

## Steady-state transport in trap-dominated relaxation semiconductors

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Steady-state transport in trap-dominated relaxation semiconductors is discussed. It is suggested that under certain conditions such transport can be of a particular simple type. In cases where carrier lifetimes do not change significantly, the steady-state conductivity ratio  $K = \sigma_n/\sigma_p$  remains essentially constant and a conductivity-locked mode of transport can occur. On the basis of this condition, a theory for the Hall effect, photoconductivity, the Dember effect, and the photomagnetolectric effect is developed. Expressions for the Hall constant, the photocurrent, the Dember photovoltage, and the photomagnetolectric short-circuit current and open-circuit photovoltage are obtained. When specialized, these expressions give a steady-state theory for these effects in the amorphous semiconductors.

### I. INTRODUCTION

van Roosbroeck has shown<sup>1</sup> that semiconductors should be separated into two distinct types according to whether carrier lifetime  $\tau$  is greater than or less than dielectric relaxation time  $\tau_d$ . The *lifetime semiconductor*, with  $\tau > \tau_d$ , is the familiar type and has been widely studied for many years. Of much current interest is the *relaxation semiconductor*, defined by  $\tau < \tau_d$ , examples of which include the amorphous semiconductors and also some of the high-resistivity crystalline materials.<sup>2,3</sup>

The above distinction is indeed appropriate, for the two types of semiconductors are fundamentally different.<sup>1-3</sup> In the lifetime regime, bulk charge neutrality is maintained at equilibrium through fast dielectric relaxation processes, whereas in the relaxation regime, over-all charge neutrality is controlled by rapid recombination. As van Roosbroeck notes, this recombination is of a particular type. It is *equality* recombination or recombination with capture of electrons and holes at equal total rates per unit volume.

As a consequence, steady-state photoconductivity in trap-dominated relaxation semiconductors is due mainly to an increase in mobile-majority-carrier concentration. The photogenerated electrons and holes thus form a mobile-carrier space charge which is just balanced by an increase in fixed (localized) space charge of opposite sign, such that an *approximate* state of overall neutrality is maintained. As carrier lifetime approaches zero this neutrality becomes exact. This is in contrast to a lifetime semiconductor in which, with fast dielectric relaxation, the photogenerated electrons and holes are often treated as a quasineutral plasma. This quasineutral plasma never forms in a relaxation semiconductor. Here, dielectric relaxation processes are slow and the carriers recombine before it can be established.

In a recent paper<sup>3</sup> in which the amorphous alloys are treated as trap-dominated relaxation semi-

conductors, van Roosbroeck, using arguments based on transport and recombination fundamentals, shows the equilibrium state of the alloys to be one of minimum conductivity, that is, one of equal electron and hole conductivities. van Roosbroeck's model, in a self-consistent way, accounts for the typically high resistivity which the alloys display, the Hall-effect-Seebeck-effect anomaly which they exhibit, and many other aspects of observed behavior. This detailed agreement between theory and experiment is here taken as strong evidence that the model is essentially correct.

In this paper steady-state transport in trap-dominated relaxation semiconductors is discussed. It is suggested that under certain conditions such transport can be of a particular simple type. Provided carrier lifetimes do not change appreciably, the steady-state conductivity ratio  $K = \sigma_n/\sigma_p$  remains essentially constant and a *conductivity-locked* mode of transport can occur. In Sec. II, a trap-dominated relaxation semiconductor with one surface uniformly illuminated is considered. An applied magnetic field is included at the outset such that the Hall effect, photoconductivity, the Dember effect, and the photomagnetolectric (PME) effect are treated simultaneously as special cases. Expressions for the Hall constant, the photocurrent, the Dember photovoltage, and the PME short-circuit current and open-circuit photovoltage are obtained. When specialized, these expressions give a steady-state theory for these effects in the amorphous semiconductors.

