Squeezed and nascent vortices in a thin normal layer with proximity induced superconductivity

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(Received 10 May 2024; revised 12 July 2024; accepted 12 July 2024; published 25 July 2024)

It is theoretically found that in-plane vortices may exist in a thin normal metal (N) layer (with thickness d_N much smaller than the coherence length ξ_N) that covers a superconductor (S). Vortices enter the N layer with proximity-induced superconductivity at a sufficiently large in-plane magnetic field. These vorticies have squeezed cores and are located (pinned) near the SN interface. At large magnetic fields, we find a nascent vortex state, which is a spatially modulated state along the finite length N layer with zero vorticity. This state does not exist in a finite length single S layer.

DOI: 10.1103/PhysRevB.110.014517

I. INTRODUCTION

The normal metal layer (N) covering the superconducting material (S) [see Fig. 1(a)] behaves in many respects like an ordinary superconductor. It screens weak magnetic field [1–8] and can carry a superconducting current density j_s comparable to the depairing current density of the adjacent superconductor [9,10]. Superconducting correlations penetrate into the N layer up to a distance of about ξ_N, which at low temperatures can be much larger than the magnetic field penetration depth λ_N [1–4]. This makes the N layer with proximity-induced superconductivity a type I superconductor. In the experiment, an abrupt breakdown of the induced superconductivity at a magnetic field B_b, its recovery at a return field $B_r < B_b$, and magnetic hysteresis have been observed [2–4], which are typical magnetic properties of type I superconductors.

Previous theories on the superconducting and magnetic properties of the N layer were based on a one-dimensional model where the superconducting characteristics depend only on the longitudinal (in direction of the normal metal) coordinate [1,3,5–8]. This approach is explained by the simplicity of the 1D model and the presumption of type I superconductivity, where the presence of vortices is not expected. In the present work, we address the question of the existence of vortices in a finite length N layer with thickness $d_N \ll \xi_N, \lambda_N$ while the S layer is a type II superconductor with thickness $d_S \gtrsim \xi_S \ll \lambda_S$. These conditions mean that the magnetic field almost does not vary across the SN bilayer. In comparison with Refs. [2–4], where micron thick N layers have been studied, this limit can be realized for small d_N (about dozens of nanometers) leading to a small mean free path $\ell \sim d_N$ and relatively high resistivity, which results in both $d_N \ll \xi_N$ and $d_N \ll \lambda_N$ even at low temperatures (so-called dirty limit).

Our main result is that vortices may exist in the N layer despite its small thickness $d_N \ll \xi_N$ in the case of a relatively large proximity-induced superconducting order parameter Δ. In some sense, it is a surprising result because even in a single S layer, vortices do not exist if its thickness is less than $\sim 1.84\xi_S$ [11]. Vortices enter the N layer at the field B_b and exit at the return field $B_r < B_b$, which is accompanied by a sudden change in magnetization, hysteretic magnetization curve $M(B)$, and a spatial oscillations of Δ along the N layer in the vortex state. Vortices enter the S layer at a much larger field $B_s \gg B_b$ when the edge barrier for vortex entry is suppressed, while at $B < B_s$ they are located near the SN interface with cores extending into both the superconductor and the normal layer. Due to $d_N \ll \xi_N$, vortex cores are squeezed in the N layer.

When the proximity-induced Δ is small, we find a nonhysteretic $M(B)$ and gradual suppression of Δ at $B > B_b$. In this case, the oscillations of Δ along the N layer [along the x axis in Fig. 1(a)] appear at $B > B_b$ despite zero vorticity $N_v = \oint \nabla\phi dl/2\pi = 0$ (ϕ is the phase of superconducting order parameter). Minima in the dependence $\Delta(x)$ could be considered as weak places where vortices enter the N layer at a larger field, and we call this state the nascent vortex state. A similar state has been predicted [12,13] for a single S layer placed in a magnetic field just below the field of suppression of the Bean-Livinston barrier for vortex entry [14], but it turned out to be an unstable saddle point state [15]. We have recently found that it can be stabilized in a current-carrying SN bridge [16], and we extend this result here to a finite-length SN bilayer in an in-plane magnetic field. We find that, in addition to nascent vortices there are also ordinary vortices that can enter and exit the N layer reversibly. We argue that this behavior resembles the properties of a thin finite length single S layer at a magnetic field exceeding the third critical magnetic field B_{c3} [17], when superconductivity survives mainly near the ends of the S layer.

