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Role of refractive-index surfaces on helicon-wave propagation

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The dispersion equation accounting for the finite phenomenological collision frequency and the electron-hole effective mass ratio in a semiconductor plasma has been derived. The refractive-index surfaces for the dimensionless collision parameter Z and the effective mass ratio d have been computed. It is shown that the refractive-index surfaces are very sensitive to changes in the collision frequency and are comparatively less sensitive to changes in effective mass ratio. The effect of the refractive-index surface on low-frequency helicon-wave propagation in a n-type InSb semiconductor plasma has been discussed. It is argued that the deformations and changes in refractive-index surfaces would be of significant diagnostic value.

I. INTRODUCTION

In the case of alkali metals the electron plasma supports the helicon waves and the phase velocity of the wave is independent of the effective mass of the carriers. However, in the case of a mobile semiconductor plasma, the effective masses of electrons and holes are comparable. The effective mass ratios of semiconducting samples are found to change with kinetic temperature and the shape of the Fermi surface. The intent of the present paper is to account for the effect of finite phenomenological collisions and for finite effective mass ratios on the refractive-index surface. In the limit of $\omega_c\tau \gg 1$ and $\omega \ll \omega_c$, the resulting dispersion equation has been used to construct the refractive-index surfaces. Variations in the effective electron-hole mass ratio and collision parameter consistent with the experimental values have been used and the corresponding changes in the refractive surfaces in the n-type semiconductor InSb have been computed (Figs. 1 and 2). It is argued that the knowledge of resulting deviations and deformations of the refractive-index surface arising from one of the two or due to a combined effect would help in analyzing the experimental data of helicon-wave propagation through semiconductors.

II. BASIC THEORY

In the presence of a strong magnetic field the charge carriers in solids do not respond completely to the electric field of an electromagnetic wave. The dominance of the Lorentz force in a highly conducting solid permits the propagation of a low-frequency electromagnetic wave. The motion of uncompensated charge carriers in response to an electromagnetic field and a static magnetic field satisfies the generalized Ohm's law. The generalized Ohm's law exhibits the finite mobility of the semiconductor plasma and is known to vary significantly with changes in ambient-plasma field parameters such as band structure, impurities, inhomogeneities, and effects of the external static magnetic field. The change in the electrical conductivity of the semiconductor plasma which finally controls the nature of helicon-wave propagation can be visualized in terms of two parameters: the phenomenological dimensionless collision parameter $Z=(\omega_c\tau)^{-1}$ and the finite effective mass ratio $d=m_e^*/m_h^*$.

The well-known dispersion equation for helicon-wave propagation in the degenerate collisionless electron plasma of metals, obtained with the help of Ohm's law and Maxwell's curl equations (neglecting the displacement currents), is written as

$$n^2 = -\,\omega_p^2/\omega(\omega \pm \omega_c)\,. \tag{1}$$

The time and space dependence of the helicon wave propagating along B_0 is taken to be $e^{-i(kz-\omega t)}$, ω_p and ω_c being, respectively, the angular plasma frequency and the cyclotron frequency. The semiconductor plasma, contrary to the inherent simplicity of an alkali-metal plasma, exhibits a band structure for which the generalized Ohm's law is satisfied and the resulting conductivity of the semiconductor plasma becomes quite different. The ratio of effective masses of electrons and holes is less than unity and is known to change significantly with changes in band structure and temperature of the semiconductor. The scattering of current carriers and consequent change in their velocity distribution function is often times represented by the relaxation time $\tau=\nu^{-1}$, where ν is considered to be the phenomenological collision frequency between charge carriers or of charge carriers with impurities, defects, and lattice oscillations. The electrons and holes in an uncompensated semiconductor plasma respond differentially and satisfy two separate equations of motion. Following the standard technique we have derived the dispersion equation for helicon-wave propagation which accounts for the effect of the electron-hole mass

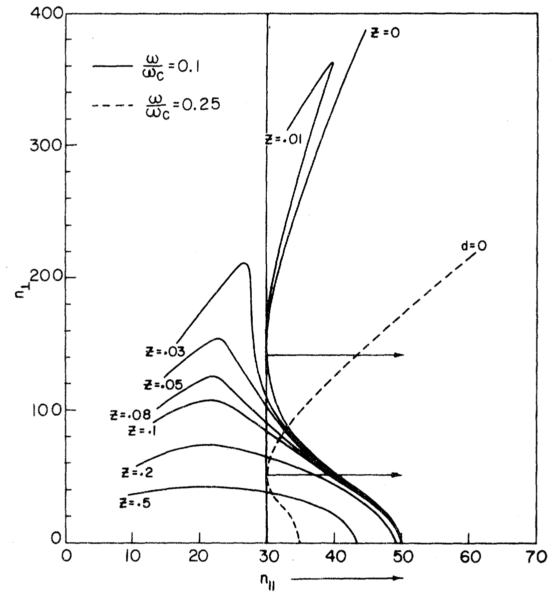

FIG. 1. Variation of refractive-index surfaces with collision parameter for n-type InSb.