### II. THEORY

Let us consider a trap-dominated relaxation semiconductor having dimensions  $x_0$ ,  $y_0$ , and unit width along  $z$  such that the lower surface at  $y=0$  is uniformly illuminated with light of intensity  $I_0$ . With an applied magnetic field  $\vec{B}$  along  $z$ , the above is a standard PME effect geometry. The applied field creates an anisotropy such that the carrier mobilities are tensors. These mobility tensors

may be expressed in matrix form in the specimen coordinate system as

$$\vec{\mu}_p = \begin{bmatrix} \mu_{pxx} & \mu_{pxy} & 0 \\ \mu_{pyx} & \mu_{pyy} & 0 \\ 0 & 0 & \mu_{pzz} \end{bmatrix}, \quad \vec{\mu}_n = \begin{bmatrix} \mu_{nxx} & \mu_{nxy} & 0 \\ \mu_{nyx} & \mu_{nyy} & 0 \\ 0 & 0 & \mu_{nzz} \end{bmatrix}, \quad (1)$$

where the subscripts  $p$  and  $n$  refer to holes and electrons, respectively. With Boltzmann statistics assumed, the corresponding diffusivities are given by

$$\vec{D}_p = (kT/e) \vec{\mu}_p, \quad \vec{D}_n = (kT/e) \vec{\mu}_n, \quad (2)$$

and conductivity tensors are

$$\vec{\sigma}_p = pe \vec{\mu}_p, \quad \vec{\sigma}_n = ne \vec{\mu}_n, \quad (3)$$

where  $p$  and  $n$  are hole and electron concentrations, respectively, and  $e$  is the (positive) electronic charge.

We seek expressions for the current  $I_{sc}$  that flows in the illuminated semiconductor when the ends along  $x$  are short-circuited and the open-circuit photovoltage per unit length  $V_{oc}$ . Steady-state continuity equations are

$$\nabla \cdot \vec{J}_p = -\nabla \cdot \vec{J}_n = e(\Delta g - \mathcal{R}) \quad (4)$$

in which  $\vec{J}_p$  and  $\vec{J}_n$  are hole and electron current densities

$$\vec{J}_p = \vec{\sigma}_p \cdot \vec{E} - e\vec{D}_p \cdot \nabla p, \quad (5)$$

$$\vec{J}_n = \vec{\sigma}_n \cdot \vec{E} + e\vec{D}_n \cdot \nabla n, \quad (6)$$

and  $\mathcal{R}$  and  $\Delta g$  are net nonequilibrium recombination and generation rates, respectively. For the geometry considered, the generation rate is given by

$$\Delta g = \alpha \beta I_0 e^{-\alpha y}, \quad (7)$$

where  $I_0$  is the incident photon flux density after reflection losses,  $\alpha$  the absorption coefficient, and  $\beta$  the quantum efficiency for electron-hole pair creation.

With mobilities in trap-dominated relaxation semiconductors typically small, the magnetic field induced anisotropy will also be small. In this case the conductivity tensors reduce to

$$\vec{\sigma}_p = \sigma_p \begin{bmatrix} 1 & a_p & 0 \\ a_p^* & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \vec{\sigma}_n = \sigma_n \begin{bmatrix} 1 & a_n & 0 \\ a_n^* & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

In the above expressions  $a_p = \sigma_{pxy}/\sigma_p$ ,  $a_p^* = \sigma_{pyx}/\sigma_p$ ,  $a_n = \sigma_{nxy}/\sigma_n$  and  $a_n^* = \sigma_{nyx}/\sigma_n$ . These quantities are, of course, directly related to carrier Hall angles.<sup>4,5</sup> Here they will simply be referred to as anisotropy factors.

For the geometry considered and with end effects neglected, hole and electron densities vary only with  $y$ , in which case  $\nabla \times \vec{E} = 0$  implies  $E_x = \text{constant}$ . Substitution of (2) and (8) into (5) and (6) then gives

$$J_{px} = \sigma_p E_x + a_p [\sigma_p E_y - (kT/e) d\sigma_p/dy], \quad (9)$$

$$J_{py} = a_p^* \sigma_p E_x + \sigma_p E_y - (kT/e) d\sigma_p/dy, \quad (10)$$

$$J_{nx} = \sigma_n E_x + a_n [\sigma_n E_y + (kT/e) d\sigma_n/dy], \quad (11)$$

$$J_{ny} = a_n^* \sigma_n E_x + \sigma_n E_y + (kT/e) d\sigma_n/dy. \quad (12)$$

Upon adding (10) and (12) to obtain  $J_y = 0$  and solving this expression for  $E_y$ , one finds

$$E_y = -\frac{(a_p^* \sigma_p + a_n^* \sigma_n) E_x}{\sigma} + \frac{kT}{e} \left( \frac{d\sigma_p}{dy} - \frac{d\sigma_n}{dy} \right) / \sigma, \quad (13)$$

where  $\sigma = \sigma_p + \sigma_n$ . Substitution of (13) into (10) and (12) then gives

$$J_{py} = -J_{ny} = \frac{a_p^* \sigma_p \sigma_n E_x}{\sigma} - \frac{kT}{e} \left( \frac{\sigma_p d\sigma_n}{dy} + \frac{\sigma_n d\sigma_p}{dy} \right) / \sigma. \quad (14)$$

