COMMENTS AND ADDENDA

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Resonant cooling of hot electrons in high magnetic fields

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The oscillatory variation of the mean electron energy with magnetic field deduced recently by Kahlert and Bauer from hot-electron Shubnikov–de Haas experiments in n-InSb at 4.2 K is explained by a resonant emission of LO phonons. Since a resonant cooling process involving only band and not impurity states is more likely to be observable in degenerately doped samples, Shubnikov–de Haas experiments are superior to magnetophonon-effect experiments with nondegenerate material.

The reappearance of magnetophonon oscillations at high electric fields and at low temperatures, where LO phonons are not thermally excited, has been attributed to a resonant cooling of the electron system by a number of authors.[1–4] This phenomenon is necessarily connected with carrier heating and resonant energy dissipation (in contrast to the momentum relaxation effects in the Ohmic regime). At magnetic fields for which the energetic distance between any two Landau levels equals the LO-phonon energy, one expects an enhanced probability for an LO-phonon-assisted transition from the higher Landau level to the lower one. The mean carrier energy or "electron temperature" should therefore exhibit a periodic dependence on the magnetic field, with minima at the resonant fields. This cooling effect and the resulting variation of the carrier mobility was first discussed theoretically by Pomortsev and Kharus[5] and later investigated experimentally[1–4] and theoretically by several authors.[6–8] All hot-electron magnetophonon experiments performed at low temperatures ($T < 40$ K) have so far been interpreted by this mechanism. However, the experiments have been mainly made with n-type InSb in a temperature range from 4.2 to 20 K and in magnetic fields of 0 to 33 kG. Under these conditions magnetic freeze-out of electrons from the conduction band to donors occurs in low-doped n-InSb, so that the observed extrema of the magnetoresistance may also result from a periodic variation of the number of conduction electrons.[9] In all theoretical models only mobility variations have been considered and no change in the free-carrier concentration was included. It is therefore desirable to perform experiments in which a change of the carrier concentration through the combined influence of electric and magnetic fields is diminished or even excluded.

Recently two of the authors[10] have observed a dependence of the electron temperature deduced from the decrease of Shubnikov–de Haas amplitudes on the magnetic field strength in n-InSb with a carrier concentration of $n_e = 6.9\times10^{16}$ cm^{-3} at liquid-He temperatures. In such highly doped material no change of the carrier concentration occurs under the influence of electric fields up to several V/cm. The purpose of this report is to analyze this situation in terms of a simple theoretical model and to discuss its implications on the analysis in terms of resonant electron-phonon

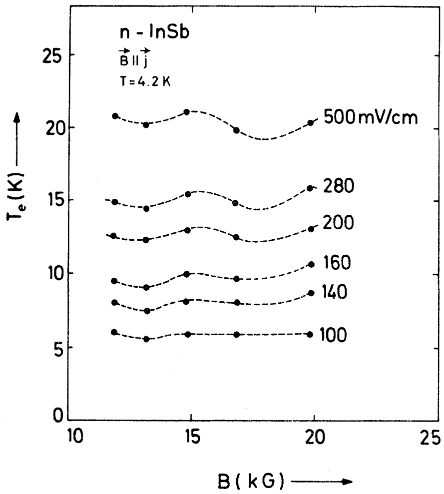

FIG. 1. Electron temperature vs magnetic field for various electric fields deduced from the damping of Shubnikov–de Haas oscillation amplitudes.

transitions, which seem to apply also to our experimental findings, indicating the first direct evidence for a resonant cooling of hot electrons under degenerate conditions. The experiments have been described in Ref. 10. The electron temperature was determined from the decrease of the extremal Shubnikov–de Haas amplitudes and is shown for five magnetic fields in Fig. 1.

