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Theory of many-body effects on conduction-electron spin resonance in anisotropic metals*

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We present the details of an earlier Letter in which we showed that many-body effects qualitatively change the behavior of conduction-electron spin resonance (CESR) in metals having a spread in g values over the Fermi surface. When scattering is too slow to motionally narrow CESR we show that electron correlations and g anisotropy combine and lead to an exchange-narrowed collective mode. This mode is Lorentzian and its width would vanish in the absence of scattering. The position of the collective mode is shifted from the position of the motionally narrowed CESR which one would observe at high temperatures. This shift offers the possibility of determining the electron-electron exchange interaction in metals for which spin waves have not yet been observed. We discuss the behavior of CESR in general and derive equations for the linewidth and resonance position in one interesting limit.

I. INTRODUCTION

In an earlier letter[1] we presented a theory of many-body effects on conduction-electron spin resonance (CESR) in anisotropic metals. This paper will give the details of that work. The theory of CESR in isotropic metals has been known for a long time.[2] It is also known that, with the exception of the alkali metals, most metals are highly anisotropic and have complex Fermi surfaces. Furthermore, in these metals the electron g value, $g(\vec{k})$,[3] is expected to be momentum dependent as a result of the spin-orbit interaction.[4] If one neglects the spin-orbit interaction then the CESR frequency is not affected by Coulomb interactions. This fact is well known[4] and the reason for this is that the CESR frequency is the rate of precession of the total magnetization $\vec{M}$ of the interacting electron gas, which is a constant of the motion (commutes with the Hamiltonian including Coulomb interactions) in the absence of spin-orbit coupling. We show that when one allows for the momentum dependence of the electron gyromagnetic ratio (which is a result of the spin-orbit interaction) and also takes account of the electron-electron exchange interaction, the situation is changed. In this case we show that these two effects combine and lead to a collective mode which we have called "collective CESR." The position of this collective mode is not that of "motionally narrowed CESR" which one would observe at high temperatures where many-body effects are unimportant. Furthermore, at low temperatures where one expects to be able to observe collective CESR, the breakdown of motional narrowing is compensated for by exchange narrowing and we find a narrow Lorentzian line whose width would vanish in the absence of scattering. On the other hand, at high temperatures, we obtain as expected a motionally narrowed[5] Lorentzian line whose position is at the average of the g distribution ($\langle g\rangle \mu_B H_0$). In order to observe the shift from $\langle g\rangle \mu_B H_0$ one must have $B\langle g\rangle \mu_B H_0 \tau/(1+B) > 1$ (B is a dimensionless interaction parameter which is similar to B_0 in the usual Landau Fermi-liquid theory; τ is a non-spin-flip scattering time). The collective CESR then provides information about the exchange parameter B in metals for which spin waves[6] are difficult to observe. Lubzens, Shanabarger, and Schultz[7] used the theory described above, and which will be fully developed in this paper, to analyze their experiments on the transmission CESR of Al, Cu, and Ag. In Al both the CESR linewidth and position were observed to be frequency and temperature dependent. The comparison of their data with the equations given by the authors[1] for the CESR linewidth and position enabled Lubzens *et al.* to make preliminary estimates of the parameter B as well as the rms spread in g values over the Fermi surface.

The following is the plan of this paper. In Sec. II we define our theoretical model and introduce the Landau-Silin[8] transport equation for the spin distribution function. In Sec. III we solve the transport equation and calculate the transverse wave-number- and frequency-dependent rf susceptibility. Section IV uses the results obtained in Sec. III to calculate the CESR linewidth and resonance position in one interesting regime. The behavior of CESR, which mathematically corresponds to a pole in the uniform ($\vec{q}=0$) transverse rf susceptibility $\chi_+(\omega)$, is fully discussed in this section. In addition we also discuss the continuum of single-particle excitations [these correspond to branch cuts in the susceptibility $\chi_+(\omega)$] which arise from combined quasiparticle spin-flip transitions and cyclotron-resonance absorptions. In the Appendix we demonstrate, for closed quasiparticle orbits, that the collective CESR always exists for $B\neq 0$ (under conditions stated in the Appendix) and also that the collective CESR always lies outside of the continuum of single-particle excitations. The possible relevance of this last fact to the failure to observe CESR in some metals is discussed in the latter part of Sec. IV. We also briefly discuss the spin-diffusion constant in several interesting limits.

II. THEORETICAL MODEL

We consider a model metal in which the conduction electrons have a momentum-dependent g value $g(\vec{k})$ as a result of the spin-orbit interaction.[4] In general one should consider a g tensor,[9] but for simplicity we consider a scalar g distribution of the form

$$g(\vec{k})=\langle g\rangle+\Delta g(\vec{k}) , \qquad (2.1)$$

where $\Delta g(\vec{k})$ is the deviation of $g(\vec{k})$ from its Fermi-surface average. In order to take into account electron correlations we follow Landau[8] and view the conduction electrons as forming a gas of quasiparticles described by the following single-particle Hamiltonian (which is to be understood as a matrix in spin space):

$$\mathcal{E}_{\mathrm{QP}}=\mathcal{E}_{\vec{k}}-\tfrac{1}{2}\gamma(\vec{k})H_0\sigma_z-\tfrac{1}{2}\gamma(\vec{k})\vec{h}\cdot\vec{\sigma}+\delta\mathcal{E}(\vec{r},\vec{\sigma},t) , \qquad (2.2)$$

where $\mathcal{E}_{\vec{k}}$ is the conduction-band Bloch energy which for most metals is highly anisotropic. The Zeeman terms describe the coupling of the quasiparticle spin to the external magnetic fields, where H_0 is a dc field oriented along the $\hat{z}$ axis and $\vec{h}(\vec{r},t)$ is an inhomogeneous rf field. The quantity $\gamma(\vec{k})=g(\vec{k})\mu_B$ (μ_B is the Bohr magneton) is the gyromagnetic ratio. The term $\delta\mathcal{E}(\vec{r},\vec{\sigma},t)$ is the interaction energy of a quasiparticle with the other excited particles of the system. In order to avoid serious complications when $\mathcal{E}_{\vec{k}}$ is anisotropic, we take $\delta\mathcal{E}$ to have the form