The structure of the paper is as follows. In Sec. II we describe our model. In Sec. III we present our results for the finite-length SN bilayer, and in Sec. IV for the superconducting layer. In Sec. V we discuss our results and in Sec. VI we make conclusions.

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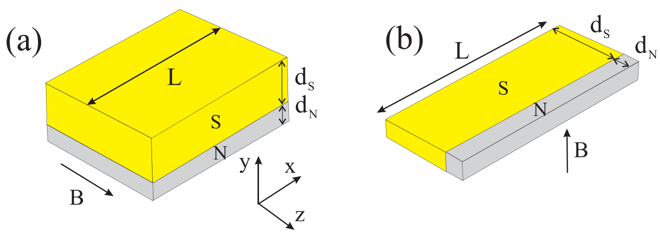

FIG. 1. (a) A superconductor-normal metal bilayer of finite length L in an in-plane magnetic field. We assume that there is no dependence of superconducting properties in the *z* direction. (b) Our results should also be applicable to a finite length superconducting strip with an N region near the edge, placed in an out-of-plane magnetic field.

II. MODEL

The calculations are based on the numerical solution of the two-dimensional Ginzburg-Landau equation for the superconducting order parameter $\Psi = \Delta e^{i\phi}$,

$$\frac{\pi \hbar D_{S,N}}{8k_B T_{cS}}\left(\nabla + i\frac{2\pi A}{\Phi_0}\right)^2 \Psi + \left(1 - \frac{T}{T_{cS,cN}} - \frac{\Delta^2}{\Delta_{GL}^2}\right)\Psi = 0, \tag{1}$$

where $D_{S,N}$ is the diffusion coefficient in the S and N layers, respectively, $\Delta_{GL} = 3.06 k_B T_{cS,cN}$, and A is the vector potential. The N layer is modeled as a superconductor with a critical temperature $T_{cN} < T_{cS}$ while the temperature is chosen from the interval $T_{cN} < T < T_{cS}$. At the SN interface, we use the boundary condition $D_S d\Psi/dy = D_N d\Psi/dy$, and at the boundaries with the vacuum, $d\Psi/dn = 0$. We do not solve the Maxwell equation for the vector potential because we assume that $d_N + d_S$ is much less than the London penetration depth, and we choose $A = (-By, 0, 0)$.

Technically, we solve Eq. (1) by adding the time derivative $d\Psi/dt$ to the right-hand side and using the Euler method for the numerical solution of the first order time-dependent equation. For any value of the magnetic field B we search for a time-independent state with $d\Psi/dt \to 0$. The method used allows us to automatically check the stability of the static state and find transient states with entering/exiting vortices. For sweeping the field up/down, we use the static state found at the previous value of B as the initial condition.

In the framework of this model, the coherence lengths in S and N layers are $\xi_S = \xi_c\sqrt{\pi/8(1 - T/T_{cS})}$ and $\xi_N = \xi_c\sqrt{\pi/8(T/T_{cN} - 1)}\sqrt{D_N/D_S}$, where $\xi_c = \sqrt{\hbar D_S/k_B T_{cS}}$.

The one-dimensional version of the GL equation has been widely used to study proximity-induced superconductivity in the N layer [1,3,8] and has demonstrated its applicability in describing experimental results, at least qualitatively. The system we consider here is shown in Fig. 1(a)—this is a finite thickness and length SN bilayer placed in an in-plane field [our results could also be applied to a finite width and length SN strip in an out-of-plane magnetic field shown in Fig. 1(b)].

