf{k}}$, that state conservation of particle number and momentum; in particular, we can evaluate the momentum flux, which comprises the average momentum current $\mathbf{u}\bar{\mathbf{p}}$, pressure P, and viscous terms as $\mathbf{\Pi} = \mathbf{u}\bar{\mathbf{p}} + \mathbf{1}P + \delta\mathbf{\Pi}$, where the last term corresponds to the viscous contribution. Decomposing the distribution functions into their equilibrium and fluctuation correction contributions, i.e., $f_\lambda = f^0_{\lambda,\mathbf{k}} + \delta f_{\lambda,\mathbf{k}}$ and $b = b^0_{\mathbf{q}} + \delta b_{\mathbf{q}}$, allow us to write

$$\delta\mathbf{\Pi} = \sum_\lambda N\int\frac{d\mathbf{k}}{(2\pi)^2}\mathbf{v}_{\lambda,\mathbf{k}}\,\mathbf{k}\,\delta f_{\lambda,\mathbf{k}} + \int\frac{d\mathbf{q}}{(2\pi)^2}\mathbf{w}_{\mathbf{q}}\mathbf{q}\delta b_{\mathbf{q}}. \quad (5)$$

We use two complementary methods to account for the fluctuations of the distribution functions coming from the collision operators: (a) the Chapman-Enskog method and (b) the the linear-response method, including a numerical solution of the BEs.

V. THE CHAPMAN-ENSKOG APPROACH

As a first approach to calculate the viscosity of the three-component fluid, we resort to the Chapman-Enskog (CE) method [49] with a Bhatnagar-Gross-Krook (BGK) approximation [50,51] for the collision operators. This is a way to effectively decouple the equations using the so-called relaxation time approximation:

$$\delta f_{\lambda,\mathbf{k}} = \tau_\lambda\mathfrak{D} f^0_\lambda \quad\text{and}\quad \delta b_{\mathbf{q}} = \tau_b\mathfrak{D} b^0. \quad (6)$$

This is the leading-order CE expansion, where the differential operator $\mathfrak{D} \doteq \partial_t + \mathbf{v}_{\mathbf{k},\lambda}\cdot\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}}\epsilon_{\mathbf{k},\lambda}\cdot\nabla_{\mathbf{k}}$ is shorthand for the LHS of Boltzmann equations, Eq. (4), and the relaxation times τ_λ and τ_b combine the collision mechanisms, providing an effective phenomenological description of the problem [52,53].

The velocity of electrons (holes) with respect to the fluid velocity reads $\mathbf{c}_\lambda = \lambda v_F\hat{\mathbf{k}} - \mathbf{u}$, which allows to rewrite the streaming term of the Boltzmann operator as

$$\mathfrak{D} f^0_\lambda = f^0_\lambda(1-f^0_\lambda)\left\{\frac{\mathbf{c}_\lambda\cdot\nabla T}{T}\left(\frac{\mathbf{c}_\lambda\cdot\mathbf{k}}{T} - \frac{3P_\lambda}{nT}\right) + \frac{\mathbf{k}}{T}\cdot\left[(\mathbf{c}_\lambda\cdot\nabla)\mathbf{u} - \frac{\mathbf{c}_\lambda - \mathbf{u}}{2}\nabla\cdot\mathbf{u}\right]\right\}, \quad (7)$$

where we made use of the Euler fluid equations to eliminate the time derivatives. This leads to a viscous tensor of the form

$$\delta\mathbf{\Pi}_\lambda = -\frac{3\tau_\lambda P_\lambda}{4}(2\dot{\boldsymbol{e}} - \mathbf{1}\,\mathrm{Tr}\,\dot{\boldsymbol{e}}), \quad (8)$$

where $\dot{e}_{ij} = (\partial_i u_j + \partial_j u_i)/2$ is the strain-rate tensor. The relaxation time τ shown here is to be suitably chosen following Eq. (6) and it is assumed to have no dependence on the momentum as a first approximation. Thus, the shear viscosity due to the fermions can be identified as $\eta = \sum_\lambda 3\tau_\lambda P_\lambda/4$. Note that there is no contribution of the bulk viscosity, as expected for systems with linear dispersion [54]. Moreover, the Fermi-liquid limit of this result agrees with the recent literature [55]. Additionally, if one wishes to account for the fermionic self-energy contributions this can easily be achieved by accounting for renormalization effects via the Fermi velocity. However, such corrections have a negligible impact on the global behavior of the viscosity (see the Supplemental Material [48]).