ratio and phenomenological collision-frequency variations. The collision frequencies ν_e and ν_h are quite different, for different semiconductors, but in the derivation of the dispersion equation algebraic simplicity has been achieved by assuming $\nu_e = \nu_h = \nu$.[1] The total conduction current in the semiconductor plasma, due to the applied field, is written as

$$i(\omega - i\nu)m_e^*\vec{J} - e^2(n_e + n_h d)\vec{E} + e(1-d)\vec{J}\times\vec{B}_0 = \frac{e^2(\vec{J}\times\vec{B}_0)\times\vec{B}_0}{i(\omega - i\nu)m_h^*} + \frac{e^3(n_h - n_e)(\vec{E}\times\vec{B}_0)}{i(\omega - i\nu)m_h^*} . \quad (2)$$

We now make use of Maxwell's field equations to eliminate $\vec{E}$ from Eq. (2). Taking the curl of Eq. (2) and assuming, in the case of n-type semiconductors, $n_e \gg n_h$, $d < 1$, and using various vector identities we simplify the resulting equation to the following form:

$$(\omega - i\nu)[\vec{\nabla}\times(\vec{\nabla}\times\vec{b})] - k\omega_{ce}(1-d)\vec{\nabla}\times\vec{b} + \frac{\omega\omega_{pe}^2}{c^2}\vec{b} = 0, \quad (3)$$

where $\vec{b}$, ω_{ce}, and ω_{pe}, respectively, are the magnetic field vector of the wave, and the cyclotron and plasma angular frequencies of the electrons. Considering the propagation of helicon waves at an angle θ with the magnetic field (assumed to be directed along the z axis) and solving the wave equation, Eq. (3), we obtain the dispersion relation

$$n^2 = \frac{\omega_{pe}^2}{\omega[\omega_{ce}(1-d)\cos\theta - \omega + i\nu]} . \quad (4)$$

It is customary to normalize the various frequencies in terms of the cyclotron frequency of the ambient plasma. We write Eq. (4) as

$$n^2 = \frac{\delta^2}{\Omega[(1-d)\cos\theta - \Omega + iZ]}, \quad (5)$$

where

$$\delta = \omega_{pe}/\omega_{ce} ,$$

$$\Omega = \omega/\omega_{ce} ,$$

and

$$Z = \nu/\omega_{ce} .$$

Rationalizing Eq. (5) and writing $n = \mu + i\psi$ we obtain the real part of refractive index as

$$\mu^2 = \frac{\delta^2[(1-d)\cos\theta - \Omega]}{\Omega\{[(1-d)\cos\theta - \Omega]^2 + Z^2\}} + \frac{\delta^2 Z^2}{4\Omega[(1-d)\cos\theta - \Omega]\{[(1-d)\cos\theta - \Omega]^2 + Z^2\}} . \quad (6)$$

This equation is very useful in studying the helicon-wave propagation in a semiconductor plasma. We find that the presence of finite collisions also sets in some instability in the semiconductor plasma, but we have ignored this feature and have considered the propagation of helicon waves.

III. REFRACTIVE-INDEX SURFACES

In the derivation of the dispersion equation for helicon-wave propagation [Eq. (6)] we included the effect of phenomenological collisions and the mobility of charge carriers. This appears in Eq. (6)

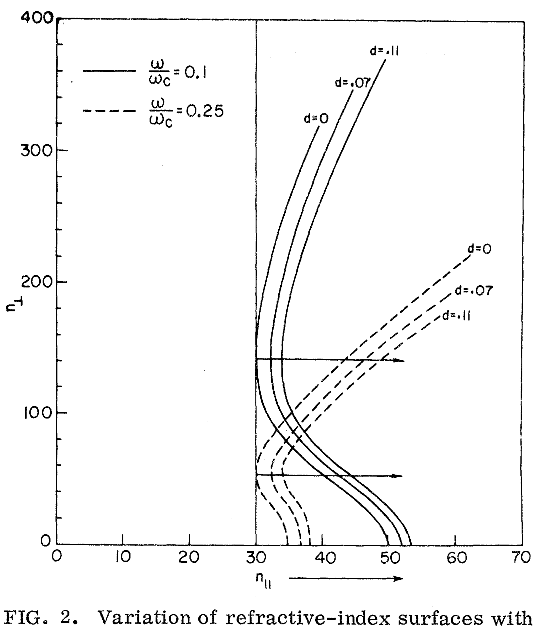

FIG. 2. Variation of refractive-index surfaces with effective electron-hole mass ratio for n-type InSb.

TABLE I. Parameters used in the computation—InSb.