Substitution of (13) into (9) plus (11) yields

$$J_x = \sigma E_x - a(kT/e)(\sigma_p d\sigma_n/dy + \sigma_n d\sigma_p/dy) / \sigma, \quad (15)$$

when second-order terms in anisotropy factors are neglected. In the above equations  $a^* = a_p^* - a_n^*$  and  $a = a_p - a_n$ .

Now the ultimate state of a strongly trap-dominated relaxation semiconductor is the minimum conductivity state, specified by  $\sigma_{n0} = \sigma_{p0}$  at equilibrium. In a minimum conductivity relaxation semiconductor, the equilibrium Fermi level is pinned at a position near midgap by *single-free-path equality recombination*.<sup>3</sup> Any small local fluctuation from  $\sigma_{n0} = \sigma_{p0}$  is rapidly eliminated by fast recombination processes, with carrier recombination times  $\approx 10^{-12}$  sec. When such a semiconductor is exposed to steady uniform-illumination equal conductivities are maintained by these same recombination processes, provided carrier lifetimes do not change.

Consider a relaxation semiconductor having large densities of equality centers (a trap-dominated relaxation semiconductor) but yet not quite sufficient densities to produce the minimum conductivity state. In this case, the conductivity ratio  $K = \sigma_{n0}/\sigma_{p0}$  is still controlled by fast recombination processes. When such a semiconductor is illuminated, this conductivity ratio is maintained by these same recombination processes, provided carrier lifetimes do not change. With large densities of equality centers present ( $\gg n_0, p_0$ ), these lifetimes will not change until *appreciable* conductivity modulation occurs. A trap-dominated relaxation semiconductor thus has a special property: Steady-state mobile carrier concentrations, in

some cases, are connected by  $\sigma_n = K\sigma_p$ . This is a most important property for it reduces the present problem involving optical injection to one of simplicity.

To further illustrate this proposed mode of transport the following comparison is appropriate: In a lifetime semiconductor with negligible trapping, excess carrier transport is *number-density-locked*. That is, the density  $\delta p$  of excess holes is everywhere equal to the density  $\delta n$  of excess electrons. The condition  $\delta n/\delta p = 1$  is controlled by fast dielectric relaxation processes such that small local departures from this ideal ratio are eliminated in times of the order of the dielectric relaxation time. It is because relaxation times are so small in lifetime semiconductors that the assumption  $\delta n/\delta p = 1$  is valid and provides theoretical results consistent with experiment. Setting  $\sigma_n/\sigma_p = K$  is thus seen to be a relaxation-case counterpart of the lifetime-case quasineutrality approximation, with small local fluctuations in  $K$  eliminated in times of the order of carrier lifetimes, which are very small in this instance. Steady-state transport in trap-dominated relaxation semiconductors, within limits, is thus *conductivity locked*.<sup>6</sup>

With  $K \equiv \sigma_n/\sigma_p = \sigma_{n0}/\sigma_{p0}$ , (14) reduces to

$$J_{py} = \left( \frac{2K}{1+K} \right) \left( \frac{1}{2} a^* \sigma_p E_x - \frac{kT}{e} \frac{d\sigma_p}{dy} \right), \quad (16)$$

$$J_{ny} = - \left( \frac{2}{1+K} \right) \left( \frac{1}{2} a^* \sigma_n E_x - \frac{kT}{e} \frac{d\sigma_n}{dy} \right). \quad (17)$$

