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Surface-generation statistics and associated thermal currents in metal-oxide-semiconductor structures*

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The non-steady-state statistics are derived for traps at the silicon-oxide interface of metal-oxide-semiconductor (MOS) structures. These results are applied to derive the surface-generation current I_g vs. temperature T characteristic associated with the generation of electron-hole pairs through the interfacial traps. It is shown that for a negligible rate of generation of carriers in the depletion layer of the semiconductor, the surface generation I_g-T characteristic reflects the trap distribution in the lower-half of the band gap. Furthermore, it is shown that the I-T curves associated with the bulk-generation and the surface-generation processes exhibit characteristics which allow the two processes to be distinguished.

I. INTRODUCTION

In a previous[1] paper the non-steady-state statistics for traps in the depletion layer of the metal-oxide-semiconductor (MOS) capacitor were derived, with the assumption that the effect of interface traps at the silicon-silicon dioxide interface could be neglected. In this paper we will derive the non-steady-state statistics for traps at the silicon-silicon dioxide interface, when the device is switched from the accumulation mode to the deep-depletion mode. In addition, we will show that a simple relationship exists between the energy distribution of the interface traps and the current-vs-temperature characteristic of the device, under the appropriate biasing and heating conditions, which expedites analysis of the experimental data. Furthermore, it will be shown that the energy distribution of most of the traps throughout the band gap can be examined by the technique.

II. NON-STEADY-STATE STATISTICS

The non-steady-state technique involves initially filling the interface traps at a low temperature by applying a forward voltage bias to the device in order to accumulate the surface of the silicon [Fig. 1(a)]. In this condition all interfacial traps below the Fermi energy are essentially full while those above are essentially empty. A reverse voltage bias is then applied to the device so that the surface of the silicon is in the non-steady-state deep-depletion mode [Fig. 1(b)]. The temperature of the device is then raised at a uniform rate β, and the current monitored as a function of temperature.

It has been shown that the rate equation which describes the occupancy of the traps at an energy E is, according to the Shockley-Read statistics, given by[2]

$$df/dt=(e_p+\bar{n})-(e_n+e_p+\bar{n}+\bar{p})f\ , \tag{1}$$

where $\bar{p}=v\sigma_p p$, $\bar{n}=v\sigma_n n$, v is the thermal velocity of the carriers, σ_n and σ_p are the capture cross sections of the traps for electrons and holes, respectively, and n and p represent the density of free electrons and holes at the interface, respectively. The emission probability rate for holes, from a trap of energy E_t above the valence band, is given by

$$e_p=v\sigma_p N_v e^{-E_t/kT}\equiv\nu_p e^{-E_t/kT} \tag{2}$$

and the corresponding emission probability rate for electrons, e_n, is given by

$$e_n=v\sigma_n N_c e^{(E_t-E_c)/kT}\equiv\nu_n e^{(E_t-E_t)/kT}\ . \tag{3}$$

Now, under reverse-biased conditions [Fig. 1(b)], the density of free electrons in the conduction band at the silicon-silicon dioxide interface is small and may be neglected. However, the density of free holes at the interface cannot be neglected when the device is approaching the inversion mode. Thus, Eq. (1) reduces to

$$df/dt=e_p-(e_n+e_p+\bar{p})f\ . \tag{4}$$

The general solution of Eq. (4) is given by[3]

$$f(t)=e^{-\lambda}\left(\int_0^t e_p e^{\lambda}\,dt+f_0\right)\ , \tag{5}$$

where f_0 represents the initial occupancy of a trap at the energy level E, and λ is given by

$$\lambda=\int_0^t (e_n+e_p+\bar{p})\,dt\ . \tag{6}$$

Now, when the device is biased initially into the accumulation mode at the low temperature [see Fig. 1(a)], the density of free *holes* at the silicon-oxide interface is virtually zero; in addition all

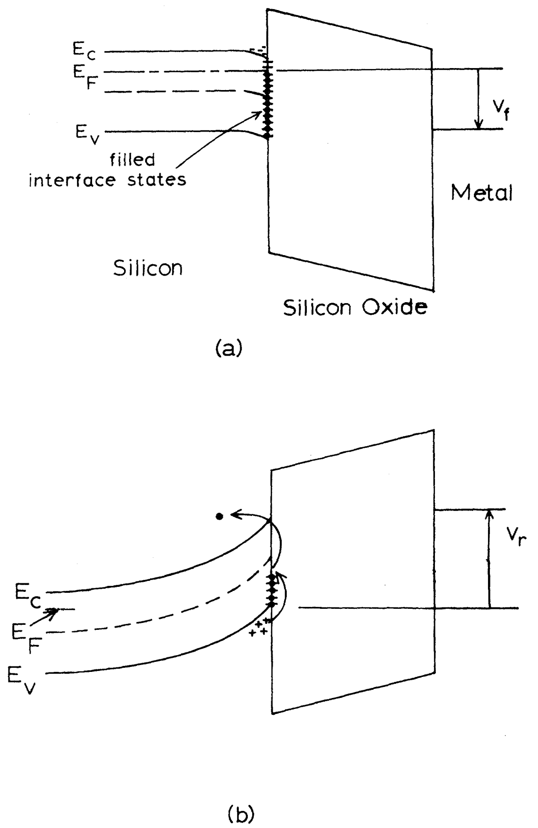

FIG. 1. Energy diagram of an n-type MOS device: (a) in the accumulation mode with all interfacial traps below the Fermi energy filled and a negligible density of free holes at the interface, (b) in the deep-depletion mode with a negligible density of free electrons at the interface.