For the plasmons, we start with the aforementioned square-root dispersion relation $\omega(q) = \sqrt{\mathcal{N}q}$, The relative velocity is defined as $\mathbf{C} = \partial_q\omega\hat{\mathbf{q}} - \mathbf{u}$, implying that $\omega(q) = 2(\mathbf{C}+\mathbf{u})\cdot\mathbf{q}$. This treatment makes the *a priori* unjustified assumption that the fluid of plasmons flows at the same velocity as the fluid of the electrons and holes. However, this is not an assumption but can in fact be derived from the underlying BE (see the Supplemental Material [48]). The streaming term now reads

$$\mathfrak{D} b^0 = b^0(1+b^0)\left\{\frac{\mathbf{q}\cdot(2\mathbf{C}+\mathbf{u})}{T^2}\mathbf{C}\cdot\nabla T + \frac{\mathbf{q}}{T}\cdot\left[(\mathbf{C}\cdot\nabla)\mathbf{u} - \frac{3}{5}\mathbf{C}\nabla\cdot\mathbf{u} + \frac{9}{20}\mathbf{u}\nabla\cdot\mathbf{u}\right]\right\}. \quad (9)$$

Choosing the correct form of the relaxation time for the plasmons is more delicate and in principle relies on a microscopic treatment of the coupled equations. However, since the coupled equations can be understood as a wave-particle interaction, it is natural to consider Landau damping as the main relaxation mechanism. From RPA calculations [42,43] the decay rate is given by

$$\frac{1}{\tau_b} = -\frac{\pi\omega(q)^2[n_f[\omega(q)/2] - n_f[-\omega(q)/2]]}{8T\log(2+2\cosh\mu/T)}, \quad (10)$$

where $n_f(\cdot)$ is the equilibrium Fermi-Dirac distribution. We used Eq. (10) to compute the bosonic contribution to viscosity. The viscous tensor is given by

$$\delta\mathbf{\Pi}_b = -\eta_b\left[2\dot{\boldsymbol{e}} - \mathbf{1}\,\mathrm{Tr}\,\dot{\boldsymbol{e}} - \mathbf{1}\frac{\nabla\cdot\mathbf{u}}{10}\right], \quad (11)$$

where the viscosity is given by the integral

$$\eta_b = \int_0^{q_c}\frac{d}{dq}\left(\frac{\mathcal{N}^2\tau_b(q)}{2}q^{5/2}\right)b^0\,dq. \quad (12)$$

We limit the integration to the cutoff at q_c in order to ensure that the contribution of plasmons is bounded within the range where they are long lived. The evaluation of the shear viscosity, arising from the plasmon sector, is showcased in Fig. 2.

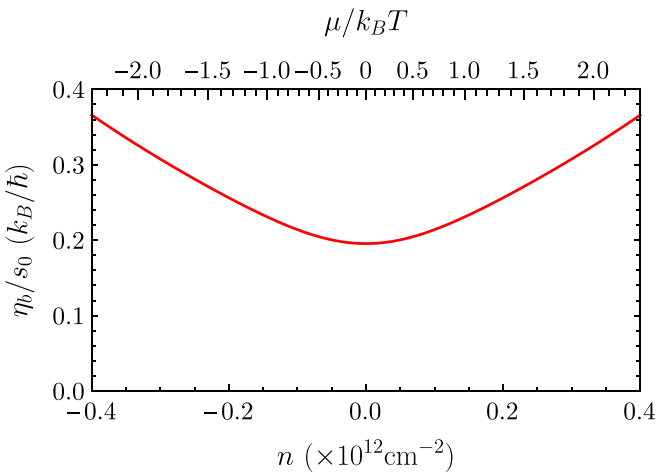

FIG. 2. Plasmonic contribution to viscosity $\alpha = 0.9$ evaluated at $T = 300$ K from Eq. (12).