$$\delta\mathcal{E}=a\,\mathrm{tr}\sum_{\vec{k}}\delta n(\vec{k},\vec{r},\vec{\sigma},t)+b\vec{\sigma}\cdot\mathrm{tr}'\times\sum_{\vec{k}}\vec{\sigma}'\,\delta n(\vec{k},\vec{r},\vec{\sigma}',t), \qquad (2.3)$$

where a and b are constants.[10] In (2.3) $\delta n(\vec{k},\vec{r},\vec{\sigma},t)=n(\vec{k},\vec{r},\vec{\sigma},t)-n^{(0)}(\mathcal{E}_{\vec{k}})$ is the deviation of the quasiparticle density matrix from its equilibrium value (a Fermi function) and describes the degree of excitation of the system. The first term in (2.3) describes the spin-independent part of the quasiparticle interaction and will not enter into our subsequent calculation of the spin susceptibility.[11] The second term in (2.3) is of the form $b\vec{\sigma}\cdot\vec{S}(\vec{r},t)$, where $\vec{S}(\vec{r},t)\equiv\mathrm{tr}\sum_k\vec{\sigma}\,\delta n(\vec{k},\vec{r},\vec{\sigma},t)$ is the local spin density of the quasiparticle gas. It then follows that the parameter b, which has the dimensions of energy, may be interpreted as an effective exchange energy coupling a quasiparticle spin to the local spin density produced by the other quasiparticles in the system.

Our object is to calculate the transverse magnetization produced by applying an rf magnetic field perpendicular to the applied dc field H_0. The transverse magnetization is then given by

$$M_+(\vec{r},t)=\tfrac{1}{2}\mathrm{tr}\int\frac{d^3k}{(2\pi)^3}\gamma(\vec{k})\sigma_+\delta n(\vec{k},\vec{r},\vec{\sigma},t) , \qquad (2.4)$$

where $\sigma_+=\sigma_x+i\sigma_y$ (σ_α for $\alpha=x,y,z$ are the Pauli spin matrices). In order to proceed further we need to determine $\delta n(\vec{k},\vec{r},\vec{\sigma},t)$. The time development of the quasiparticle density matrix is given by [see Eq. (14) of Silin's paper, Ref. 8]

$$\frac{\partial n}{\partial t}+\{n,\mathcal{E}_{\mathrm{QP}}\}+i[\mathcal{E}_{\mathrm{QP}},n]=\left(\frac{\partial n}{\partial t}\right)_{\mathrm{coll}} , \qquad (2.5)$$

where $\{n,\mathcal{E}_{\mathrm{QP}}\}$ is the symmetrized Poisson bracket and is given by

$$\{n,\mathcal{E}_{\mathrm{QP}}\}\equiv\tfrac{1}{2}\left(\frac{\partial n}{\partial\vec{r}}\cdot\frac{\partial\mathcal{E}_{\mathrm{QP}}}{\partial\vec{k}}+\frac{\partial\mathcal{E}_{\mathrm{QP}}}{\partial\vec{k}}\cdot\frac{\partial n}{\partial\vec{r}}\right)-\tfrac{1}{2}\left(\frac{\partial n}{\partial\vec{k}}\cdot\frac{\partial\mathcal{E}_{\mathrm{QP}}}{\partial\vec{r}}+\frac{\partial\mathcal{E}_{\mathrm{QP}}}{\partial\vec{r}}\cdot\frac{\partial n}{\partial\vec{k}}\right), \qquad (2.6)$$

and $[\mathcal{E}_{\mathrm{QP}},n]$ is the quantum-mechanical commutator. The effects of quasiparticle collisions with other quasiparticles, impurities, and phonons are contained in the collision term $(\partial n/\partial t)_{\mathrm{coll}}$. The derivation of the linearized kinetic equation follows readily from the above equations, but is lengthy and has been given elsewhere.[12] For this reason we will move quickly to the result and refer the reader interested in the details to the above references.

In order to take into account collisions, we choose the following phenomenological linearized collision integral:

$$\left(\frac{\partial n}{\partial t}\right)_{\mathrm{coll}}=-\frac{\delta\tilde{n}}{\tau}(\vec{k},\vec{r},t)+\frac{1}{\nu}\int\frac{d^3k'}{(2\pi)^3}\delta(\mathcal{E}_{\vec{k}}-\mathcal{E}_{\vec{k}'})\times\left(\frac{2}{\tau_0}\delta\tilde{n}(\vec{k}',\vec{r},t)+\frac{1}{\tau_s}\mathrm{tr}\,\delta\tilde{n}(\vec{k}',\vec{r},t)\right), \qquad (2.7)$$

where $1/\tau = 1/\tau_0 + 1/\tau_s$; τ_0 is a non-spin-flip scattering time, τ_s is the spin-flip scattering time, and ν is the density of states per unit volume for $\mathcal{E} = \mathcal{E}_F$. It is easy to demonstrate that the collision integral in (2.7) has the properties that (i) the total number of quasiparticles in the system is conserved by the collisions, and (ii) the magnetization relaxes with the relaxation time τ_s. In (2.7) we have introduced the deviation from local equilibrium

$$\delta\tilde{n}(\vec{k},\vec{r},\vec{\sigma},t) = n(\vec{k},\vec{r},\vec{\sigma},t) - n^{(0)}(\mathcal{E}_{QP}). \tag{2.8}$$

It is this object which appears in (2.7) because it is the full (including interactions) local quasiparticle energy which is conserved in a quasiparticle collision. If we compare (2.8) with the definition of $\delta n(\vec{k},\vec{r},\vec{\sigma},t)$ (see 2.3) then we can write

$$\delta n = \delta\tilde{n} - \delta(\mathcal{E}_k - \mathcal{E}_F)[\delta\mathcal{E} - \tfrac{1}{2}\gamma(\vec{k})H_0\sigma_z - \tfrac{1}{2}\gamma(\vec{k})\vec{h}\cdot\vec{\sigma}], \tag{2.9}$$

where we have used a low-temperature property of the Fermi function and written $\partial f_0/\partial\mathcal{E}_k = -\delta(\mathcal{E}_k - \mathcal{E}_F)$.