interfacial traps below the Fermi level are full. Thus, when a reverse voltage bias is applied to the device, the semiconductor becomes deeply depleted [Fig. 1(b)], and the initial value of the free-hole density, p_0, is zero. Furthermore, surface generation at the low temperature is negligibly small and may be equated to zero. Hence, the contribution of $\bar{p}$ to λ in (4) is insignificant at the low temperatures. Nevertheless, $\bar{p}$ will be retained in the evaluation of Eq. (4) since at the higher temperatures at which generation occurs, $\bar{p}$ plays a dominant role. Now let us suppose that the temperature is increased at a uniform rate β given by

$$T=\beta t+T_0 \ , \tag{7}$$

where T_0 represents the initial temperature. Then substituting (7) into (5) gives

$$f=e^{-\lambda}\left(\beta^{-1}\int_{T_0}^{T} e_p e^{\lambda}\, dT+f_0\right), \tag{8}$$

and substituting (7) into (6) gives

$$\lambda=\beta^{-1}\int_{T_0}^{T}(e_n+e_p+\bar{p})\, dt \ . \tag{9}$$

The approximate solution of (5) is given by[1]

$$f=\left(1-\frac{e_p}{e_n+e_p}\right)e^{-\lambda}+\frac{e_p}{e_n+e_p+\bar{p}} \ . \tag{10}$$

In Fig. 2, the factor $e^{-\lambda}$ in (10) is shown plotted as a function of energy for various values of temperature. It will be noted that in the upper-half of the band gap, it has a similar functional dependence on energy as that of a Fermi-Dirac distribution function. For energy levels in the upper-half of the bandgap, that is, for $e_n \gg e_p$, the integral of (9) can be shown to be given approximately by[4]

$$\lambda \simeq (kT^2/E)\, e_n(E) \ . \tag{11}$$

Now for energy levels above the energy E_{tn} defined by $\lambda=1$, that is,

$$(kT^2/E_{tn})\, e_n(E_{tn}) \simeq 1 \ , \tag{12}$$

the traps are essentially empty, and below E_{tn} they are essentially full. Thus it follows that for E_{tn} greater than about $2kT$ above midgap, the second term on the right-hand side of (10) may be neglected. Hence, in the upper-half of the bandgap

$$f \simeq e^{-\lambda} \ , \tag{13}$$

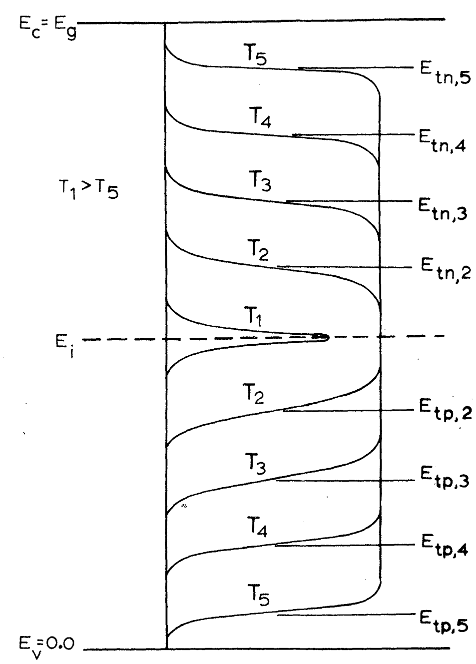

FIG. 2. Term $e^{-\lambda}$ is shown plotted as a function of energy with temperature as the parameter. The quasi-Fermi levels E_{tn} and E_{tp} are shown approaching the midgap with increasing temperature.

and the statistics are governed solely by the emission process, the generation process being negligible. This may be verified by solving (4) with only the emission term included.

Figure 2 illustrates (13) as a function of energy, with temperature as a parameter. It is noted that as the temperature is increased, E_{tn} approaches midgap (see Fig. 2) which means that trap levels above E_{tn} have been emptied by the emission process. In the process of being swept out of the depletion layer the emitted electrons give rise to the emission current. At some temperature T_1, at which E_{tn} reaches midgap, the function becomes sharply peaked about E_{tn} (here ν_n is assumed to be equal to ν_p), and collapses rapidly about E_i for any further increase in temperature. Hence, above T_1 the first term of (10) becomes negligible in comparison with the second term, which means that the emission process ceases and the generation process takes over. It is interesting to note that when emission has ceased, the occupancy function expressed by (10) collapses to a form that is identical to the quasiequilibrium statistic. We will now consider the emission and generation processes quantitatively.

III. EMISSION CURRENT

In this section we will consider the current that arises from the emission of electrons from interfacial traps in the upper-half of the band gap to the conduction band.