Serial No.	Parameters	Values	Remarks
1	ω_{pe}	4.6×10^{12} sec^{-1}	$4.2\,^\circ K$ (Ref. 5)
		9.0×10^{12} sec^{-1}	$77\,^\circ K$ (Ref. 6)
2	ω_{ce}	3.0×10^{11} sec^{-1}	$\omega<\omega_{ce}$ has been assumed
		6.0×10^{11} sec^{-1}	
3	ν	0.90×10^{10} sec^{-1}	$4.2\,^\circ K$, corresponding $Z=0.03$ (Ref. 1)
		0.60×10^{11} sec^{-1}	$77\,^\circ K$, corresponding $Z=0.1$ (Ref. 1)
4	m_e^*	$0.013m_0$	$4.2\,^\circ K$ (Ref. 7)
		$0.021m_0$	$77\,^\circ K$ (Ref. 8)
5	m_h^*	$0.18m_0$	$4.2\,^\circ K$, corresponding $d=0.07$ (Ref. 7)
		$0.20m_0$	$77\,^\circ K$, corresponding $d=0.11$ (Ref. 8)

in terms of the dimensionless parameters Z and d. The collision parameter Z in semiconductor specimen is governed by various factors such as size and shape, defects and dislocations, concentration of carriers and impurities, the nature of interactions between uncompensated charge carriers and carriers interaction with the lattice.[1-4] Despite the role of these parameters the temperature dependence of semiconductor specimen produces a marked change in the effective Z value and thus controls the helicon-wave propagation. We have set aside the details of various processes controlling the effective relaxation time and have computed the refractive index for different values of the dimensionless parameter Z. The increasing Z values correspond to increasing temperature of the specimen. We have chosen InSb for studying the detailed features of refractive-index surfaces. Using Eq. (6) and the parameters shown in Table I, we have computed the variation in the refractive-index surfaces with varying collisional parameter Z and effective mass ratio d. Figure 1 shows the refractive-index surface deformation with increasing Z values. The refractive-index surfaces for $\omega \ll \omega_{ce}$ and $Z=0$, $d=0$ show points of inflection which lie strictly on a straight line perpendicular to the static magnetic field. The points of inflection shown in Fig. 1 correspond to a minimum value of $n_{\parallel}$ and is independent of the helicon-wave frequency. Thus, the phase velocity of the helicon wave in the limit of $Z=0$, $d=0$ is independent of frequency and the wave propagation is strictly parallel to the external magnetic field (as shown by the arrows). These features of wave propagation essentially give rise to a focusing effect of helicon waves and result in attenuationless propagation along the externally applied magnetic field. This property of low-frequency wave propagation through the terrestrial plasma is well known. We find that the point of inflection (for a fixed $\Omega=0.1$) changes with increasing values of Z (from zero onwards). For higher values of Z, the point of inflection loses its significance and the refractive-index surfaces are distorted. The depicted change in the refractive-index surface results in an increase in the phase velocity along the magnetic field. It is obvious that the presence of finite phenomenological collisions in semiconductors, and their variation, affects the helicon-wave propagation significantly.

In metals and gases the electron-ion mass ratio is often times ignored while analyzing the results of helicon-wave propagation. However, in semiconductors the ratio of the effective masses of charge carriers plays an important role and cannot be ignored. The dispersion analysis of the InSb specimen shows that effective hole mass $m_h^* =0.2m_0$. The energy dependence of m_h^* is not precisely measured. The effective mass of the electron is energy dependent due to the nonparabolic nature of the conduction band. The measurements of the effective electron mass for two concentrations[9] are

$$N_e=2.8\times10^{18}\ \mathrm{cm}^{-3},\quad m_e^*=0.040m_0,$$

$$N_e=4.0\times10^{18}\ \mathrm{cm}^{-3}\quad m_e^*=0.041m_0.$$

Taking these values of the effective masses we find that the parameter $d=m_e^*/m_h^*=0.25$ in the case of InSb, and varies slightly with electron concentration. Using Eq. (6) we have computed the refractive-index surfaces and the results of varying the effective mass ratio on the refractive-index surfaces are shown in Fig. 2. With increasing values of d, the refractive-index surfaces are found to shift towards higher values of $n_{\parallel}$. The position and shape of the point of inflection remains almost unaffected. The refractive-index surfaces for two normalized frequencies are shown in Fig. 2. Unlike the effect of the phenomenological collisions parameter, the refractive-index surfaces due to changing d values are not deformed. The variation of effective mass ratio brings in a corresponding variation in the phase velocity of the helicon waves, although the direction of propagation remains parallel to the applied magnetic field. It seems that these features might play an important role in the interpretation of helicon waves transmitted through metal and semiconductor plasmas. The role of finite collisions and effective mass ratio, or both together, may result in further deformation of refractive-index surfaces and complicate the analysis of helicon waves transmitted through semiconductor samples. The interference effect due to changing phase velocity produces a corresponding modulation in the intensity of the transmitted helicon waves. This effect is likely to be confused with the collision-induced instability mechanism resulting in decaying and growing helicon waves.[10] With precise control of plasma and field parameters the propagation features of helicon waves and the role of instabilities can be easily ascertained.

ACKNOWLEDGMENT

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