Note that, contrary to the case of fermions, when taking the divergence of this tensor, one will get both shear viscosity and bulk viscosity $\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_b = -\eta_b[\nabla^2\mathbf{u} - \frac{1}{10}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{u})]$, the latter being, however, small compared to the shear part.

We conclude that there is an effect of the plasmons on the total viscous forces.

While the relaxation time approximation is very useful to understand the qualitative features of the system, it does not allow for quantitative statements, because it relies on a particular choice of the relaxation time value and behavior. Therefore, we will now devise a strategy to determine the relaxation time based on a linear-response theory solution of the Boltzmann equation.

VI. BOLTZMANN EQUATION AND LINEAR-RESPONSE THEORY

The BGK procedure has the relaxation time as a free parameter. A technique complementary to the CE procedure is to solve the coupled Boltzmann equations, Eqs. (4), within linear response. This gives direct access to the relaxation times from a microscopic theory. The collision integrals $\mathcal{I}^b[f,b]$ and $\mathcal{I}^f_\lambda[f,b]$ describe collision processes involving two fermions and a single plasmon and are detailed in Ref. [43] as well as the Supplemental Material [48] with their corresponding Feynman diagrams.

In the presence of a small velocity gradient, the streaming terms on the left-hand side assume the form $\mathbf{w}_\mathbf{q}\cdot\boldsymbol{\nabla}_\mathbf{x} b^0_\mathbf{q} = \frac{\omega_\mathbf{q}}{2T} b^0_\mathbf{q}(1+b^0_\mathbf{q})[(\hat{q}_i\hat{q}_j - \delta_{ij})\frac{X_{ij}}{2} + \frac{\boldsymbol{\nabla}\cdot\mathbf{u}}{2}]$ and $\mathbf{v}_{\lambda,\mathbf{k}}\cdot\boldsymbol{\nabla}_\mathbf{x} f^0_{\lambda,\mathbf{k}} = \frac{\epsilon_{\lambda,\mathbf{k}}}{T} f^0_{\lambda,\mathbf{k}}(1-f^0_{\lambda,\mathbf{k}})[(\hat{k}_i\hat{k}_j - \delta_{ij})\frac{X_{ij}}{2} + \frac{\boldsymbol{\nabla}\cdot\mathbf{u}}{2}]$ in linear response where we introduced $\boldsymbol{X}_{ij} = 2\dot{\boldsymbol{e}} - \mathbf{1}\,\mathrm{Tr}\,\dot{\boldsymbol{e}}$. Let us note that the last term in the square brackets proportional to $\boldsymbol{\nabla}\cdot\mathbf{u}$ is responsible for the bulk viscosity, which we disregard from now on [56].

The driving terms motivate a parametrization of the deviation of the distribution functions from their equilibrium values $f^0_\lambda(\mathbf{k})$ and $b^0(\mathbf{q})$ in the standard way. We write $f_\lambda(\mathbf{x},\mathbf{k},t) = f^0_\lambda(\mathbf{k}) + \delta f_\lambda(\mathbf{k})$ and $b(\mathbf{q},\mathbf{x},t) = b^0(\mathbf{q}) + \delta b(\mathbf{q})$, where

$$\delta b(\mathbf{q}) = \frac{g^b(\mathbf{q})}{T}\frac{\omega_\mathbf{q}}{2T} b^0(\mathbf{q})[1+b^0(\mathbf{q})]\sum_{ij}\left(\frac{\mathbf{q}_i\mathbf{q}_j}{q^2} - \mathbf{1}_{ij}\right)\mathbf{X}_{ij},$$
$$\equiv b^0(\mathbf{q})[1+b^0(\mathbf{q})]\sum_{ij} g^b_{ij}(\mathbf{q})\mathbf{X}_{ij}, \tag{13}$$