For the case of distributed traps above the midgap, the emission current is proportional to the rate at which traps are emptied, that is [cf. Appendix of Ref. (1)]

$$I_e \propto qAN_{st}(E_{tn})\,\beta\,\frac{dE_{tn}}{dT} \ , \tag{14}$$

or [cf. Appendix of Refs. (1) and (3)]

$$I_e = \frac{C_0}{C_0 + C_d}\{qA\beta N_{st}(E_{tn})10^{-4}[1.92\log_{10}(\nu/\beta) + 3.2]\} \ , \tag{15}$$

where $N_{st}(E_{tn})$ is the interfacial trap density at the non-steady-state Fermi energy, E_{tn}, for electrons. C_0 and C_d are the capacitances of the oxide and depletion layer of the silicon, respectively. In practical MOS devices, $C_0 \gg C_d$ with the device in the deep-depletion mode so that (15) reduces to

$$I_e = 10^{-4}\,qA\beta[1.92\log_{10}(\nu/\beta) + 3.2]N_{st}(E_{tn}) \ , \tag{16}$$

which shows that the emission current is proportional to the interfacial trap density at E_{tn}. Also, it can be shown[3] that

$$E_c - E_{tn} = 10^{-4}T[1.92\log_{10}(\nu/\beta) + 3.2] - 0.0155 \ ; \tag{17}$$

hence the temperature axis can be transformed into an axis representing energy below the conduction-band edge. Thus, it will be apparent from (16) and (17) that an $I_e - T$ plot is a direct image[3] of the interfacial trap distribution in the upper-half of the band gap.

IV. GENERATION CURRENT

In the subsections that follow we will derive the expression for the surface-generation current I_{gs} in terms of the temperature T and the interfacial trap density N_{st}. Furthermore, we will show that for trap distributions that vary slowly with energy, the relationship between the non-steady-state Fermi level for holes (E^*_{tp}) and temperature may be expressed by a linear approximation.

A. Relation between E^*_{tp} and temperature

We saw in an earlier section that when the temperature is such that interfacial traps in the upper-half of the bandgap are empty, the occupancy f

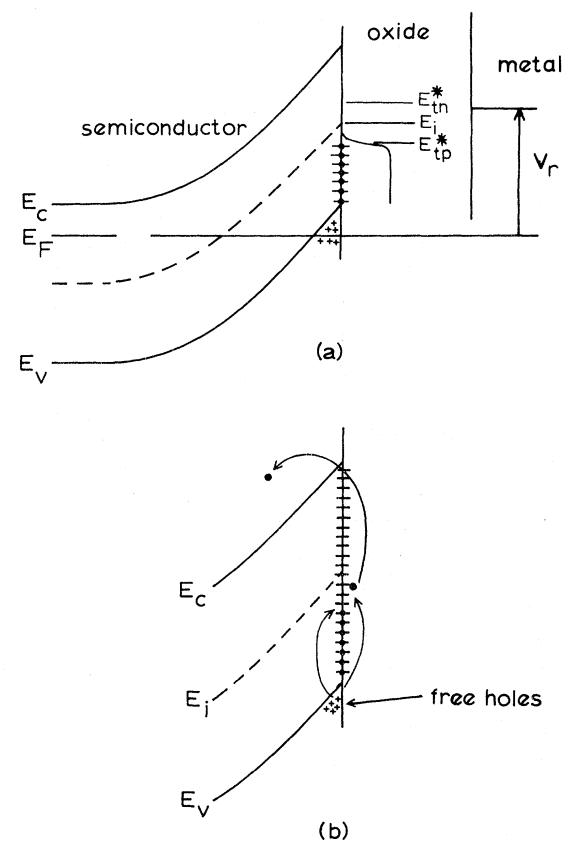

FIG. 3. Energy diagram of an n-type MOS device: (a) in the deep-depletion mode with the occupancy function $f = e_p/(e_n + e_p + \bar{p})$ superimposed; (b) illustrating the generation of electron-hole pairs through interface traps and the recapture of free holes by traps at E^*_{tp}.

is expressed in terms of the generation statistics [the second term of (10)] given by

$$f=\frac{e_p}{e_n+e_p+\bar{p}} \quad . \tag{18}$$

Figure 3(a) shows the energy diagram of the device in the non-steady-state deep-depletion mode, and superimposed on it is the occupancy function for traps at the interface given by (18). The energy level E_{tn}^* and E_{tp}^* on the figure represent the non-steady-state Fermi levels for trapped electrons and holes at the interface, respectively, and are defined by

$$e_n(E_{tn}^*)=\bar{p} \text{ and } e_p(E_{tp}^*)=\bar{p} \ , \tag{19}$$

since, as will be seen later, E_{tn}^* and E_{tp}^* define essentially the energy range of trap levels involved in the generation process.

Now, as the temperature is increased, the free-hole density at the interface increases rapidly as a result of the generation of electron-hole pairs through the traps at the Si-SiO_2 interface. However, at the same time, the density of free holes at the interface is diminished through recombination with electrons in the interface traps. Thus, the net effect is that electrons are released from interface traps in the lower-half of the bandgap to the conduction band. This two-step process is indicated by the arrows in Fig. 3(b).