We calculate the magnetic moment of the SN bilayer $\mathbf{M}$ as

$$\mathbf{M} = \frac{1}{2c}\iint [\mathbf{r} \times \mathbf{j_s}] dx dy, \tag{2}$$

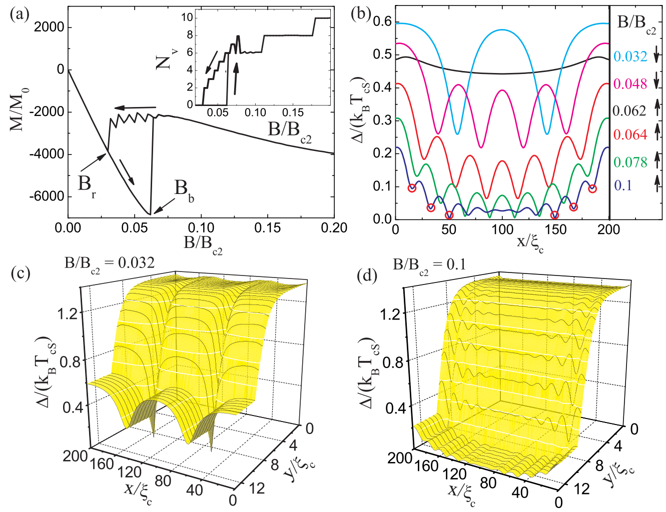

FIG. 2. (a) Magnetization curve of the SN bilayer at $T/T_{cN} = 1.05$. In the inset, we show the change in the vortex number with sweeping up and down of the magnetic field. (b) Spatial variation of Δ along the SN bilayer on the SN interface at different magnetic fields. Minima in $\Delta(x)$ correspond to the locations of vortex cores at $B < 0.08B_{c2}$, while at larger fields the number of minima is larger than the number of vortices. Red circles indicate minima of $\Delta(x)$ occupied by vortices at $B/B_{c2} = 0.1$. (c), (d) Two-dimensional distribution of Δ in the SN bilayer at two magnetic fields: $B = 0.032B_{c2}$ (just before transition to the vortex-free state when sweeping field down) and $B = 0.1B_{c2}$.

where the superconducting current density

$$j_s = -\frac{1}{\rho_{S,N}|e|}\frac{\pi|\Delta|^2 q}{4k_B T_{cS}} = -\frac{c}{4\pi\lambda_{S,N}^2}(\Phi_0\nabla\phi/2\pi + A), \tag{3}$$

where $\rho_{S,N} = 1/(2e^2 D_{S,N} N(0))$ is the resistivity of the S and N layers [$N(0)$ is a density of states of electrons on the Fermi level, which is taken to be the same in the S and N layers to reduce the number of free parameters], $q = \nabla\phi + 2\pi A/\Phi_0$, $\Phi_0 = \pi\hbar c/|e|$ is the magnetic flux quantum.

In our calculations, we choose $T/T_{cS} = 0.8$ (which gives $\xi_S \sim 1.4\xi_c$), the ratio $D_N/D_S = 100$, and two values for $T/T_{cN} = 1.05$ ($\xi_N \sim 28\xi_c$) and $T/T_{cN} = 1.2$ ($\xi_N \sim 14\xi_c$). These values correspond to cases with relatively large and small induced Δ in the N layer. The other parameters are as follows: $d_S = 10\xi_c \sim 7\xi_S$, $d_N = 3\xi_c \ll \xi_N$, and $L = 200\xi_c$. The magnetic moment is normalized in units of $M_0 = \xi_c^3 j_{\rm dep}/2c$, where $j_{\rm dep}$ is the depairing current density in the superconductor. The magnetic field is given in units of $B_{c2} = \Phi_0/2\pi\xi_S^2$.

If we consider Al as a normal metal with resistivity $\rho_N \simeq 2\ \mu\mathrm{Ohm\cdot cm}$ at low temperatures for films with $d_N \lesssim 50$ nm [18] then we find $\lambda_N \sim \rho_N^{1/2}/\Delta \sim 21\xi_c$ for $T/T_{cN} = 1.05$, and $\lambda_N \sim 43\xi_c$ for $T/T_{cN} = 1.2$ at $B = 0$. Therefore, for the chosen parameters, we have type II superconductivity in the N layer.