and

$$\delta f_\lambda(\mathbf{k}) = \frac{g^f_\lambda(\mathbf{k})}{T}\frac{\epsilon_{\lambda,\mathbf{k}}}{T} f^0_\lambda(\mathbf{k})\big[1-f^0_\lambda(\mathbf{k})\big]\sum_{ij}\left(\frac{\mathbf{k}_i\mathbf{k}_j}{k^2} - \mathbf{1}_{ij}\right)\mathbf{X}_{ij},$$
$$\equiv f^0_\lambda(\mathbf{k})\big[1-f^0_\lambda(\mathbf{k})\big]\sum_{ij} g^f_{\lambda,ij}(\mathbf{k})\mathbf{X}_{ij}. \tag{14}$$

We proceed to the linearization of the collision integrals and find that

$$\mathcal{I}^b_{(1)}[f,b] = -N\sum_{\lambda,\lambda'}\int\frac{d\mathbf{k}}{(2\pi)^2}\mathcal{M}^{\mathbf{k}+\mathbf{q},\mathbf{k}}_{\lambda\lambda'}\delta(\omega_\mathbf{q}+\epsilon_{\lambda,\mathbf{k}}-\epsilon_{\lambda',\mathbf{k}+\mathbf{q}})\big[f^0_\lambda(\mathbf{k})\big[1-f^0_{\lambda'}(\mathbf{k}+\mathbf{q})\big]b^0(\mathbf{q})\big]\big[g^f_{\lambda,ij}(\mathbf{k}) - g^f_{\lambda',ij}(\mathbf{k}+\mathbf{q}) + g^b_{ij}(\mathbf{q})\big]\mathbf{X}_{ij}, \tag{15}$$

$$\mathcal{I}^f_{(1)\lambda}[f,b] = -\sum_{\lambda'}\int\frac{d\mathbf{q}}{(2\pi)^2}\mathcal{M}^{\mathbf{k}+\mathbf{q},\mathbf{k}}_{\lambda\lambda'}\delta(\omega_\mathbf{q}+\epsilon_{\lambda,\mathbf{k}}-\epsilon_{\lambda',\mathbf{k}+\mathbf{q}})\big[f^0_\lambda(\mathbf{k})\big[1-f^0_{\lambda'}(\mathbf{k}+\mathbf{q})\big]b^0(\mathbf{q})\big]\big[g^f_{\lambda,ij}(\mathbf{k}) - g^f_{\lambda',ij}(\mathbf{k}+\mathbf{q}) + g^b_{ij}(\mathbf{q})\big]\mathbf{X}_{ij}$$
$$-\sum_{\lambda'}\int\frac{d\mathbf{q}}{(2\pi)^2}\mathcal{M}^{\mathbf{k}-\mathbf{q},\mathbf{k}}_{\lambda\lambda'}\delta(-\omega_\mathbf{q}+\epsilon_{\lambda,\mathbf{k}}-\epsilon_{\lambda',\mathbf{k}-\mathbf{q}})\big[\big[1-f^0_\lambda(\mathbf{k})\big]f^0_{\lambda'}(\mathbf{k}-\mathbf{q})b^0(\mathbf{q})\big]$$
$$\times\big[g^f_{\lambda',ij}(\mathbf{k}-\mathbf{q}) - g^f_{\lambda,ij}(\mathbf{k}) + g^b_{ij}(\mathbf{q})\big]\mathbf{X}_{ij}. \tag{16}$$

The remainder is to find the solution of the linearized Boltzmann equations for the function $g^b_{ij}(\mathbf{q})$ and $g^f_{\lambda,ij}(\mathbf{k})$ by means of a variational method. This method is standard and has been intensively used for solving a Boltzmann equation (see, for example, Ref. [57]).