If we write

$$\delta\tilde{\psi}_+ = \tfrac{1}{2}\,\mathrm{tr}\sigma_+\delta\tilde{n} \equiv \psi(\vec{k},\vec{r},t)\,\delta(\mathcal{E}_F - \mathcal{E}_{\vec{k}}) \tag{2.10}$$

then the "spin density" $\psi(\vec{k},\vec{r},t)$ satisfies the Landau-Silin equation

$$\frac{d\psi}{dt} + i\gamma^* H_0\psi = -\frac{\gamma^*}{2}\frac{\partial h_+}{\partial t} + \frac{2}{\nu}\int\frac{d^3k'}{(2\pi)^3}\,\delta(\mathcal{E}_{\vec{k}'} - \mathcal{E}_F)\times\left(\frac{1}{\tau_0} + \frac{B}{1+B}\frac{\partial}{\partial t}\right)\psi(\vec{k}') - \frac{\psi(\vec{k})}{\tau}, \tag{2.11}$$

where the derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_{\vec{k}}\cdot\frac{\partial}{\partial\vec{r}} + \frac{e}{c}(\vec{v}_{\vec{k}}\times\vec{H}_0)\cdot\frac{\partial}{\partial\vec{k}} \tag{2.12}$$

is along the quasiparticle trajectory in $\vec{k}$ space. We have defined the dimensionless interaction parameter $B = \nu b$ (B is similar to B_0 in the usual Landau theory of isotropic systems). In the alkali metals Na and K the Landau parameters B_0 can be obtained from spin-wave experiments[6] and have the values $B_0 = -0.21$ (Na) and $B_0 = -0.28$ (K) (the negative values of these parameters indicate that the quasiparticle exchange interaction in these metals is "ferromagnetic" in character leading to an enhancement of the static Pauli susceptibility). In (2.11) we have introduced a renormalized gyromagnetic ratio

$$\gamma^*(\vec{k}) = \mu_B\left(\frac{\langle g\rangle}{1+B} + \Delta g(\vec{k})\right). \tag{2.13}$$

The renormalization of the gyromagnetic ratio in (2.13) tells us that the energy required to reverse the spin of a *single* quasiparticle is altered by the presence of other quasiparticles. In general the term involving $\Delta g(\vec{k})$ in (2.13) will also be renormalized by the exchange interaction, but because we have taken the exchange energy b in (2.3) to be a constant this renormalization is absent in our model.

III. SOLUTION OF THE LANDAU-SILIN EQUATION

If we integrate over t we can convert Eq. (2.11) into the following integral equation:

$$\psi(\vec{k},\vec{r},t) = -\tfrac{1}{2}\int_{-\infty}^{t}dt'\exp\left[-i\left(\omega_L^* - \frac{i}{\tau}\right)(t-t') - i\int_{t'}^{t}dt''\,\Delta\omega_L(\vec{k}(t''))\right]\gamma^*(\vec{k}(t'))\frac{\partial h_+(\vec{r},t')}{\partial t'}$$
$$+\frac{2}{\nu}\int\frac{d^3k'}{(2\pi)^3}\,\delta(\mathcal{E}_{k'} - \mathcal{E}_F)\int_{-\infty}^{t}dt'\exp\left[-i\left(\omega_L^* - \frac{i}{\tau}\right)(t-t') - i\int_{t'}^{t}dt''\,\Delta\omega_L(\vec{k}(t''))\right]\left(\frac{1}{\tau_0} + \frac{B}{1+B}\frac{\partial}{\partial t'}\right)\psi(\vec{r},\vec{k}',t'), \tag{3.1}$$

where we have found it convenient to introduce the following quantities:

$$\omega_L^* = \frac{\langle g\rangle}{1+B}\mu_B H_0, \tag{3.2a}$$

and

$$\Delta\omega_L(\vec{k}) = \Delta g(\vec{k})\mu_B H_0. \tag{3.2b}$$

In the following it will be useful to have the space-time Fourier transforms

$$\psi(\vec{r},\vec{k},t)=\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,e^{i(\vec{q}\cdot\vec{r}-\omega t)}\psi(\vec{q},\vec{k},\omega) \tag{3.3}$$

and the inverse

$$\psi(\vec{q},\vec{k},\omega)=\int d^3r\int_{-\infty}^{\infty}dt\,e^{-i(\vec{q}\cdot\vec{r}-\omega t)}\psi(\vec{r},\vec{k},t)\,,$$

and similar transforms for the rf field $\vec{h}(\vec{r},t)$. If we then take the Fourier transform of (3.1) and perform some simple manipulations we find

$$\begin{aligned}\psi(\vec{q},\vec{k},\omega)={}&\frac{i\omega}{2}\int d^3r\,e^{-i\vec{q}\cdot\vec{r}}\int\frac{d^3q'}{(2\pi)^3}\int_0^{\infty}dt'\Big[\exp[i\vec{q}\,'\cdot\vec{r}(-t')+i(\omega-\omega_L^*+i/\tau)t']\\&\times\exp\Big(-i\int_0^{t'}dt''\,\Delta\omega_L(\vec{k}(-t''))\Big)\gamma^*(\vec{k}(-t'))\,h_+(\vec{q}\,',\omega)\Big]\\&+\frac{2}{\nu}\int\frac{d^3k'}{(2\pi)^3}\,\delta(\mathcal{E}_{k'}-\mathcal{E}_F)\int d^3r\,e^{-i\vec{q}\cdot\vec{r}}\int\frac{d^3q'}{(2\pi)^3}\int_0^{\infty}dt'\Big[\exp[i\vec{q}\,'\cdot\vec{r}(-t')+i(\omega-\omega_L^*+i/\tau)t']\\&\times\exp\Big(-i\int_0^{t'}dt''\,\Delta\omega_L(\vec{k}(-t''))\Big)\Big(\frac{1}{\tau_0}-i\omega\frac{B}{1+B}\Big)\psi(\vec{q}\,',\vec{k}',\omega)\Big].\end{aligned} \tag{3.4}$$