Likewise, (15) becomes

$$\begin{aligned} J_x &= (1+K)\sigma_p E_x - a \left( \frac{kT}{e} \right) \left( \frac{2K}{1+K} \right) \frac{d\sigma_p}{dy}, \\ &= \left( \frac{1+K}{K} \right) \sigma_n E_x - a \left( \frac{kT}{e} \right) \left( \frac{2}{1+K} \right) \frac{d\sigma_n}{dy}. \end{aligned} \quad (18)$$

Continuity equations are

$$\begin{aligned} \frac{kT}{e} \left( \frac{2K}{1+K} \right) \mu_p \frac{d^2 \delta p}{dy^2} - \frac{1}{2} a^* \left( \frac{2K}{1+K} \right) \mu_p E_x \frac{d\delta p}{dy} \\ - \frac{\delta p}{\tau_p} + \alpha \beta I_0 e^{-\alpha y} = 0, \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{kT}{e} \left( \frac{2}{1+K} \right) \mu_n \frac{d^2 \delta n}{dy^2} - \frac{1}{2} a^* \left( \frac{2}{1+K} \right) \mu_n E_x \frac{d\delta n}{dy} \\ - \frac{\delta n}{\tau_n} + \alpha \beta I_0 e^{-\alpha y} = 0, \end{aligned} \quad (20)$$

where  $\delta p$  and  $\delta n$  are concentrations of nonequilibrium holes and electrons, respectively, and  $\tau_p$  and  $\tau_n$  their lifetimes. These lifetimes will here be treated as constants.

By defining

$$\mu_p^* = [2K/(1+K)]\mu_p, \quad \mu_n^* = [2/(1+K)]\mu_n, \quad (21)$$

$$D_p^* = (kT/e)\mu_p^*, \quad D_n^* = (kT/e)\mu_n^*, \quad (22)$$

$$L^* = (D_p^* \tau_p)^{1/2} \equiv (D_n^* \tau_n)^{1/2}, \quad (23)$$

and setting  $Y = y/L^*$ ,  $A = \alpha L^*$  in (19) and (20), these equations become

$$\frac{d^2 \delta p}{dY^2} - \frac{a^* e L^*}{2kT} E_x \frac{d\delta p}{dY} - \delta p + \alpha \beta I_0 \tau_p e^{-AY} = 0, \quad (24)$$

$$\frac{d^2 \delta n}{dY^2} - \frac{a^* e L^*}{2kT} E_x \frac{d\delta n}{dY} - \delta n + \alpha \beta I_0 \tau_n e^{-AY} = 0. \quad (25)$$

As is otherwise apparent, these equations are equivalent since  $\delta p/\tau_p = \delta n/\tau_n$  and one generates the other. What these equations describe are the characteristics of a mobile carrier space charge, the effects of which are included in the modified hole and electron transport coefficients. Both types of carriers have the same diffusion length  $L^*$ . Either equation may be used to solve the problem at hand. Here, we choose to keep track of the holes.

Upon substituting (22) and (23) into (18), the short-circuit current ( $E_x = 0$ ) is found to be

$$I_{sc} = -eD_p^* \int_0^{Y_0} a \left( \frac{d\delta p}{dY} \right) dY. \quad (26)$$

Under open-circuit conditions no current flows, in which case  $V_{oc} \equiv -E_{x(oc)}$  is

$$V_{oc} = -eD_p^* \int_0^{Y_0} a \left( \frac{d\delta p}{dY} \right) dY / (1+K)L^* \int_0^{Y_0} \sigma_p dY. \quad (27)$$

We therefore seek an expression for  $\delta p$  that satisfies appropriate surface boundary conditions. Substitution of such an expression into (26) and (27) will then yield the desired results.

The boundary conditions for the holes are

$$J_{py} = -es_{p1} \delta p \quad (\text{at } y=0) \quad (28)$$

$$= es_{p2} \delta p \quad (\text{at } y=y_0). \quad (29)$$

In (28) and (29),  $s_{p1}$  and  $s_{p2}$  are surface recombination velocities for holes at the illuminated and dark surfaces, respectively. Using (16), (21), (22), and (23) the boundary conditions become

$$\frac{d\delta p}{dY} - \frac{a^* e L^*}{2kT} E_x \delta p = S_1 \delta p \quad (\text{at } Y=0), \quad (30)$$

$$\frac{d\delta p}{dY} - \frac{a^* e L^*}{2kT} E_x \delta p = -S_2 \delta p \quad (\text{at } Y=Y_0), \quad (31)$$

where  $S_1 = s_{p1} L^*/D_p^*$ ,  $S_2 = s_{p2} L^*/D_p^*$  are dimensionless surface recombination velocities. These actually distinguish the surfaces with respect to the space charge since  $\delta p/\tau_p = \delta n/\tau_n$  and  $p/\tau_p = n/\tau_n$ . Substitution of the above into (30) and (31) gives the same boundary conditions for the electrons. That is,  $s_p \tau_p \equiv s_n \tau_n$  relate the hole and electron surface recombination velocities.