VII. RESULTS

For a slowly varying velocity field, the correction to the stress tensor can be expanded in gradients of $\mathbf{u}(\mathbf{x})$. The shear viscosity characterizes the diffusive relaxation of transverse

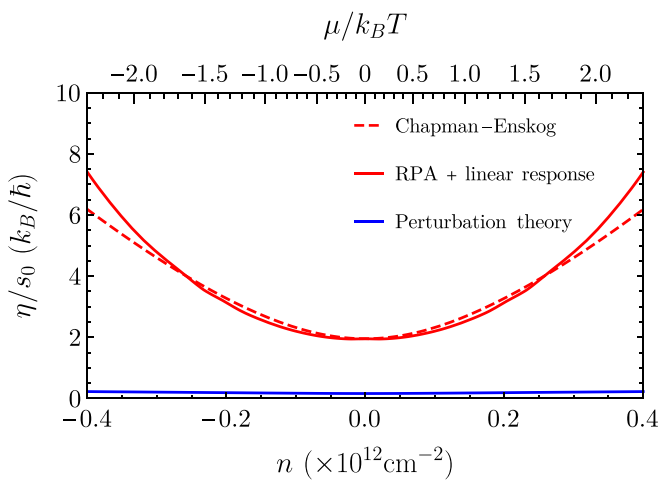

FIG. 3. The ratio of shear viscosity η to s_0 evaluated at the room temperature $T = 300$ K as a function of charge density n. Here, η/s_0 is plotted in the unit of $k_B/\hbar$ where s_0 is electron entropy density at the Dirac point. In this plot, the fine-structure constant $\alpha \approx 0.9$.

momentum density fluctuations measuring the resistance of a fluid to establish a transverse velocity gradient. In the model of the three-component fluid of electrons, holes, and plasmons, Eq. (5) describes the momentum flux which can be related to the shear viscosity according to $\delta\Pi_{ij} = -\eta_{ijkl}\dot{e}_{kl}$. Microscopically, this leads to

$$\eta_{ijkl} = \frac{1}{8}\left(\sum_\lambda N \int \frac{d\mathbf{k}}{(2\pi)^2}\frac{(\epsilon_{\lambda,\mathbf{k}})^2}{T^2} g^f_\lambda(\mathbf{k}) f^0_{\lambda,\mathbf{k}}\left(1 - f^0_{\lambda,\mathbf{k}}\right) + \int \frac{d\mathbf{q}}{(2\pi)^2}\frac{(\omega_\mathbf{q})^2}{4T^2} g_b(\mathbf{q}) b^0_\mathbf{q}\left(1 + b^0_\mathbf{q}\right)\right) \times (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}), \tag{17}$$

which is what we calculated numerically. In order to estimate the size of the effect of collective plasmon modes, we also calculated the viscosity in perturbation theory following Ref. [22]. For a value of $\alpha \approx 0.9$, the result is plotted in Fig. 3. As a function of the density, the red curve shows the total viscosity including fermions and plasmons, whereas the blue curve shows the viscosity of the fermions only calculated in perturbation theory in α. We find that there is a drastic increase in viscous effects due to plasmons.

VIII. CONCLUSIONS

In this paper, we computed the contribution to the shear viscosity from the plasmons in the charge fluid of graphene. We employed a multifluid model describing electrons, holes, and plasmons in line with Refs. [41–43]. Following both relaxation time approximation and linear-response theory methods, we arrive to agreeing results that show a possibly strong enhancement of the viscosity due to the electron/hole-plasmon interactions. Comparing our results to experiment will shed light on the interaction physics of graphene and might allow for an alternative determination of the fine-structure constant. Beyond graphene, we believe that our result is generic to two-dimensional metals which will allow for insights into the physics of two-dimensional Fermi liquids [58,59].