The rate at which the number of free holes per unit area p_s at Si-SiO_2 interface changes with temperature is equal to the rate of generation of electron-hole pairs at the interface minus the rate at which the free holes are captured by interface traps. The rate equation is therefore given by

$$\frac{dp_s}{dt}=\int_{E_v}^{E_c}\frac{e_ne_pN_{st}(E)\,dE}{e_n+e_p+\bar{p}}-\int_{E_v}^{E_c}f'N_{st}(E)\,dE \ , \tag{20}$$

where $N_{st}(E)$ represents the number of interfacial traps per unit area per unit energy. It will be recalled that the non-steady-state occupancy is given by $f\cong e_p/(e_n+e_p+\bar{p})$, and f', which represents the rate of change of trap occupancy at energy E, is obtained by taking the derivative of the occupancy f with respect to time using (19), and is given by

$$f'=\frac{f(1-f)}{kT}\frac{dE_{tp}^*}{dt} \, . \tag{21}$$

On substituting (7) and (21) into (20) we obtain

$$\beta\frac{dp_s}{dT}=\int_{E_v}^{E_c}\frac{e_ne_p}{e_n+e_p+\bar{p}}N_{st}(E)\,dE-\int_{E_v}^{E_c}\frac{f(1-f)}{kT}N_{st}(E)\left(-\frac{dE_{tp}^*}{dt}\right)dE \ . \tag{22}$$

Now, for trap distributions that are not too strong a function of energy, i.e., $dN(E)/dE<N(E)/kT$, the second integral on the right-hand side of (22) may be approximated by

$$\int_{E_v}^{E_c}\frac{f(1-f)}{kT}N_{st}(E)\frac{dE_{tp}^*}{dt}dE\simeq N_{st}(E_{tp}^*)\frac{dE_{tp}^*}{dt} \ ,$$

where $N_{st}(E_{tp}^*)$ is the trap density at the non-steady-state Fermi level E_{tp}^*. Hence (22) becomes

$$\beta\frac{dp_s}{dT}=\int_{E_v}^{E_c}\frac{e_ne_p}{e_n+e_p+\bar{p}}N_{st}(E)dE-\beta N_{st}(E_{tp}^*)\left(-\frac{dE_{tp}^*}{dT}\right) . \tag{23}$$

Furthermore, let us examine the factor $G(E,T)=e_ne_p/(e_n+e_p+\bar{p})$ in the above integral. The factor is shown plotted in Fig. 4 as a function of energy for a given value of temperature. We observe that between the energy levels E_{tn}^* and E_{tp}^* (where $\bar{p}\gg e_n, e_p$), $G(E,T)$ is approximately constant in energy, and is given by

$$G(E,T)=e_ne_p/\bar{p} \ . \tag{24}$$

Also, for energies $E<E_{tp}^*-2kT$ and $E>E_{tn}^*+2kT$, it will be apparent from Fig. 4 that $G(E,T)$ may be approximated by zero. In view of these comments and noting that $e_ne_p/\bar{p}$ is independent of energy, (20) reduces to

$$\beta\frac{dp_s}{dT}\cong\frac{e_ne_p}{\bar{p}}\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE-\beta N_{st}(E_{tp}^*)\left(-\frac{dE_{tp}^*}{dT}\right) . \tag{25}$$

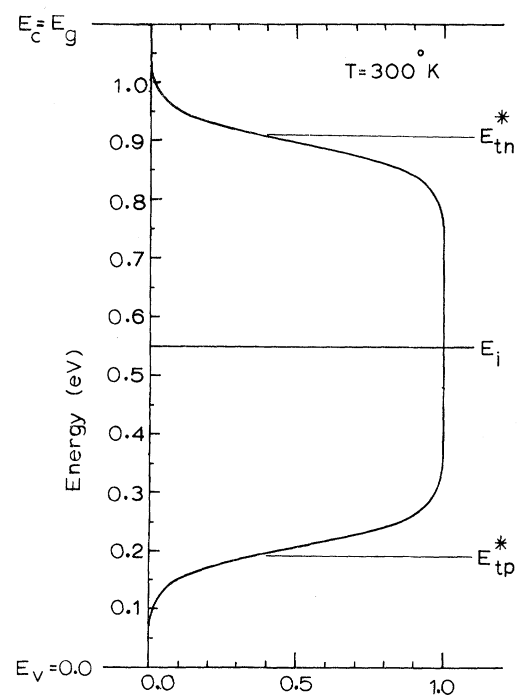

FIG. 4. The function $G=e_ne_p/(e_n+e_p+\bar{p})$ is plotted as a function of energy at a temperature of 300 °K. For energy levels 2kT above E_{tp}^* and 2 kT below E_{th}^*, the function is approximated by the constant $e_ne_p/\bar{p}$, and is shown normalized to $e_ne_p/\bar{p}$ in the plot above.

Let us suppose that the distribution of free holes in the valence band of the silicon under reverse conditions is given by Maxwell-Boltzmann statistics, that is ,

$$p(x)=p(0)\,e^{[\phi(x)-\phi_s]/kt}\ , \tag{26}$$

where $p(0)$ is the density of free holes at the interface, ϕ_s is the interfacial surface potential of the semiconductor, and ϕ is the potential of the bottom of the conduction band with respect to its value in the bulk of the silicon. Substituting (26) into Poisson's equation and integrating, we obtain the following expression for the electric field $\mathcal{E}(x)$:

$$\mathcal{E}(x)=(q/\epsilon_s)^{1/2}\{kT\,p(0)[e^{-\phi_s/kT}+e^{[\phi(x)-\phi_s]/kT}]+N_d\phi(x)\}^{1/2}\ . \tag{27}$$