At low electric fields around 100 mV/cm there is only one dip visible at approximately 13 kG. As the electric field is increased, an additional dip occurs at about 17 kG. An analysis of the relevant energy-loss mechanisms reveals that above 200 mV/cm polar scattering of LO phonons ($\hbar\omega_{\mathrm{LO}}=23.9$ meV) becomes important.[10] Accordingly, the occurrence of a structure for fields as low as 100 mV/cm might at a first view be attributed to the 2TA mechanism, resembling the scattering of an optical phonon of energy $\hbar\omega_{2\mathrm{TA}}=10.3$ meV, first proposed by Stradling and Wood.[2] Baumann[11] has shown theoretically that the strength of this process may become comparable to that of the LO mechanism under conditions appropriate to InSb at liquid-He temperatures.

For reasons of simplicity the following discussion will be in terms of the electron-temperature concept, because the conclusions drawn from our expression for the energy-loss rate will not depend on the detailed shape of the carrier distribution function. For the same reasons a standard band model is used. The energy-loss rate for transitions between Landau states $|\nu\rangle=|n,k_y,k_z\rangle$, where $\vec{k}$ denotes the electron wave vector and n the Landau level number, through absorption and emission of phonons (energy $\hbar\omega_q^{(i)}$, wave vector $\vec{q}$, e-ph-coupling constant $C_q^{(i)}$) of mode i (= LO, LA, TA, ...), for $\vec{E}\parallel\vec{B}\parallel z$, is

$$P^{(i)}=\frac{1}{V}\frac{2\pi}{\hbar}\sum_{\vec{q}}\hbar\omega_q^{(i)}C_q^{(i)2}\,2\sum_{\nu}\sum_{\nu'}\{E_{\nu\nu'}^{(i)}(\vec{q})f(\epsilon_{\nu'},T_e)\times[1-f(\epsilon_\nu,T_e)]-A_{\nu\nu'}^{(i)}(\vec{q})f(\epsilon_{\nu'},T_e)[1-f(\epsilon_\nu,T_e)]\},\tag{1}$$

where the emission and absorption terms are given through Fermi's golden rule as

$$E_{\nu\nu'}^{(i)}(\vec{q})=[N(\hbar\omega_q^{(i)},T)+1]\,|\langle\nu|e^{-i\vec{q}_{\mathrm{em}}\cdot\vec{r}}|\nu'\rangle|^2\times\delta(\epsilon_{\nu'}-\epsilon_\nu-\hbar\omega_q^{(i)})\tag{2}$$

and

$$A_{\nu\nu'}^{(i)}(\vec{q})=N(\hbar\omega_q^{(i)},T)\,|\langle\nu|e^{i\vec{q}_{\mathrm{ab}}\cdot\vec{r}}|\nu'\rangle|^2\times\delta(\epsilon_\nu-\epsilon_{\nu'}-\hbar\omega_q^{(i)})\;.\tag{3}$$

V is the crystal volume and the two arguments of the Planck distributions N for phonons and heated Fermi distributions f for the carriers denote energy and temperature, respectively. The Landau states $|\nu\rangle$ have energies

$$\epsilon_\nu=(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m\;,\tag{4}$$

where $\omega_c=eB/mc$ is the cyclotron frequency for carriers of effective mass m. The absorption amplitude is given by[12]

$$\langle\nu|e^{i\vec{q}\cdot\vec{r}}|\nu'\rangle\equiv M_{nn'}(c\hbar q_\perp^2/2eB)\delta_{k_y,k_y'+q_y}\delta_{k_z,k_z'+q_z}\;,\tag{5}$$

with

$$q^2=q_z^2+q_\perp^2\tag{6}$$

and

$$M_{nn'}\left(\frac{c\hbar q_\perp^2}{2eB}\right)=\left(\frac{n_<!}{n_>!}\right)^{1/2}\exp\left(\frac{-c\hbar q_\perp^2}{4eB}\right)\times\left(\frac{c\hbar}{2eB}q_\perp^2\right)^{(n_>-n_<)/2}L_{n_<}^{n_>-n_<}\left(\frac{c\hbar}{2eB}q_\perp^2\right),\tag{7}$$

where the L_l^λ are generalized Laguerre polynomials A similar expression holds for phonon emission.