III. VORTICES IN A THIN N LAYER

In Fig. 2 we show our results for the SN bilayer when $T/T_{cN} = 1.05$. With increasing field, a vortex chain consisting

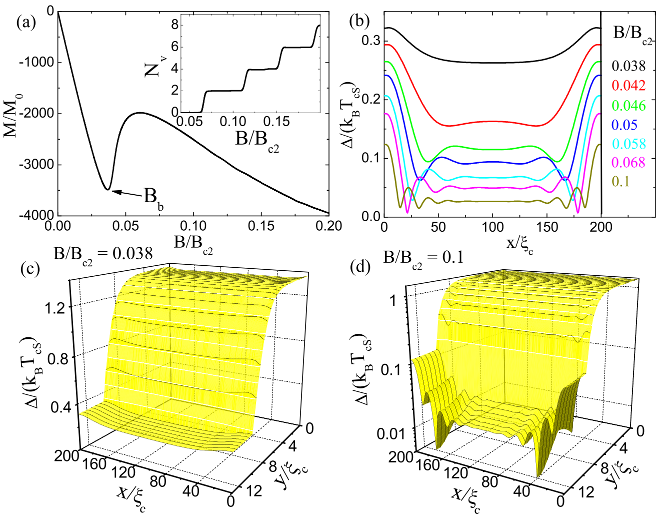

FIG. 3. (a) The magnetization curve of the SN bilayer with the same parameters as in Fig. 2 and $T/T_{cN} = 1.2$. In the inset we show the dependence $N_v(B)$. (b) Variation of Δ along the SN bilayer at the SN interface at different magnetic fields. Minima in $\Delta(x)$ correspond to the location of nascent vortices at $B < 0.063B_{c2}$. For $B > 0.063B_{c2}$ there are two vortices located in the minima closest to the ends of bilayer. (c), (d) The two-dimensional distribution of Δ across the SN bilayer at two magnetic fields: $B = 0.038B_{c2}$ (at this field strong suppression of proximity-induced superconductivity in the N layer starts) and $B = 0.1B_{c2}$.

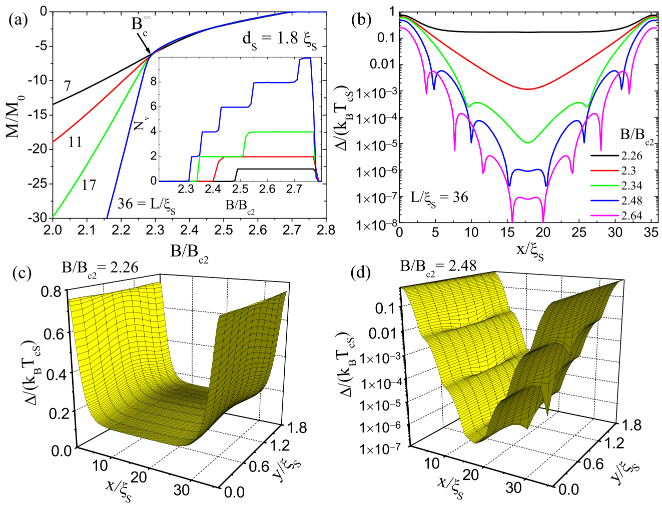

FIG. 4. (a) The magnetization curves of the finite length superconducting layer with a thickness $d_S = 1.8\xi_S$ and different lengths L at large magnetic fields are shown. In the inset, we plot the field-dependent number of vortices N_v. (b) Dependence $\Delta(x)$ in the center of the S layer with length $L = 36\xi_S$ at different magnetic fields is depicted. Minima in $\Delta(x)$ correspond to the locations of the vortex cores. (c), (d) Dependencies $\Delta(x, y)$ in the S layer with $L = 36\xi_S$ at $B = 2.26B_{c2}$ (just below B_c^∞), see (c); and at $B = 2.48B_{c2}$, when there are six vortices in the S layer, see (d).