Now the total number of free holes p_s in the inversion layer is found by integrating Eq. (26) with respect to x, and is given by

$$p_s=\int_0^{x_d} p(0)\,e^{[\phi(x)-\phi_s]/kT}\,dx\ . \tag{28}$$

If we assume that the free-hole density at the surface is sufficiently large that the second term within the brackets of (27) can be neglected, then on making use of (27) in (28), and integrating, we obtain

$$p_s=2[\epsilon_s p(0)kT/q]^{1/2}\ . \tag{29}$$

On the other hand, if the first term within the brackets of (27) is negligible, p_s is given approximately by

$$p_s\cong kT\,p(0)(\epsilon_s/qN_d\phi_s)^{1/2}\ . \tag{30}$$

From Eqs. (25) and (29) we obtain the following differential equation:

$$\frac{dp(0)}{dT}=e_ne_p\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE\Bigg/\beta v\sigma_p kT\left[\left(\frac{\epsilon_s p(0)}{qkT}\right)^{1/2}+N_{st}(E_{tp}^*)\right]\ , \tag{31}$$

When $N_{st}(E_{tp}^*)\gg[\epsilon_s p(0)/qkT]^{1/2}$, which is normally the case except under strong inversion conditions, (31) simplifies to

$$\frac{dp(0)}{dT}=\frac{v\sigma_nN_cN_v}{\beta kT}\,e^{-E_g/kT}\left(\frac{\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE}{N_{st}(E_{tp}^*)}\right). \tag{32}$$

In a similar manner, Eq. (32) is obtained from Eqs., (25) and (30) with the assumption $N_{st}(E_{tp}^*)\gg p(0)(\epsilon_s/qN_d\phi_s)^{1/2}$. The approximate solution of Eq. (22) is given by (see Appendix)

$$p=\frac{v\sigma_nN_cN_vT}{\beta E_g}\left(\frac{\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE}{N_{st}(E_{tp}^*)}\right)e^{-E_g/kT}\ . \tag{33}$$

The temperature dependence of E_{tp}^* can now be derived by substituting (20) into (33)

$$E_{tp}^*=E_g-kT\ln\left(\frac{v\sigma_nN_cT}{\beta E_g}\,\frac{\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE}{N_{st}(E_{tp}^*)}\right). \tag{34}$$

If we take the usually adopted temperature dependences for σ_n ($\propto T^{-2}$), v ($\propto T^{1/2}$) and N_c ($\propto T^{3/2}$), an examination of (34) reveals that the factor $v\sigma_nN_cT/\beta$ in the argument of the logarithm is linearly dependent on the temperature. Furthermore, when we consider the factor R, given by

$$R=\int_{E_{tp}^*}^{E_{tn}^*}N_{st}(E)\,dE\Big/N_{st}(E_{tp}^*)\ , \tag{35}$$

in the logarithm of (34), we find that for trap distributions which vary relatively slowly with energy, the ratio R changes only slowly with temperature. Thus, assuming the argument of the logarithmic term to be essentially independent of temperature, Eq. (34) predicts an approximately linear dependence of E_{tp}^* or temperature.

B. Relation between J_g and trap density N_{st}

In Sec. IV A it was pointed out that the interfacial traps in the lower-half of the band gap release their electrons to the conduction band by means of a two-step process [see Fig. 3(b)]. However, it is only the surface-generated electrons that give rise to the current in the external circuit since only these carriers are transported through the system. It must be pointed out here that in practical devices, electron-hole pairs are generated in the depletion layer of the silicon as well. However, in the analysis that follows, it will be assumed that the density of traps in the depletion layer is small in comparison to the interfacial trap density so that any contribution to the current from bulk generation can be neglected (however, see Sec. VI C). The rate of surface generation of electron-hole pairs through interfacial traps in the range of energy dE is given by the product of the emission probability rate e_n, the interface trap density $N_{st}(E)$, and the trap occupancy $e_p(E)/[e_n(E)+e_p(E)+\bar{p}]$. Thus, the total rate of generation of electrons is given by

$$\frac{dn}{dt}=\int_{E_v}^{E_c}e_n(E)\,\frac{e_p(E)}{e_n(E)+e_p(E)+\bar{p}}\,N_{st}(E)\,dE\ . \tag{36}$$

Hence, the generation current J_g is given by

$$J_g=\frac{qC_0}{C_0+C_0}\,\frac{dn}{dt}\ , \tag{37}$$

where the factor $C_0/(C_0+C_d)$ arises because the electrons are swept through the depletion layer only, and not the oxide as well. In the deep-depletion mode, C_0 is usually much larger than C_d so

$$J_g \simeq q\,\frac{dn}{dt}\ . \tag{38}$$

From an inspection of the integrals in (22) and (25) it is seen that (36) may be expressed in the following form:

$$\frac{J_g}{q} \simeq \frac{e_n e_p}{\bar{p}} \int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\,dE\ . \tag{39}$$

From Eqs. (33), and (39) we obtain the following expression for the generation current, J_g:

$$J_g = (q\beta E_g/T)\,N_{st}(E_{tp}^*)\ , \tag{40}$$

or

$$N_{st}(E_{tp}^*) = J_{gs}T/q\beta E_g\ . \tag{41}$$

Thus, the interfacial trap density at energy E_{tp}^* *is proportional to the product of the generation current and the temperature.*