ACKNOWLEDGMENTS

K.P. and L.F. acknowledge former collaborations and discussions with H. T. C. Stoof, T. Ludwig, S. Grubinskas, R. A. Duine, E. I. Kiselev, A. Lucas, J. Lux, A. Mitchell, M. Müller, A. Rosch, S. Sachdev, J. Schmalian, H. Teras, M. Vojta, G. Wagner, T. Meng, and J. Waissman. K.P. thanks the Institute for the Promotion of Teaching Science and Technology (IPST) of Thailand for support from a Ph.D. scholarship. One of the authors (P.C.) acknowledges the funding provided by Fundação para a Ciência e a Tecnologia (FCT-Portugal) through Grant No. PD/BD/150415/2019 and Project No. PTDC/FIS-OUT/3882/2020. This work is part of the D-ITP consortium, a program of the Netherlands Organisation for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW).

K.P. and P.C. contributed equally to this work.

[1] L. Landau and E. Lifshitz, *Fluid Mechanics: Course of Theoretical Physics* (Pergamon, New York, 1987).

[2] S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Theory of the Nernst effect near quantum phase transitions in condensed matter and in dyonic black holes, Phys. Rev. B **76**, 144502 (2007).

[3] A. V. Andreev, S. A. Kivelson, and B. Spivak, Hydrodynamic description of transport in strongly correlated electron systems, Phys. Rev. Lett. **106**, 256804 (2011).

[4] J. Zaanen, Electrons go with the flow in exotic material systems, Science **351**, 1026 (2016).

[5] L. Levitov and G. Falkovich, Electron viscosity, current vortices and negative nonlocal resistance in graphene, Nat. Phys. **12**, 672 (2016).

[6] A. Lucas and K. C. Fong, Hydrodynamics of electrons in graphene, J. Phys.: Condens. Matter **30**, 053001 (2018).

[7] B. N. Narozhny, Electronic hydrodynamics in graphene, Ann. Phys. **411**, 167979 (2019).

[8] L. Fritz and T. Scaffidi, Hydrodynamic electronic transport, arXiv:2303.14205.

[9] P. Kovtun, Lectures on hydrodynamic fluctuations in relativistic theories, J. Phys. A: Math. Theor. **45**, 473001 (2012).

[10] K. Damle and S. Sachdev, Nonzero-temperature transport near quantum critical points, Phys. Rev. B **56**, 8714 (1997).

[11] V. B. Bulchandani, B. Hsu, C. P. Herzog, and S. L. Sondhi, Hydrodynamics of quantum spin liquids, Phys. Rev. B **104**, 235412 (2021).

[12] C. Ulloa, A. Tomadin, J. Shan, M. Polini, B. J. van Wees, and R. A. Duine, Nonlocal spin transport as a probe of viscous magnon fluids, Phys. Rev. Lett. **123**, 117203 (2019).

[13] R. Gurzhi, Minimum of resistance in impurity-free conductors, Sov. Phys. JETP **44**, 771 (1963).

[14] J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene, Science **351**, 1058 (2016).

[15] D. A. Bandurin, A. V. Shytov, L. S. Levitov, R. K. Kumar, A. I. Berdyugin, M. Ben Shalom, I. V. Grigorieva, A. K. Geim, and G. Falkovich, Fluidity onset in graphene, Nat. Commun. **9**, 4533 (2018).

[16] B. A. Braem, F. M. D. Pellegrino, A. Principi, M. Röösli, C. Gold, S. Hennel, J. V. Koski, M. Berl, W. Dietsche, W. Wegscheider, M. Polini, T. Ihn, and K. Ensslin, Scanning gate microscopy in a viscous electron fluid, Phys. Rev. B **98**, 241304(R) (2018).

[17] J. A. Sulpizio, L. Ella, A. Rozen, J. Birkbeck, D. J. Perello, D. Dutta, M. Ben-Shalom, T. Taniguchi, K. Watanabe, T. Holder, R. Queiroz, A. Principi, A. Stern, T. Scaffidi, A. K. Geim, and S. Ilani, Visualizing Poiseuille flow of hydrodynamic electrons, Nature (London) **576**, 75 (2019).