In order to solve (3.4) it will be convenient to choose a coordinate system with the dc field H_0 along the $\hat{z}$ axis and the wave vector $\vec{q}$ in the x-z plane making an angle Δ with H_0. If we integrate the equation of motion $\dot{\vec{k}}=(eH_0/c)(\dot{\vec{r}}\times\hat{z})$ back along the quasiparticle trajectory from $\vec{k}$ we find

$$x(-t)=x+(c/eH_0)[k_y-k_y(-t)]\quad\text{and}\quad z(-t)=z-v_zt\ ,$$

so that we can write

$$\vec{q}\cdot\vec{r}(-t)=\vec{q}\cdot\vec{r}+(q_\perp c/eH_0)[k_y-k_y(-t)]-q_\parallel v_zt\ , \tag{3.5}$$

where we have introduced $q_\perp=q\sin\Delta$ and $q_\parallel=q\cos\Delta$, which are the components of the wave vector perpendicular and parallel, respectively, to the dc field H_0. Equation (3.5) is general and is valid for a Fermi surface of arbitrary geometry. If we substitute (3.5) into (3.4) and do the integrations over $\vec{r}$ [which gives $\delta(\vec{q}-\vec{q}\,')$] and $\vec{q}\,'$ we find

$$\begin{aligned}\psi(\vec{q},\vec{k},\omega)={}&e^{iq_\perp ck_y/eH_0}\int_0^{\infty}dt'\Big\{\exp\Big(\frac{-iq_\perp ck_y(-t')}{eH_0}+i(\omega-q_\parallel v_z-\omega_L^*+i/\tau)t'\Big)\\&\times\exp\Big(-i\int_0^{t'}dt''\,\Delta\omega_L(\vec{k}(-t''))\Big)\Big[\frac{i\omega}{2}h_+(\vec{q},\omega)\gamma^*(\vec{k}(-t'))\\&+\Big(\frac{1}{\tau_0}-\frac{i\omega B}{1+B}\Big)\frac{2}{\nu}\int\frac{d^3k'}{(2\pi)^3}\,\delta(\mathcal{E}_{\vec{k}'}-\mathcal{E}_F)\psi(\vec{q},\vec{k}',\omega)\Big]\Big\}.\end{aligned} \tag{3.6}$$

For any function $F(\vec{k})$ defined on the Fermi surface it will be convenient to define its Fermi-surface average

$$\langle F\rangle=\frac{2}{\nu}\int\frac{d^3k}{(2\pi)^3}\,\delta(\mathcal{E}_{\vec{k}}-\mathcal{E}_F)F(\vec{k})\ , \tag{3.6$'$}$$

so that we can write (3.6) in the compact form

$$\psi(\vec{q},\vec{k},\omega)=\frac{i\omega h_+}{2}G_1(\vec{q},\vec{k},\omega)+\Big(\frac{1}{\tau_0}-\frac{i\omega B}{1+B}\Big)\langle\psi\rangle\,G_0(\vec{q},\vec{k},\omega)\ , \tag{3.7}$$

where we have defined the functions

$$G_n(\vec{q},\vec{k},\omega)=\exp\Big(\frac{iq_\perp ck_y}{eH_0}\Big)\int_0^{\infty}dt'\Big[\exp\Big(\frac{-iq_\perp ck_y(-t')}{eH_0}+i(\omega-q_\parallel v_z-\omega_L^*+i/\tau)t'-i\int_0^{t'}dt''\,\Delta\omega_L(\vec{k}(-t''))\Big)\gamma^{*n}(\vec{k}(-t'))\Big]\ , \tag{3.8}$$

for $n=0,1$. The solution of (3.7) is then trivial and we find

$$\psi(\vec{q},\vec{k},\omega)=\frac{i\omega h_+}{2}\left[G_1(\vec{q},\vec{k},\omega)+\left(\frac{1}{\tau_0}-\frac{i\omega B}{1+B}\right)\frac{\langle G_1\rangle G_0(\vec{q},\vec{k},\omega)}{1-[1/\tau_0-i\omega B/(1+B)]\langle G_0\rangle}\right]. \tag{3.9}$$

If we recall Eqs. (2.4), (2.9), and (2.10) we find that the transverse magnetization can be written as follows:

$$M_+(\vec{q},\omega)=\tfrac{1}{2}\nu\langle\gamma^*\psi\rangle+\tfrac{1}{4}\nu h_+\langle\gamma^*\gamma\rangle\ , \tag{3.10}$$

or by making use of (3.9) we have for the transverse wave vector and frequency-dependent susceptibility

$$\chi_+(\vec{q},\omega)=\frac{i\omega\nu}{4}\left[\langle\gamma^*G_1\rangle+\left(\frac{1}{\tau_0}-\frac{i\omega B}{1+B}\right)\frac{\langle\gamma^*G_0\rangle\langle G_1\rangle}{1-[1/\tau_0-i\omega B/(1+B)]\langle G_0\rangle}\right]+\frac{\nu}{4}\langle\gamma^*\gamma\rangle\ . \tag{3.11}$$

The last term in (3.11) is the static Pauli susceptibility of the quasiparticles and in our model

$$\chi_{\text{Pauli}}=\frac{\nu}{4}\langle\gamma^*\gamma\rangle=\frac{\nu\mu_B^2}{4}\left(\frac{(\langle g\rangle)^2}{1+B}+\langle(\Delta g)^2\rangle\right)\ ,$$

so that if there was no g anisotropy $[\Delta g(\vec{k})=0]$ we would have the result $\chi_{\text{Pauli}}=\chi_{\text{Pauli}}^{(0)}/(1+B)$ (here $\chi_{\text{Pauli}}^{(0)}$ is the susceptibility of the noninteracting electron system), which is the well-known result for the effects of Coulomb interactions on the spin susceptibility of an interacting electron gas.[13]

IV. CALCULATION OF CESR POSITION AND LINEWIDTH

The CESR position and linewidth may be obtained by considering the uniform ($q=0$) transverse susceptibility $\chi_+(\omega)$.[14] The susceptibility $\chi_+(\omega)$ has a pole at the complex frequency ω_0 such that

$$\omega_0\equiv\omega_{\text{CESR}}-i/T_2^*\ , \tag{4.1}$$

where $\omega_{\text{CESR}}\equiv g_{\text{obs}}\mu_B H_0$ and $1/T_2^*$ give the position and linewidth, respectively, of the resonance. From (3.11) we see that $\chi_+(\omega)$ has poles whenever

$$1-\left(\frac{1}{\tau_0}-\frac{i\omega_0 B}{1+B}\right)\langle G_0(\omega_0)\rangle=0\ . \tag{4.2}$$