C. Surface initially inverted

Let us assume that a large reverse voltage bias has been applied to the device at room temperature so that it is in the inversion mode, and then cooled in this mode to some low temperature. Under the condition of strong inversion, the concentration of free holes p_{s0} at the silicon-silicon dioxide interface is sufficiently large that $\bar{p}_{s0}$ is comparable with e_n and e_p, and hence we retain the first term of Eq. (A8). Now, it will be recalled from Sec. IV that the function $e_ne_p/(e_n+e_p+\bar{p}_s)$ is approximately constant in energy for $E_{tn}^* > E > E_{tp}^*$ (see Fig. 4); hence (37) for the non-steady-state generation current may be written in the following approximate form:

$$I_{gs} \cong qAC_0e_ne_p\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\,dE\Big/(C_0+C_d)\left(\bar{p}_{s0}+\left[\nu_n\nu_pT\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\,dE\Big/\beta E_g N_{st}(E_{tp}^*)\right]e^{-E_g/kT}\right)\ . \tag{42}$$

From an examination of (42) it is observed that at low temperatures the second term within the large parentheses of the denominator is small in comparison with $\bar{p}_{s0}$. This means that at the low temperatures at which the rate of generation of electron-hole pairs is very low, the increase in the free-hole density at the interface is only a small fraction of the total free-hole density at the initial temperature. Hence, the surface-generation current expressed by (42) may be expressed by the approximate linear expansion:

$$I_{gs} \simeq \frac{qAC_0e_ne_p}{(C_0+C_d)\bar{p}_{s0}}\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\,dE\left[1-\left(\nu_n\nu_pT\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\,dE\Big/\beta E_g\bar{p}_{s0}N_{st}(E_{tp}^*)\right)e^{-E_g/kT}\right]\ , \tag{43}$$

that is, at a given temperature, the surface-generation current is inversely proportional to the free-hole density p_{s0} at the initial temperature. As the temperature is increased, the surface generation increases with the temperature and falls to zero when the system reaches the steady-state inversion condition. The area under the curves is proportional to the net positive charge accumulated in the inversion layer (for n-type silicon), which, in turn, is directly related to the increase in the reverse voltage bias $(V_f - V_i)$. Figure 5(a) illustrates a series of theoretical curves obtained for increasing values of V_f and V_i while maintaining $V_f - V_i$ constant, and assuming a uniform trap distribution. It is observed that the peaks shift to higher temperatures as V_f is increased, while the area under each peak remains constant. This is in contrast with the peaks that are obtained when bulk-generation is the dominant process [see Ref. 1 and Fig. 5(b)] in which the temperatures at which the maxima occur are relatively independent of V_f for a given value of $V_f - V_i$. Thus the experiment provides a straightforward method for determining which of the two generation processes is dominant.

V. COMPUTED CURVES

If we examine (34), which relates the non-steady-state Fermi level for trapped holes (E_{tp}^*) to the temperature, we observe that there is no tractable solution of the equation. However, a plot of the factor R [given by (35)] as a function of E_{tp}^*, using the trap distribution of Fig. 7(a), shows that the change in R is very small in comparison with the factor $v\sigma_nN_cT/\beta E_g$ in the argument of the logarithm. The computed $E_{tp}^* - T$ curve, indicated by the *broken* line in Fig. 6, was obtained using (34) and the energy distribution of the interfacial traps shown in Fig. 7(a); this distribution was found to give the greatest deviation from linearity. A parabolic distribution and distributions that changed slowly with energy were found to give more linear $E_{tp}^* - T$ curves. However, all the curves can be linearized without incurring serious error. Thus we may write (34) in the following approximate form:

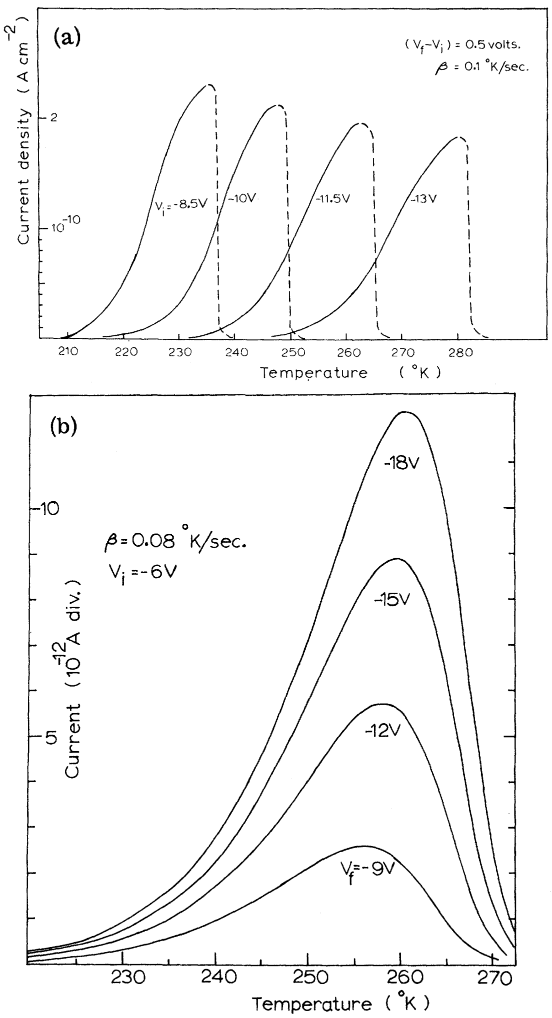

FIG. 5. $I_g - T$ characteristic for (a) surface generation shows a strong dependence of the position of the peak on the voltage bias. (b) For bulk generation there is a weak dependence on the voltage bias.