The integration over the y and z components of the carrier momenta is straightforward by use of the momentum and energy δ functions, and we obtain

$$P^{(i)}=\frac{emB}{n_e(2\pi)^3\hbar^4c}\sum_n\sum_{n'}\int_{Q_{nn'}^2}dq_z^2\frac{1}{q_z^2}\times\int_0 dq_\perp^2\,\hbar\omega_q^{(i)}C_q^{(i)2}[N(\hbar\omega_q,T_e)-N(\hbar\omega_q,T)]\times[f(\epsilon^{(i)},T_e)-f(\epsilon^{(i)}+\hbar\omega_q^{(i)},T_e)]\left|M_{n'n}\left(\frac{\hbar c}{2eB}q_\perp^2\right)\right|^2,\tag{8}$$

where

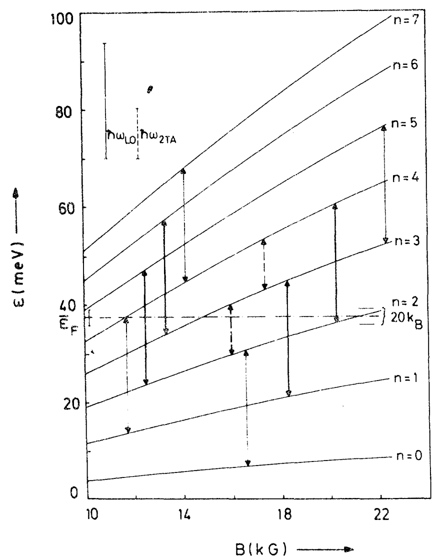

FIG. 2. Landau levels vs magnetic field for nonparabolic band structure, spin splitting ignored. Arrows show either LO or 2TA transitions. $\bar{\epsilon}_F$ denotes a mean value of the magnetic-field-dependent Fermi energy, and brackets around $\bar{\epsilon}_F$ indicate thermal broadening.

$$\epsilon^{(i)} = (n+\tfrac{1}{2})\hbar\omega_c + \epsilon_z^{(i)} \tag{9}$$

and

$$\epsilon_z^{(i)} = (1/8m\hbar^2 q_z^2)\{2m[\hbar\omega_q^{(i)} + (n-n')\hbar\omega_c] - \hbar^2 q_z^2\}^2 . \tag{10}$$

Here a low-longitudinal-momentum cutoff Q was introduced to incorporate all those mechanisms [represented by our experimentally determined Dingle temperature T_D (Ref. 13)], which will necessarily damp out the following resonance behavior for $q_z \to 0$: For $Q\to 0$, $P_{\rm LO}$ and $P_{\rm 2TA}$ diverge at the lower integration limit if $(n'-n)\hbar\omega_c = \hbar\omega_q$, as can be seen from Eqs. (8) and (9), because otherwise $\epsilon_\nu^{(i)} \to \infty$ and therefore $f\to 0$, compensating the pole at $q_z^2=0$. In $P_{\rm LA}$ and $P_{\rm TA}$, a resonance condition would require $n'=n$, but even then f remains finite only for $q\to 0$, in which limit the singularity is cancelled by $C_q \to 0$, since we have included static screening (taken independent of B) for i=TA (piezoacoustic scattering). Since $P\to\infty$ means $T_e \to T$, resulting sharp dips in the T_e-vs-B curve should occur.

For finite cutoff we might expect these minima to become less pronounced and to disappear for sufficiently high Dingle temperatures.[13] Indeed, under magnetophonon-resonance conditions, the cutoff Q can be directly related to T_D, and a preliminary numerical analysis (including LO, LA and TA modes) revealed that the weak (because only logarithmic) LO singularities discussed above are smeared out. It should be noted that additional singularities due to the $q^2=0$ pole in $C_q^{(\rm LO)2}$ cannot occur for our experimental situation, even for zero cutoff; our magnetic fields give resonances at $n_> - n_< \geq 2$, so that the factor $q_\perp^{2(n_>-n_<)}$ in $|M|^2$ eliminates the $q^2=0$ singularity. However, it should be stressed that our analysis is based on the assumption of heated Fermi functions, whereas one has strong reasons for noticeable distortions of the carrier distribution due to the strong inelasticity of LO processes, at least for the nondegenerate case.[7]