We can solve (4.2) for the complex frequency ω_0 in one interesting regime, namely, for

$$\frac{|\Delta g_0|\mu_B H_0}{|\omega-\langle g\rangle\mu_B H_0/(1+B)+i/\tau|}<<1$$

and (4.3)

$$\frac{|\Delta g_m|\mu_B H_0}{m\omega_c}<<1,\quad m\neq 0$$

where the Δg_m's come from expanding $\Delta g(\vec{k})$ in a Fourier series in a phase variable ϕ for fixed k_z,

$$\Delta g(\vec{k}(-t))=\Delta g_0(k_z)+\sum_{m=-\infty}^{\infty}{}'\,\Delta g_m(k_z)e^{im[\phi-\omega_c(k_z)t]}, \tag{4.4}$$

where we have chosen as coordinates on the Fermi surface k_z, the momentum along the field H_0, and a phase variable ϕ which is measured from the k_x-k_z plane and which locates a quasiparticle on its orbit. We note that the Δg_m's are functions of k_z and therefore a general Fermi surface can be expected to vary with the direction of the dc field H_0. Also because of the anisotropy of the Fermi surface quasiparticles on different orbits will have different cyclotron frequencies so that $\omega_c(k_z)$ is also a function of k_z.[15] The prime on the summation in (4.4) indicates that we are excluding the $m=0$ term. If we expand the exponential in $\langle G_0(\omega)\rangle$ to lowest nonvanishing order in the small quantities appearing in (4.3) and solve (4.2) for the complex frequency ω_0, we find

$$\omega_0=\langle g\rangle\mu_B H_0-\frac{i(1+B)}{\tau_s}+\frac{(1+B)\langle(\Delta g_0)^2\rangle(\mu_B H_0)^2\tau^*}{\langle g\rangle\mu_B H_0 B\tau^*/(1+B)+i}$$

$$+2(1+B)\left(\frac{\langle g\rangle\mu_B H_0 B\tau^*}{1+B}+i\right)\sum_{m=1}^{\infty}\left\langle\frac{(\Delta g_m)^2}{[\langle g\rangle\mu_B H_0 B\tau^*/(1+B)+i]^2-(m\omega_c\tau^*)^2}\right\rangle(\mu_B H_0)^2\tau^*\ , \tag{4.5}$$

where we have used the expansion (4.4) and have defined

$$\frac{1}{\tau^*}=\frac{1}{\tau_0}-\frac{B}{\tau_s}\ . \tag{4.6}$$

Taking the real and imaginary parts of (4.5) and recalling (4.1) we find

$$g_{\rm obs}=\langle g\rangle\left(1+\frac{B\langle(\Delta g_0)^2\rangle(\mu_B H_0)^2\tau^{*2}}{1+X^2}+2B\tau^{*2}\sum_{m=1}^{\infty}\left\langle\frac{|\Delta g_m|^2(\mu_B H_0)^2[1+X^2-(m\omega_c\tau^*)^2]}{[1-X^2+(m\omega_c\tau^*)^2]^2+4X^2}\right\rangle\right) \tag{4.7}$$

and

$$\frac{1}{T_2^*}=\frac{(1+B)}{\tau_s}+\frac{(1+B)\langle(\Delta g_0)^2\rangle(\mu_B H_0)^2\tau^*}{1+X^2}+2(1+B)\tau^*\sum_{m=1}^{\infty}\left\langle\frac{|\Delta g_m|^2(\mu_B H_0)^2[1+X^2+(m\omega_c\tau^*)^2]}{[1-X^2+(m\omega_c\tau^*)^2]^2+4X^2}\right\rangle, \tag{4.8}$$

where we have defined the quantity $X=[B/(1+B)]\langle g\rangle\mu_B H_0\tau^*$. Note that for $\omega_c\tau^*\ll 1$ we can sum the series in (4.7) and (4.8) and find

$$g_{\rm obs}=\langle g\rangle\left(1+\frac{B\langle(\Delta g)^2\rangle(\mu_B H_0)^2\tau^{*2}}{1+X^2}\right) \tag{4.9}$$

and

$$\frac{1}{T_2^*}=\frac{1+B}{\tau_s}+\frac{(1+B)\langle(\Delta g)^2\rangle(\mu_B H_0)^2\tau^*}{1+X^2}, \tag{4.10}$$

provided that an integer m_0 exists such that $\Delta g_{m_0}\approx 0$ for $m\geq m_0$ and $m_0\omega_c\tau^*\ll 1$. In the opposite limit $\omega_c\tau^*\gg 1$, which is more interesting in practice, the terms involving Δg_m are negligible compared to those involving Δg_0. In the strong scattering limit $|X|\ll 1$ we see from the above equations that many-body effects are not important. The linewidth is characteristic of a motionally narrowed Lorentzian line [the factors $(1+B)$ can be absorbed into τ^* and τ_s, which already contain many-body effects]. The center of the resonance which we have called motionally narrowed CESR is at $g_{\rm obs}=\langle g\rangle$, the average of the g distribution. In the weak scattering limit $|X|\gg 1$ many-body effects are dominant and the linewidth is characteristic of an exchange-narrowed Lorentzian line whose width would vanish as $\tau^*\to\infty$ if there were no spin-lattice relaxation ($\tau_s\to\infty$). The center of this resonance which we have called collective CESR is shifted from the position of motionally narrowed CESR. This shift can provide information about the many-body parameter B in metals for which spin waves are difficult to observe.[7] We also note from (4.10) that in the regime $|X|\gg 1$, if we neglect the variation of τ_s with temperature then the linewidth should pass through a maximum with increasing temperature. When $|X|\gg 1$ we conclude that many-body effects qualitatively change the spectrum arising from the free-electron picture. For noninteracting electrons, motional narrowing ceases when $|\Delta g|\mu_B H_0\tau\geq 1$, and we should expect a spectrum reflecting the full g-value distribution [Fig. 1(a)]. In fact, for $|X|\gg 1$ the spectrum consists of the free-electron continuum shifted in frequency by $-[B/(1+B)]\langle g\rangle\mu_B H_0$, and the collective CESR [see Fig. 1(b)]. The narrow line must be understood as a collective mode; its width vanishes in the no scattering ($\tau\to\infty$) limit. In the Appendix we shall prove that the collective CESR always exist for $B\neq 0$.