of six vortices enters the N layer at $B = B_b \simeq 0.064B_{c2}$, leading to a sharp increase in M [see Fig. 2(a)]. Vortices cannot enter the S layer because there is an energy barrier similar to the Bean-Livingston barrier [14], due to the difference in Δ in the S and N layers. This barrier is suppressed only at $B = B_s \simeq 0.34B_{c2}$, and vortices remain pinned at the SN interface. Because of the vortices, there are spatial oscillations of Δ along the SN bilayer, as shown in Figs. 2(b)–2(d). With increasing magnetic field, the number of vortices increases up to eight, then two vortices exit, and at high fields, vortices enter in pairs. At $B/B_{c2} > 0.08$, the number of oscillations of Δ is larger than the number of vortices. Minima in $\Delta(x)$ that are "empty" of vortices act as potential entry points for ordinary vortices entry at higher fields, and we refer to these as nascent vortices.

At sweeping field down, vortices exit the N layer one by one at $B < 0.064B_{c2}$ and in pairs at larger fields. The last vortex exits at the field $B_r \simeq 0.03B_{c2}$. The magnetization curve is hysteretic in the field range $B_r < B/B_{c2} < 0.064$, and $M(B)$ is nonhysteretic in the field range $0.064 < B/B_{c2} < B_s$, where vortex entry/exit is reversible. Due to small thickness of the N layer, the vortices have squeezed cores as shown in Figs. 2(c) and (d). Partially, their cores are located in the S layer, where they are much smaller in size, as can be seen in Fig. 2(c).

In Fig. 3 we show results when $T/T_{cN} = 1.2$, which corresponds to a much smaller induced Δ (almost half compared to $T/T_{cN} = 1.05$) in the N layer. In this case, the magnetization curve is reversible [see Fig. 3(a)] at all fields up to $B_s \simeq 0.34B_{c2}$ when vortices enter the S layer. Strong suppression of superconductivity in the N layer starts at $B = B_b \simeq 0.038B_{c2}$, leading to an increase in magnetization. Up to the field $B = 0.068B_{c2}$, there are no vortices, but Δ oscillates along the N layer as shown in Fig. 3(b). For $B > 0.068B_{c2}$, vortices enter the N layer in pairs via local minima in $\Delta(x)$ which are closest to the layer ends. Their entry/exit is reversible. The number of oscillations of Δ is larger than the number of vortices, resembling the situation with nascent vortices considered above.

IV. VORTICES IN A THIN SINGLE S LAYER

Before discussing our results, it makes sense to consider vortex states in a finite length thin single S layer. It is known that when $d_S \lesssim 1.84\xi_S$, there are no vortices in the infinitely long S layer at any magnetic field [11], up to $B_c^\infty/B_{c2} \simeq 2\sqrt{3}\xi_S/d_S$, at which point superconductivity vanishes [19]. Because of absence of the vortices, the magnetization curve of such a superconductor is nonhysteretic. However, we find that in the finite length S layer, vortices may exist up to $d_S \sim 1.4\xi_S$, and their entry/exit is reversible. In Fig. 4 we show typical magnetization curves of the superconducting layer with a thickness $d_S = 1.8\xi_S$ and different lengths. The superconductivity survives at $B > B_c^\infty$ due to the presence of corners at the layer ends, which makes the effective thickness of the superconductor smaller [20], leading to a larger critical magnetic field according to the expression for B_c^∞. Note that the same effect exists in superconducting wedges, squares, or triangles [20–24].

Above B_c^∞, vortex pairs enter the S layer in the middle (one vortex from each edge) where Δ is minimal, and then they relocate toward the ends of the layer. The longer the layer, the larger the number of vortices that can enter the superconductor before its transition to the normal state. This vortex entry/exit process is reversible, providing nonhysteretic $M(B)$.