The detailed numerical evaluation of the energy-loss rate seems to us computationally manageable only if the cutoff Q^2 can be chosen independent of the magnetic quantum numbers n and n' for each electronic transition, which is actually only the case for the "resonant" transitions. On the other hand, the numerical results are naturally very sensitive to Q, especially for the acoustic losses, which dominate the power loss at all but the highest electrical fields of our analysis. For this reason, it is easily possible that the structure found in our T_e-vs-B curves is at least partially caused by the magnetic field dependence of the cutoff. Moreover, the B dependence of the collision-broadening mechanism itself as well as of the static screening in the piezoelectric interaction and of possible phonon disturbances might introduce additional magnetic field effects. It should be noted that phonon heating[14] may be an additional damping mechanism. In our approximate analysis we replaced the actual stationary phonon distribution by $N(\hbar\omega_q, T)$ in Eqs. (2) and (3); in the limit of complete phonon heating, $N(\hbar\omega_q, T)\to N(\hbar\omega_q, T_e)$ and $P\to 0$, as is also obvious on physical grounds. In view of the many experimental evidences for "hot magnetophonon resonances," our foregoing critical remarks should emphasize the urgent need for a quantitative theoretical treatment of the relevant damping mechanisms in such processes.

We now turn to a discussion of Fig. 1. The most striking feature is the electric field dependence of the two dips at 13 and 17 kG. As mentioned before, the appearance of the 13-kG structure at the lowest electrical fields resembles a resonant 2TA process, whereas the appearance of the second minimum at much higher electric fields suggests that it should be associated with LO transitions. For a more detailed investigation of this point we calculated (for the actual nonparabolic band structure) the position of those resonant LO and 2TA transitions, which take place between electron states in Landau levels on opposite sides of the Fermi energy ϵ_F; in such cases the difference between the two distribution functions in Eq. (8) is large ($\simeq 1$). These transitions are shown as bold arrows in Fig. 2, where we have also indicated the thermal broadening of the Fermi level by a $20k_B$ region around ϵ_F. Also shown are some weak transitions (thin arrows), where both electron states are either below or above ϵ_F, with one level near ϵ_F, so that the difference of the two Fermi distributions will be $\ll 1$, but still finite. It is obvious that the 13-kG minimum cannot be associated with 2TA transitions across the Fermi level. Since we exclude any LO effects in this minimum for the above-mentioned energetical reasons, this structure seems to be connected with the low-momentum cutoff for the acoustic losses mentioned before. In view of this conclusion, one might also question the explanation of the 17-kG minimum as an LO-phonon effect, but here the appearance of the structure at those electron temperatures, which are necessary for the onset of noticeable electron–LO-phonon scattering in n-InSb, strongly supports the interpretation as an LO-phonon resonance. Moreover, it follows from Eq. (7) that within the range of q_1^2 values dominating the energy-loss rate [Eq. (8)] the transition amplitude for the 3-1 transition at 18.4 kG is much larger than the 4-2 amplitude near 20 kG, in good agreement with the experimental dip around 17 kG.

In conclusion, the experiment on the dependence of the electron temperature on the magnetic field in n-InSb deduced from the damping of Shubnikov–de Haas oscillations in the longitudinal magnetic field configuration can be explained by a resonant transition of electrons between individual Landau levels. LO phonons are emitted, leading to a resonant cooling of the electron system, which is reflected in the nonmonotonic behavior of the Shubnikov–de Haas oscillations with magnetic field. In contrast to experiments with low-doped n-InSb, no change of the carrier concentration occurs in the high-field experiments. Thus the observed effect is entirely due to the properties of conduction electrons subjected to high electric and quantizing magnetic fields.

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