The functions $\langle G_n(\omega)\rangle$ and therefore the rf susceptibility have a series of branch cuts corresponding to power absorption by simple quasiparticle spin-flip transitions. To see this more clearly consider (we put $1/\tau\to 0$ for the purpose of studying elementary excitations) the function

$$\langle G_0(\omega)\rangle=\left\langle\int_0^\infty dt'\exp\left[i\left(\omega-\langle g\rangle\frac{\mu_B H_0}{1+B}-\Delta g_0\mu_B H_0\right)t'\right]P(k_z,\phi,t')\right\rangle, \tag{4.11}$$

where we have used (3.8) (for $\vec{q}=0$) and (4.4) and have defined the function

$$P(k_z,\phi,t)=\exp\left({\sum_{m=-\infty}^{\infty}}'\frac{\Delta g_m\mu_B H_0}{m\omega_c(k_z)}e^{im\phi}(e^{-im\omega_c(k_z)t}-1)\right). \tag{4.12}$$

We note that $P(k_z,\phi,t)$ is a periodic function of t with period $2\pi/\omega_c$ and may be expanded in a Fourier series

$$P(k_z,\phi,t)=\sum_{n=-\infty}^{\infty}\bar{P}_n(k_z,\phi)e^{-in\omega_c(k_z)t}\ . \tag{4.13}$$

If we substitute (4.13) into (4.11) and do the integral over t we find

$$\langle G_0(\omega)\rangle=i\left\langle\sum_{m=-\infty}^{\infty}\frac{\bar{P}_m(k_z,\phi)}{\omega-\langle g\rangle\mu_B H_0/(1+B)-\langle\Delta g\rangle_{k_z}\mu_B H_0-m\omega_c(k_z)}\right\rangle, \tag{4.14}$$

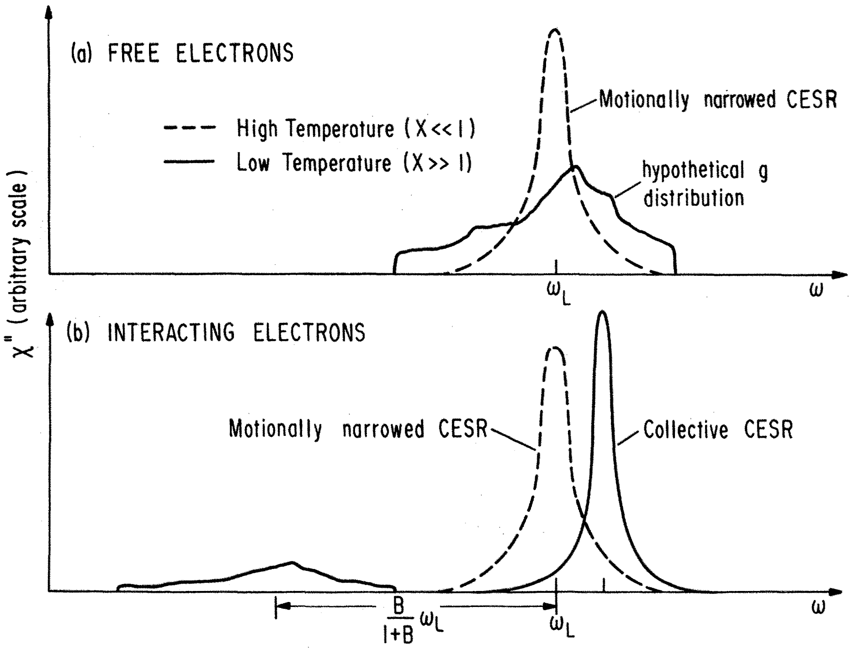

FIG. 1. Schematic absorption spectra for (a) free electrons and (b) interacting electrons, drawn to a common scale. $\omega_L = \langle g\rangle \mu_B H_0$. For free electrons X should be understood as $\omega\tau$. For interacting electrons $X = B\omega\tau/(1+B)$; motional narrowing requires the stronger condition $\omega\tau \ll 1$.

where $\langle \Delta g\rangle_{k_z} \equiv \Delta g_0(k_z)$ is the average of Δg around an orbit at fixed k_z. We see from (4.14) that $\langle\langle G_0(\omega)\rangle\rangle$ [and therefore $\chi_+(\omega)$] has branch cuts whenever

$$\omega = \langle g\rangle \mu_B H_0/(1+B) + \langle \Delta g\rangle_{k_z} \mu_B H_0 + m\omega_c(k_z), \tag{4.15}$$

so that the shape of the single-particle absorption spectrum reflects the detailed distributions of g values and cyclotron frequencies. However, this interesting structure, intrinsic to the material, is blurred by collisions when $\Delta\omega \equiv \omega - \langle g\rangle \mu_B H_0 B/(1+B) \sim 1/\tau$. Physically the excitations described by (4.15) are combined single-quasiparticle spin flips and cyclotron-resonance absorptions and Eq. (4.15) is nothing more than a statement of conservation of energy for these processes.

When $|\Delta g| \mu_B H_0 \tau/|X| = (|\Delta g|/g)[(1+B)/|B|] \ll 1$, almost all of the oscillator strength (area under the absorption line) is in the collective CESR (see Freedman, Ref. 12); this is the circumstance for which (4.7) and (4.8) are valid. If we imagine reducing $|B|$, we should find that the collective mode approaches the edge of the continuum, which shifts to meet it; at the same time, the oscillator strength of the collective CESR would decrease, and the integrated strength of the continuum would increase. In the limit $|B| \to 0$, the collective CESR can no longer be excited; this limit cannot be studied with (4.7) and (4.8) because these equations are only valid if $|B|/(1+B) \gg |\Delta g|/g$. That the collective CESR always lies outside the continuum may have an important experimental consequence. Let

$$\min g_0(k_z) = \langle g\rangle - \Delta g_- \tag{4.16}$$

and

$$\max g_0(k_z) = \langle g\rangle + \Delta g_+ , \tag{4.17}$$

where $g_0(k_z)$ is the average of $g(\vec{k})$ around an orbit at fixed k_z. Since the collective CESR always lies outside the continuum it occurs at frequency ω such that

$$\omega/\mu_B H_0 > \langle g\rangle/(1+B) + \Delta g_+ \quad \text{for } B > 0 \tag{4.18}$$

and

$$\omega/\mu_B H_0 < \langle g\rangle/(1+B) - \Delta g_- \quad \text{for } B < 0 . \tag{4.19}$$

This is shifted from the position of the "motionally narrowed CESR," observed at high temperature, by

$$|\omega/\mu_B H_0 - \langle g\rangle| > \Delta g_\pm - B\langle g\rangle/(1+B) \sim \Delta g_\pm \tag{4.20}$$

for $|B|/(1+B) < \Delta g_\pm/\langle g\rangle$. Therefore when the scattering is sufficiently weak, the collective CESR may be shifted substantially from the position of high-temperature CESR; this large shift may be responsible for the failure to observe CESR in some metals.