$$E_g - E^*_{tp} = T[2.3k \log_{10}(\nu/\beta) + M] + N , \qquad (44)$$

where M and N are constants to be determined from the linearized $E^*_{tp} - T$ curve. From a knowledge of the slope of the linearized curve of Fig. 6, the value of M was found to be 4.6×10^{-4}. The value of N was determined by choosing a value of temperature T and substituting the corresponding value of E^*_{tp} obtained from Fig. 6 into (44), and was found to be 0.025. Hence, in the linear approximation equation, (34) reduces to

$$E_g - E^*_{tp} = T\times 10^{-4}[1.98 \log_{10}(\nu/\beta) + 4.6] + 0.025 . \qquad (45)$$

Thus, using the approximately linear relation between E^*_{tp} and temperature given by (45), the temperature axis can be transformed into an axis representing energy above the top of the valence band. Hence, it will be apparent from (41) and (45) that an $I_{gs}T - T$ *plot is a direct image of the interfacial trap distribution in the lower-half of the band gap*.

In order to demonstrate the accuracy of the technique, we transformed the $J_{gs}T$ product-vs.-temperature (T) curve [computed using (36) and (37)] into the $N_{st} - E$ curve using the transformation equations (41) and (45), and compared it with the original trap distribution. The broken line of Fig. 7(a) indicates the original trap distribution, and the solid line represents the transformed $J_{gs} - T$ curve. The value of β used in the computation was 0.1 °K/sec, and the trap cross section was 10^{-15} cm^2. Figure 7(b) shows a distribution with maxima at 0.3 and 0.47 eV above the top edge of the valence band. Again, the broken line indicates the actual distribution, while the solid line represents the transformed $J_{gs} - T$ curve. The comparison between the actual distribution and the transformed curve is seen to be good in both cases. Changing the heating rate and the trapping cross section, within practical values, gave similarly good correlation.

VI. DISCUSSION

In the derivation of the expression for the surface-generation current that was presented above, it was assumed that the density of bulk traps in the depletion layer of the silicon was sufficiently small that the bulk-generation current could be neglected in comparison with the surface-generation current. Nevertheless, in many cases this may not be a

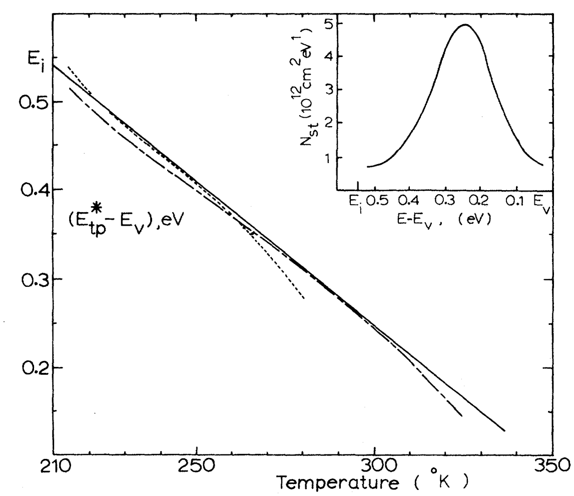

FIG. 6. Energy E^*_{tp} is shown plotted as a function of temperature (dashed line) using the trap distribution shown in the insert. The broken line represents the $E^*_{tp} - T$ characteristic using a similar distribution but with the maximum at 0.4 eV. The solid line represents the linearized $E^*_{tp} - T$ curves.