The same effect exists in an S layer of finite length with $d_S > 1.84\xi_S$ at a magnetic field exceeding third critical magnetic field of an infinitely long S layer $B_{c3}^{\infty}(d_S)$ ($B_{c3}^{\infty} \simeq 1.695B_{c2}$ when $d_S \gg \xi_S$ [17]). At $B > B_{c3}^{\infty}$ superconductivity survives near the ends of the S layer, and vortex entry/exit is reversible in the field range $B_{c3}^{\infty} < B < B_{c3}^{L}$ ($B_{c3}^{L}(d_S \gg \xi_S) \sim 2B_{c2}$ [24]). In this case the number of vortices scales with the length of the superconductor.

V. DISCUSSION

Let us start the discussion of our results concerning the effects associated with the finite length of the N layer. The finite length is responsible for the appearance of ordinary and nascent vortex states at large magnetic fields, when proximity-induced superconductivity in the N layer is strongly suppressed. This corresponds to fields $B > B_b$ for both cases ($T/T_{cN} = 1.05$ and 1.2) considered in Sec. III. At these fields vortex entry and exit are reversible (when $B < B_s \simeq 0.34B_{c2}$ and vortices do not enter the S layer), $M(B)$ is nonhysteretic, and it resembles the properties of a finite length S layer at $B > B_c^{\infty}$ ($1.4\xi_S \lesssim d_S \lesssim 1.84\xi_S$) or $B > B_{c3}^{\infty}$ ($d_S \gtrsim 1.84\xi_S$).

The new feature, which is absent in a single S layer, is the existence of nascent vortices in the N layer due to the influence of the adjacent superconductor. Indeed, in a single S layer, Δ exponentially decays far from the ends at large B [see Figs. 4(b) and 4(d)], while in the N layer, the amplitude of oscillations of Δ rapidly decays but Δ saturates [see Figs. 3(b) and 3(d)]. In a single S layer, the number of vortices scales with the length [see Fig. 4(b)], while in the N layer the sum of ordinary and nascent vortices scales with L (this has been verified for lengths $L = 100 - 300\xi_c$). Nascent vortices could be considered as an analog of ordinary vortices, which cannot fully enter the N layer at large B due to relatively large proximity-induced Δ.

Properties of the N layer with a large induced Δ (case with $T/T_{cN} = 1.05$) in the field range $B_r < B < B_b$ resemble the properties of a single S layer with $d_S \gtrsim 1.84\xi_S$ at $B < B_{c3}^{\infty}$. As in the S layer, the vortex entry and exit are irreversible, $M(B)$ is hysteretic, nascent vortices are absent, and the number of vortices scales with the length of the N layer (this is verified for lengths $L = 100 - 300\xi_c$). However, in the N layer, vortices may exist even when $d_N \ll \xi_N$ due to the adjacent superconductor with $\xi_S \ll \xi_N$, where part of the vortex core is located.

We find similar results for different values of T/T_{cN}. Hysteresis in $M(B)$ appears at $T/T_{cN} \simeq 1.15$ (this value is not universal and depends on the ratio D_N/D_S and d_N). With increasing T/T_{cN} the value of Δ decreases in the N layer, leading to a smaller B_b above which nascent and ordinary vortices appear in the N layer. With decreasing T/T_{cN}, hysteresis increases and in the field range $B_r < B < B_b$ there are only ordinary vortices in the N layer, while nascent vortices exist at large fields when Δ becomes strongly suppressed.

Similar nascent vortices were found theoretically in a current-carrying SN bridge with normal or superconducting leads [16] where the name "nascent vortex" was adopted from Ref. [13] (see discussion in Ref. [16]). In [16] it was proven that the nascent vortex state is a finite length effect and appears in an inhomogeneous superconductor. Additionally, it was argued that nascent vortices have similarities with different types of saddle point states in current-carrying superconductors or superconductors placed in a magnetic field. This statement is also valid for the system considered here.