From (3.11) we see that $\chi_+(\vec{q}, \omega)$ has a branch of singularities in the ω-q plane whenever the function

$$S(\vec{q}, \omega) = 1 - \left(\frac{1}{\tau_0} - \frac{iB\omega}{1+B}\right) \langle\langle G_0(q, \omega)\rangle\rangle$$

vanishes. In the regime where Eqs. (4.3) are valid, the above condition becomes (to order q^2)

$$\omega = \langle g \rangle \mu_B H_0 - \frac{i(1+B)}{\tau_s} + \frac{(1+B)\langle(\Delta g_0)^2\rangle(\mu_B H_0)^2}{\omega - \langle g \rangle \mu_B H_0/(1+B) + i/\tau} + iq^2 D , \qquad (4.21)$$

where D is the complex spin-diffusion constant. We have calculated (for details see Freedman, Ref. 12) D for a metal having a spherical Fermi surface and for simplicity have also taken $g(\vec{k})$ to have azimuthal symmetry about the dc field H_0 [this is equivalent to neglecting the terms Δg_m for $m \neq 0$ in (4.4)]. For free electrons with an isotropic g value [i.e., $B = 0$ and $\Delta g(\vec{k}) = 0$] we obtain the well-known result

$$D = \tfrac{1}{3} v_F^2 \tau^0 \left(\frac{\sin^2\Delta}{1 + (\omega_c \tau_0)^2} + \cos^2\Delta \right) \qquad (4.22)$$

of a real anisotropic spin-diffusion constant. The asymmetry with respect to the dc field is telling us that in a charged Fermi system it is more difficult for the magnetization to diffuse across the field ($\Delta = \pi/2$) than along the field ($\Delta = 0$). In a neutral Fermi liquid like ^{3}He the spin-diffusion constant is isotropic and one obtains the correct result by putting $\omega_c = 0$ in (4.22). For a metal with an isotropic g value [i.e., $\Delta g(\vec{k}) = 0$], but with exchange interactions, we find

$$D = \frac{i}{3} (1+B) v_F^2 \tau^* \left(\frac{\sin^2\Delta (X+i)}{(X+i)^2 - (\omega_c \tau^*)^2} + \frac{\cos^2\Delta}{X+i} \right), \qquad (4.23)$$

where as defined earlier $X = [B/(1+B)] \langle g \rangle \mu_B H_0 \tau^*$. This agrees with Eq. (4) of Platzmann-Wolff (Ref. 6) if in their equation we set $B_1 = 0$. Note that in the limit where many-body effects are important, namely, $|X| \gg 1$, then the term in large parentheses in (4.23) becomes nearly real. In this limit $\mathrm{Im}D \gg \mathrm{Re}D$ and from (4.21) we see that this gives rise to a traveling wave instability in $\chi_+(q, \omega)$. This is the paramagnetic spin wave. There is yet another limit to consider, free electrons with g anisotropy [i.e., $B = 0$ and $\Delta g(\vec{k}) \neq 0$]. In this limit we recover essentially the results of Kaplan and Glasser,[16] who considered the effects on spin diffusion of a momentum-dependent g value and spin-lattice relaxation time. We find "spin-wave-like" terms in D (terms which make D complex), but these terms are of order $\Delta g_0 \mu_B H_0 \tau \ll 1$ and are in no way responsible for the spin-wave resonances observed in Na and K.[6]

V. CONCLUSIONS

We have shown that Coulomb interactions can lead to observable effects on CESR in anisotropic metals for which the electron g factor is momentum dependent as a result of the spin-orbit interaction. Our results can most easily be seen by considering Fig. 1. For free electrons [Fig. 1(a)] at low temperatures, such that scattering is too slow to motionally narrow the g distribution, one finds a spectrum reflecting the full spread in g values. On the other hand, at high temperatures (i.e., for $\omega\tau \ll 1$) one finds a motionally narrowed Lorentzian line centered at the average of the g distribution ($\langle g \rangle \mu_B H_0$). This picture is qualitatively changed for interacting electrons [Fig. 1(b)]. At low temperatures when motional narrowing ceases and when $|X| = |B| \omega\tau/(1+B) \gg 1$, one finds instead of the full g distribution a narrow Lorentzian line which is narrowed by the electron-electron exchange interaction and whose width would vanish in the absence of scattering ($\tau \to \infty$). The position of this narrow mode, which we call collective CESR, is shifted from the motionally narrowed CESR observed at high temperatures. As explained in Sec. IV of this paper, this shift can provide information about the electron-electron exchange interaction (the parameter B) in metals for which spin waves[6] have not yet been observed. We also see from Fig. 1(b) that the free-electron continuum is shifted in frequency by $-[B/(1+B)] \times \langle g \rangle \mu_B H_0$ from the average of the g distribution. In this paper we have discussed the behavior of CESR in general and in one interesting limit [see (4.3)] have given equations for the CESR linewidth and position.

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APPENDIX

In this appendix we prove, for the case where all the quasiparticle orbits in k space are closed, that the collective CESR always exist for $B \neq 0$ and moreover that it always lies outside of the continuum of single-quasiparticle spin-flip excitations. It is well known[17] that for all real metals with the exception of the alkalis that there exists open orbits in k space for certain orientations of the dc field H_0. The cyclotron mass $m_c(k_z)$ may become infinite for quasiparticles on these open orbits.[18] As we shall see our proof will require that m_c^+, the maximum cyclotron mass for quasiparticles on the Fermi surface, be finite. This clearly restricts the validity of the proof given here to the alkali metals and to other metals only when the dc field H_0 is along a direction for which all orbits in k space are closed.