Under short-circuit conditions the drift terms in (24), (30), and (31) are zero, since  $E_x = 0$ . We shall assume these terms to be negligible under open-circuit conditions as well. This approximation is here an excellent one: In a trap-dominated relaxation semiconductor effective carrier mobilities are small, thus the anisotropy factor  $a^*$  is necessarily small. In addition, with lifetimes typically  $\approx 10^{-12}$  sec, the diffusion length  $L^*$  is also very small. With this approximation, the solution to (24) is

$$\delta p = \delta p_0^* (C_1 e^{-Y} + C_2 e^Y + e^{-AY}), \quad (32)$$

where

$$\delta p_0^* = \alpha \beta I_0 \tau_p / (1 - A^2), \quad (33)$$

$$C_1 = - \frac{(S_1 + A)(S_2 + 1)e^{-Y_0} - (S_2 - A)(S_1 - 1)e^{-AY_0}}{2(S_1 S_2 + 1) \sinh Y_0 + 2(S_1 + S_2) \cosh Y_0}, \quad (34)$$

$$C_2 = \frac{(S_1 + A)(S_2 - 1)e^{-Y_0} - (S_2 - A)(S_1 + 1)e^{-AY_0}}{2(S_1 S_2 + 1) \sinh Y_0 + 2(S_1 + S_2) \cosh Y_0}. \quad (35)$$

Substitution of (32) into (26) and (27) then yields the desired results:

$$I_{sc} = a J_0^* L^*, \quad (36)$$

$$V_{oc} = \frac{a J_0^*}{\sigma_0 Y_0 + \Sigma_0^*}. \quad (37)$$

In (36) and (37),  $J_0^*$  is a current density given by

$$J_0^* = [\epsilon \beta A I_0 / (1 - A^2)] [C_1 (1 - e^{-Y_0}) + C_2 (1 - e^{Y_0}) + (1 - e^{-AY_0})], \quad (38)$$

and  $\Sigma_0^*$  is a photoconductivity given by

$$\Sigma_0^* = \frac{(1 + K)^2 \epsilon^2 \beta A L^* I_0}{2K kT(1 - A^2)} \times \left( C_1 (1 - e^{-Y_0}) + C_2 (e^{Y_0} - 1) + \frac{1 - e^{-AY_0}}{A} \right). \quad (39)$$

With  $a = 0$  in (18) (zero magnetic field) and  $E_x = E_0$  (an applied electric field), an expression for the photocurrent  $I_{pc}$  may be obtained. That is

$$I_{pc} = (1 + K) E_0 L^* \int_0^{Y_0} \sigma_p dY, \quad (40)$$

which, using (21), (22), (23), and (32) reduces to

$$I_{pc} = (\sigma_0 Y_0 + \Sigma_0^*) L^* E_0. \quad (41)$$

In (41),  $E_0$  is assumed to be small, such that the conduction is Ohmic.<sup>3</sup>

By setting  $a_p^* = a_n^* = 0$  (zero magnetic field), an expression for the Dember photovoltage  $V_D$  may be obtained from (13). That is,

$$V_D = - \int_0^{Y_0} E_y dy, \quad (42)$$

which reduces to

$$V_D = \frac{kT}{e} \left( \frac{1 - K}{1 + K} \right) \int_{Y_0}^0 \frac{(d\sigma_p/dY) dY}{\sigma_p}. \quad (43)$$

After integration and substitution from (32), the above may be expressed as

$$V_D = (kT/e) \left( \frac{1 - K}{1 + K} \right) \ln \left( \frac{G_0^*(0)}{G_0^*(Y_0)} \right). \quad (44)$$

In (44),  $G_0^*(Y)$  is the net rate at which mobile carrier space charge is generated at the respective surfaces. The expression for  $G_0^*(Y)$  is given by

$$G_0^*(Y) = g_0 + \frac{\alpha \beta I_0}{1 - A^2} (C_1 e^{-Y} + C_2 e^Y + e^{-AY}), \quad (45)$$

where  $g_0$  is the thermal equilibrium generation rate.