[18] M. J. H. Ku, T. X. Zhou, Q. Li, Y. J. Shin, J. K. Shi, C. Burch, L. E. Anderson, A. T. Pierce, Y. Xie, A. Hamo, U. Vool, H. Zhang, F. Casola, T. Taniguchi, K. Watanabe, M. M. Fogler, P. Kim, A. Yacoby, and R. L. Walsworth, Imaging viscous flow of the Dirac fluid in graphene, Nature (London) **583**, 537 (2020).

[19] A. B. Kashuba, Conductivity of defectless graphene, Phys. Rev. B **78**, 085415 (2008).

[20] L. Fritz, J. Schmalian, M. Müller, and S. Sachdev, Quantum critical transport in clean graphene, Phys. Rev. B **78**, 085416 (2008).

[21] M. Müller, L. Fritz, and S. Sachdev, Quantum-critical relativistic magnetotransport in graphene, Phys. Rev. B **78**, 115406 (2008).

[22] M. Müller, J. Schmalian, and L. Fritz, Graphene: A nearly perfect fluid, Phys. Rev. Lett. **103**, 025301 (2009).

[23] B. N. Narozhny, I. V. Gornyi, M. Titov, M. Schütt, and A. D. Mirlin, Hydrodynamics in graphene: Linear-response transport, Phys. Rev. B **91**, 035414 (2015).

[24] A. Lucas, J. Crossno, K. C. Fong, P. Kim, and S. Sachdev, Transport in inhomogeneous quantum critical fluids and in the Dirac fluid in graphene, Phys. Rev. B **93**, 075426 (2016).

[25] E. I. Kiselev and J. Schmalian, Lévy flights and hydrodynamic superdiffusion on the Dirac cone of graphene, Phys. Rev. Lett. **123**, 195302 (2019).

[26] E. I. Kiselev and J. Schmalian, Nonlocal hydrodynamic transport and collective excitations in Dirac fluids, Phys. Rev. B **102**, 245434 (2020).

[27] A. N. Afanasiev, P. S. Alekseev, A. A. Greshnov, and M. A. Semina, Ballistic-hydrodynamic phase transition in flow of two-dimensional electrons, Phys. Rev. B **104**, 195415 (2021).

[28] Z.-Y. Xian, S. Danz, D. Rodríguez Fernández, I. Matthaiakakis, C. Tutschku, R. L. Klees, J. Erdmenger, R. Meyer, and E. M. Hankiewicz, Hall viscosity and hydrodynamic inverse Nernst effect in graphene, Phys. Rev. B **107**, L201403 (2023).

[29] D. Pines and D. Bohm, A collective description of electron interactions: II. Collective vs individual particle aspects of the interactions, Phys. Rev. **85**, 338 (1952).

[30] D. Bohm and D. Pines, A collective description of electron interactions: III. Coulomb interactions in a degenerate electron gas, Phys. Rev. **92**, 609 (1953).

[31] A. A. Vlasov, The vibrational properties of an electron gas, Sov. Phys. Usp. **10**, 721 (1968).

[32] D. Pines, A collective description of electron interactions: IV. Electron interaction in metals, Phys. Rev. **92**, 626 (1953).

[33] F. Stern, Polarizability of a two-dimensional electron gas, Phys. Rev. Lett. **18**, 546 (1967).

[34] B. Wunsch, T. Stauber, F. Sols, and F. Guinea, Dynamical polarization of graphene at finite doping, New J. Phys. **8**, 318 (2006).

[35] S. Das Sarma and Q. Li, Intrinsic plasmons in two-dimensional Dirac materials, Phys. Rev. B **87**, 235418 (2013).

[36] Y. Takada, Plasmon mechanism of superconductivity in two- and three-dimensional electron systems, J. Phys. Soc. Jpn. **45**, 786 (1978).

[37] J. Ruhman and P. A. Lee, Pairing from dynamically screened Coulomb repulsion in bismuth, Phys. Rev. B **96**, 235107 (2017).

[38] H. W. Wyld and D. Pines, Kinetic equation for plasma, Phys. Rev. **127**, 1851 (1962).