When N is a true normal metal with $T_{cN} = 0$ or a superconductor with $T_{cN} \ll T_{cS}$, the change of T/T_{cN} at a fixed T/T_{cS} corresponds to the change of the temperature when $T \ll T_{cS}$, and the superconducting parameters of the S layer are almost temperature independent. From the Usadel and Eilenberger models, it follows that with lowering T, the induced Δ (or anomalous Green function) increases in the N layer, and $\xi_N = \sqrt{\hbar D_N/6\pi k_B T}$ (dirty limit) or $\xi_N = \hbar v_{FN}/2\pi k_B T$ (pure limit) increases too. Therefore, we expect that at high temperature, the magnetization of the N layer will be reversible with no ordinary or nascent vortices—if we neglect finite length effect, as in the case of a superconducting cylinder covered by thin normal layer. At low T the magnetization curve will be hysteretic, with squeezed vortices located near the SN interface in the field range $B_r \lesssim B \lesssim B_b$.

Our calculations show that to observe the predicted effect, one needs $D_N \gg D_S$ ($\rho_N \ll \rho_S$). This provides a large difference in Δ in the S and N layers, which favors vortex pinning at the SN interface. Dirty superconductors like NbN, MoSi, and NbTiN, and low resistive thin films of Au, Ag, Al, and Cu with thicknesses of about dozens of nanometers are preferable candidates.

There is an interesting question about how the results would change when the thickness of the N layer greatly exceeds λ_N, but we still have $d_N \lesssim \xi_N$, which means that the N layer is a type I superconductor. When vortices enter the N layer, they strongly suppress Δ there [see Figs. 2(b) and 2(c)]. Because $\lambda_N \sim 1/\Delta$ in the Ginzburg-Landau model (which assumes a local relation between the superconducting current and the vector potential), the magnetic field penetrates deeper into the N layer in the vortex state. This makes the system closer to the model studied here, at least for not extremely small Ginzburg-Landau parameter κ. Note that the nonlocal relation between current density and vector potential in the N layer, which has been considered in several works [5–7], makes the problem more complex from the point of view of the appearance of vortices.

Nevertheless, we speculate that our results could be considered as a possible mechanism for the peculiar diamagnetic and paramagnetic responses of the normal metal layer found in Refs. [25,26], which covers the superconducting cylinder. These results were observed at extremely low temperatures, when the thickness of the N layer is comparable with ξ_N. At such a low T, one may expect a relatively large value of the induced Δ in the N layer needed for the appearance of vortices. The superconductor was a dirty Nb which has a short coherence length $\xi_S \sim 10$ nm $\ll \xi_N \sim 5 - 10$ µm, diffusion coefficient $D_S \ll D_N$ and a large pinning current density, which assumes large intrinsic inhomogeneity. The last property may provide pinning of the vortex cores that partially penetrate the superconductor, according to our results. In the experiment, $M(B)$ was hysteretic at low temperature and nonhysteretic at high temperature. Therefore, we suppose that frozen (pinned) vortices may appear at the SN interface at low T when the magnetic field is swept up and down, leading to a paramagnetic response of the N layer. Indeed, we found that a local defect in the superconductor placed close to the SN interface (modeled as a local suppression of T_{cS}) decreases B_r. However, to make a quantitative comparison with an experiment, one needs to consider the minimal model where the N layer is a superconductor with a small London penetration depth $\lambda_N \ll \xi_N$, thickness $d_N \lesssim \xi_N$, and the S layer is a type II superconductor with a small coherence length $\xi_S \ll \xi_N$.

VI. CONCLUSION

We theoretically find that at sufficiently large in-plane magnetic field, vortices may exist in a thin N layer that covers the superconductor. The cores of the vortices are located in both the superconductor and the normal metal, and in the N layer they are strongly squeezed due to its small thickness $d_N \ll \xi_N$. At large magnetic fields, nascent vortices may exist in a finite length N layer, leading to a state with spatial oscillations of Δ along the N layer and zero vorticity.

ACKNOWLEDGMENT

The work is supported by the Russian Science Foundation (Project No. 23-72-30004).

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