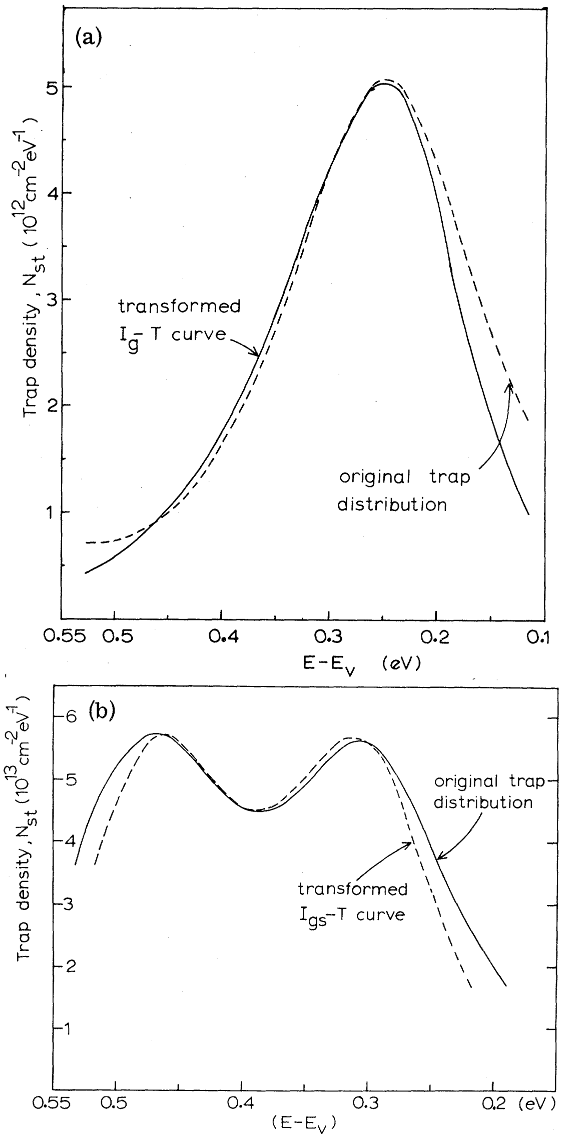

FIG. 7. (a) Dashed line represents the trap distribution and the solid line represents the exact $I_{gs} - T$ curve that has been transformed using (41) and (45). (b) Illustrates the accuracy when a distribution that exhibits two peaks is used.

valid assumption; indeed, the bulk trap density in the depletion layer may be so large that the converse may be true. Hence, it is important to be able to distinguish between the two processes.

In both cases the net generation current ceases to flow when the surface of the silicon reaches the equilibrium condition. Now, it has been shown in a previous paper[1] that for a given heating rate the size of the bulk-generation $I_{gs} - T$ curve increased with increasing reverse voltage bias, but that the temperature at which the maximum occurs remains approximately the same. Furthermore, the temperature at which the generation current ceases to flow is weakly dependent on the voltage bias. In contrast with this observation[1,5] of the bulk-generation process, when the surface-generation process dominates, the temperature at which the current ceases to flow shifts by a significant amount to higher values as the reverse voltage bias on the device is increased [Fig. 5(a)].

If the interfacial trap density is large in comparison with the bulk density, then under normal biasing conditions surface-generation will mask any bulk-generation effects. However, it will be recalled that the surface-generation process becomes less efficient with increasing free-hole density at the interface. Thus, if an appropriately large reverse-voltage bias is applied to the device so that a large density of free holes exist at the interface at the start of the heating process, the surface-generation current can be made arbitrarily small in comparison with the bulk-generation current, thus allowing one to study the bulk-generation effects alone.

APPENDIX: RELATION BETWEEN FREE-HOLE DENSITY AT THE INTERFACE AND TEMPERATURE

The differential equation that relates the free-hole concentration at the surface to the temperature is given by (32) of the text, namely,

$$\frac{dp(0)}{dT} = \frac{v\sigma_n N_c N_v}{\beta k T} e^{-E_g/kT} \left(\frac{\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\, dE}{N_{st}(E_{tp}^*)} \right) . \qquad \text{(A1)}$$

Now, it is found that the ratio within the large parentheses of (A1) is very weakly dependent on the temperature; hence, (A1) may be simply written as

$$\int dp = \frac{v\sigma_n N_c N_v}{\beta k} \left(\frac{\int_{E_{tp}^*}^{E_{tn}^*} N_{st}(E)\, dE}{N_{st}(E_{tp}^*)} \right) \int \frac{e^{-E_g/kT}}{T}\, dT . \qquad \text{(A2)}$$

Let us consider the integral on the right-hand side of (A2). On substituting $x = E_g/kT$ into the integrand and simplifying, the integral becomes

$$\int \frac{e^{-E_g/kT}}{T}\, dT = -\int \frac{e^{-x}}{x}\, dx . \qquad \text{(A3)}$$

Integrating by parts and collecting terms gives

$$-\int \frac{e^{-x}}{x} \left(1 + \frac{1}{x}\right) dx = \frac{e^{-x}}{x} . \qquad \text{(A4)}$$

Now, for reasonable values of temperature $E_g/kT \gg 1$; hence $1/x$ $(= kT/E_g) \ll 1$ and (A4) may be simplified to

$$-\int \frac{e^{-x}}{x}\, dx \simeq \frac{e^{-x}}{x} . \qquad \text{(A5)}$$

Substituting this result into (A2) gives the following expression for the free-hole density at the surface:

$$p - p_0 \simeq \frac{v\sigma_n N_c N_c}{\beta k}\left(\frac{\int_{E^*_{tp}}^{E^*_{tn}} N_{st}(E)\,dE}{N_{st}(E^*_{tp})}\right) \times \left(T e^{-E_g/kT} - T_0 e^{-E_g/kT_0}\right)\frac{k}{E_g}\,, \qquad (A6)$$

when the device is biased into the accumulation mode at the low temperature, p_0 is negligible in comparison with p for temperatures $T(\gg T_0)$ at which generation begins. Hence, (A6) simplifies to

$$p \cong \frac{v\sigma_n N_c N_v}{\beta k T}\left(\frac{\int_{E^*_{tp}}^{E^*_{tn}} N_{st}(E)\,dE}{N_{st}(E^*_{tp})}\right) e^{-E_g/kT}\,. \qquad (A7)$$

When the device is biased into the strong-inversion mode, then for $T \gg T_0$, (A6) simplifies to

$$p \simeq \frac{v\sigma_n N_c N_v T}{\beta E_g}\left(\frac{\int_{E^*_{tp}}^{E^*_{tn}} N_{st}(E)\,dE}{N_{st}(E^*_{tp})}\right) e^{-E_g/kT} + p_0\,. \qquad (A8)$$

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