[39] F. Rana, J. H. Strait, H. Wang, and C. Manolatou, Ultrafast carrier recombination and generation rates for plasmon emission and absorption in graphene, Phys. Rev. B **84**, 045437 (2011).

[40] M. J. Klug, M. S. Scheurer, and J. Schmalian, Hierarchy of information scrambling, thermalization, and hydrodynamic flow in graphene, Phys. Rev. B **98**, 045102 (2018).

[41] K. Pongsangangan, S. Grubinskas, and L. Fritz, Thermoelectric response in two-dimensional Dirac systems: Role of particle-hole pairs, Phys. Rev. Res. **4**, 043107 (2022).

[42] K. Pongsangangan, T. Ludwig, H. T. C. Stoof, and L. Fritz, Hydrodynamics of charged two-dimensional Dirac systems. I. Thermoelectric transport, Phys. Rev. B **106**, 205126 (2022).

[43] K. Pongsangangan, T. Ludwig, H. T. C. Stoof, and L. Fritz, Hydrodynamics of charged Dirac electrons in two dimensions. II. Role of collective modes, Phys. Rev. B **106**, 205127 (2022).

[44] J. M. Link, D. E. Sheehy, B. N. Narozhny, and J. Schmalian, Elastic response of the electron fluid in intrinsic graphene: The collisionless regime, Phys. Rev. B **98**, 195103 (2018).

[45] D. Pines and J. R. Schrieffer, Approach to equilibrium of electrons, plasmons, and phonons in quantum and classical plasmas, Phys. Rev. **125**, 804 (1962).

[46] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, The electronic properties of graphene, Rev. Mod. Phys. **81**, 109 (2009).

[47] V. N. Kotov, B. Uchoa, V. M. Pereira, F. Guinea, and A. H. Castro Neto, Electron-electron interactions in graphene: Current status and perspectives, Rev. Mod. Phys. **84**, 1067 (2012).

[48] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.110.045443 for the discussion about the locking of electrons, holes, and plasmon velocities as a consequence of entropic constraints. The collision integrals of the Boltzmann equations are given explicitly there. In addition, we discuss the velocity renormalization of fermions within the approximation of our model.

[49] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases: An Account of the Kinetic Theory of Viscosity, Thermal Conduction, and Diffusion in Gases* (Cambridge University Press, Cambridge, UK, 1954).

[50] P. L. Bhatnagar, E. P. Gross, and M. Krook, A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems, Phys. Rev. **94**, 511 (1954).

[51] E. P. Gross and M. Krook, Model for collision processes in gases: Small-amplitude oscillations of charged two-component systems, Phys. Rev. **102**, 593 (1956).

[52] M. Schütt, P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Coulomb interaction in graphene: Relaxation rates and transport, Phys. Rev. B **83**, 155441 (2011).

[53] V. Kozii and L. Fu, Thermal plasmon resonantly enhances electron scattering in Dirac/Weyl semimetals, Phys. Rev. B **98**, 041109(R) (2018).

[54] A. Gabbana, D. Simeoni, S. Succi, and R. Tripiccione, Relativistic lattice Boltzmann methods: Theory and applications, Phys. Rep. **863**, 1 (2020).

[55] W. Chen and W. Zhu, Viscosity of disordered Dirac electrons, Phys. Rev. B **106**, 014205 (2022).

[56] As we have seen, the CE method shows that the bulk viscosity of the fermionic sector should be identically null and that for the plasmon sector it is $\sim 1/10$ of the shear counterpart.

[57] J. Ziman, *Electrons and Phonons* (Oxford University Press, Oxford, UK, 2000).

[58] Y. Liao and V. Galitski, Drag viscosity of metals and its connection to Coulomb drag, Phys. Rev. B **101**, 195106 (2020).

[59] D. Zverevich and A. Levchenko, Transport signatures of plasmon fluctuations in electron hydrodynamics, Low Temp. Phys. **49**, 1376 (2023).