Finally, by setting  $I_0 = 0$  and neglecting anisotropic "pinch" effects,<sup>7,8</sup> (13) gives the Hall field  $E_H$  as

$$E_H = -(a_p^* \sigma_p + a_n^* \sigma_n) E_x / \sigma, \quad (46)$$

in which case the Hall coefficient  $R_H = E_H / (J_x B)$  reduces to

$$R_H = (a_p + K a_n) / (1 + K) \sigma_0 B. \quad (47)$$

In (49) the substitutions  $a_p = -a_p^*$ ,  $a_n = -a_n^*$  hold, since the anisotropy is induced by a magnetic field in this case.<sup>3,4</sup>

Note that in (36), (37), (41), and (44) quantities pertaining to the holes that have been treated do not appear explicitly, and that (47) gives an ambipolar Hall coefficient.<sup>9</sup> These equations describe the steady-state properties of a mobile carrier space charge. This space charge is characterized by an equilibrium conductivity  $\sigma_0$ , a conductivity-locked diffusion length  $L^*$ , absorption parameters  $\alpha$ ,  $\beta$ , and five numbers. These are a conductivity ratio  $K$  which defines a state, two dimensionless surface recombination velocities  $S_1$  and  $S_2$  which indicate how the space charge interacts with surfaces, and two anisotropy factors  $a_p$  and  $a_n$  which take into account applied magnetic field.

The amorphous semiconductors, with equal-free-path equality recombination, are minimum conductivity relaxation semiconductors. That is, they reside in the  $K = 1$  state. With this specialization, (36), (37), (41), and (47) give steady-state PME, photoconductivity, and Hall effect theories for this amorphous state. As is indicated by (44), the Dember effect vanishes for this special case.

The general validity and range of applicability of the conductivity-locked mode of transport proposed here can be checked by experiment. For example, the theory predicts a linear variation in the PME short-circuit current with illumination intensity. An observed departure from strict linearity would thus signal departure from this simple type of transport. Recent investigations<sup>10-12</sup>

of the PME effect in polycrystalline CdTe films, in high resistivity GaP, and in the layer compound GaSe have detected a linear variation in PME short-circuit current for a wide range of illumination intensities and appear to support the present model. The materials used in the above studies are certainly relaxation-case semiconductors and must be analyzed as such.

The linear variation in photoconductivity with illumination intensity exhibited by many of the amorphous chalcogenides,<sup>13-15</sup> particularly at low temperatures, is predicted by (39). This, together with positive thermoelectric power<sup>15</sup> and negative Hall coefficient,<sup>3</sup> is consistent with their being  $K=1$  state relaxation semiconductors, as has been proposed by van Roosbroeck. The field activation of conductivity which occurs in certain amorphous alloys below a switching threshold, a process that

can be normalized and represented by a single curve over a wide temperature range, is accounted for on the basis of electron and hole conductivities that remain equal as they are increased.<sup>3</sup> This is another example of conductivity-locked behavior for  $K=1$ .

It must be emphasized, however, that conductivity-locked transport is not the exclusive steady-state mode in relaxation semiconductors nor even the characteristic one. In cases of recombinative injection of minority carriers, for example, the condition  $\mathcal{R} \approx 0$ , or quasizero net recombination rate ( $n\dot{p} \approx n_0\dot{p}_0$ ), can prevail on a local basis and the transport then is certainly not conductivity-locked. In fact, when it applies, the condition  $\mathcal{R} \approx 0$  is the characteristic feature of the relaxation regime that distinguishes it from the more familiar lifetime regime.

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<sup>6</sup>In an alternate but equivalent approach, one may define  $R_f$  as the ratio of electron-to-hole mean free paths,  $R_f \equiv L_{fn}/L_{fp}$ . Single-free-path equality recombination then gives  $K=R_f^2$  when arguments used in Ref. 3 above are employed. That is,  $K \neq 1$  results when carrier mean free paths are unequal.

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