From (3.11) we see that the transverse susceptibility $\chi_+(\omega)$ has poles whenever there exist frequencies ω such that

$$1 = -\frac{i\,\omega B}{1+B}\langle\langle G_0(\omega)\rangle\rangle\,, \tag{A1}$$

where we have let $\tau \to \infty$ in order to study collective behavior and

$$\langle\langle G_0(\omega)\rangle\rangle = \left\langle \int_0^\infty dt\, \exp\left[i\left(\omega - \frac{\langle g\rangle\mu_B H_0}{1+B}\right)t - i\int_0^t dt'\Delta g(k(-t'))\,\mu_B H_0\right]\right\rangle, \tag{A2}$$

where $\omega \to \omega + i0^+$. If we proceed as in Eqs. (4.11)–(4.13) we arrive at

$$\langle\langle G_0(\omega)\rangle\rangle = i\left\langle \sum_{m=-\infty}^{\infty} \frac{\overline{P}_m(k_z,\phi)}{\omega - \langle g\rangle\mu_B H_0/(1+B) - \Delta g_0(k_z)\mu_B H_0 - m\,\omega_c(k_z)}\right\rangle, \tag{A3}$$

which is Eq. (4.14) of this paper and which we repeat here for convenience. As discussed earlier the function $\langle\langle G_0(\omega)\rangle\rangle$ is an analytic function of real ω except for an infinite series of branch cuts for

$$\omega = \frac{\langle g\rangle\mu_B H_0}{1+B} + \Delta g_0(k_z)\mu_B H_0 + m\,\omega_c(k_z), \tag{A4}$$

where m is any integer. For $m \neq 0$ these cuts correspond to the simultaneous excitation of a quasiparticle spin flip and cyclotron resonance absorption for a quasiparticle on an orbit characterized by k_z. In general Eq. (A1) will have many solutions, each of which corresponds to a possible collective mode of the system. A discussion of all of these collective modes would require an appendix of prohibitive length. Therefore we will restrict our discussion to the collective CESR which is the mode which lies nearest in frequency to the branch points of the cut corresponding to $m=0$. Our proof will require that this branch cut be separated in frequency from the branch cuts corresponding to $m=\pm 1$. It is easy to see from (A4) that these cuts will be well separated in frequency provided that $m_e/m_c^+ \gg \Delta g/g$, where m_e is the rest mass of a free electron and m_c^+ is the maximum cyclotron mass for quasiparticles on the Fermi surface. For metals for which[7] $\Delta g/g \sim 10^{-2}$ the above condition will be well satisfied for $m_c^+ \leq 10\, m_e$.

If we substitute (A3) into (A1) we are led to study the equation

$$-\frac{1+B}{B\omega} = \left\langle \sum_{m=-\infty}^{\infty} \frac{\overline{P}_m(k_z,\phi)}{\langle g\rangle\mu_B H_0/(1+B) + \Delta g_0(k_z)\mu_B H_0 + m\,\omega_c(k_z) - \omega}\right\rangle. \tag{A5}$$

For purposes of discussion it is convenient to define the functions $F_1(\omega)$ and $F_2(\omega)$ where

$$F_1(\omega) = -(1+B)/B\omega \tag{A6a}$$

and

$$F_2(\omega) = \left\langle \sum_{m=-\infty}^{\infty} \frac{\overline{P}_m(k_z,\phi)}{\langle g\rangle\mu_B H_0/(1+B) + \Delta g_0(k_z)\mu_B H_0 + m\,\omega_c(k_z) - \omega}\right\rangle \tag{A6b}$$

are the left- and right-hand sides, respectively, of (A5). We will show below that for $B>0$ (corresponding to antiferromagnetic exchange) the collective CESR is at a frequency ω_0^+ such that $\omega_R(0) < \omega_0^+ < \omega_L(1)$, where we have introduced $\omega_R(n)$ and $\omega_L(n)$ to denote the right- and left-hand branch points, respectively, of the branch cut corresponding to the integer n. For the case of ferromagnetic exchange $-1<B<0$ (we note that for B negative it is necessary that $B>-1$ for Fermi-liquid theory to be valid) the collective CESR is at a frequency ω_0^- such that $\omega_R(-1) < \omega_0^- < \omega_L(0)$.

In the following discussion we will only consider the collective modes in the frequency range $\omega_R(0) < \omega < \omega_L(1)$; the case $\omega_R(-1) < \omega < \omega_L(0)$ can be similarly discussed. A few observations are in order. For $\omega_R(0) < \omega < \omega_L(1)$ we see from (A6a) that the function $F_1(\omega)$ is a continuous monotonically increasing function of ω which assumes finite negative (for $B > 0$) values at both end points of this interval. From (A6b) it is not difficult to see that $F_2(\omega)$ is also a continuous monotonically increasing function of ω for $\omega_R(0) < \omega < \omega_L(1)$. This observation follows from the fact that for all integers n

$$A_n(k_z) \equiv \int_0^{2\pi} \frac{d\phi}{2\pi} \overline{P}_n(k_z, \phi) \geq 0, \tag{A7}$$

so that each term in $F_2(\omega)$ [i.e., Eq. (A6b)] is itself a continuous monotonically increasing function of ω for $\omega_R(0) < \omega < \omega_L(1)$. A proof of (A7) can be given as follows. From (4.12) we can write

$$P(k_z, \phi, t) = e^{q(\phi - \omega_c t) - q(\phi)} \equiv Q(\phi - \omega_c t)/Q(\phi), \tag{A8}$$

where

$$q(\phi) = \sum_{m=-\infty}^{\infty}{}' \frac{\Delta g_m \mu_B H_0}{m\omega_c} e^{im\phi} \quad \text{and} \quad Q(\phi) = e^{q(\phi)}.$$

Using (4.12) and (4.13) together with (A8) we can write

$$\begin{aligned} A_n(k_z) &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{2\pi} \frac{d\phi'}{2\pi} \frac{Q(\phi - \phi')}{Q(\phi)} e^{in\phi'} \\ &\equiv \left(\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{e^{-in\phi}}{Q(\phi)} \right) \left(\int_0^{2\pi} \frac{d\phi'}{2\pi} Q(\phi') e^{in\phi'} \right). \end{aligned} \tag{A8a}$$

We note that $\Delta g_m^* = \Delta g_{-m}$ since $\Delta g(\phi)$ is real. Therefore $q^*(\phi) = -q(\phi)$, so that $Q^*(\phi) = e^{q^*(\phi)} = e^{-q(\phi)} = 1/Q(\phi)$. From (A8a) we have that

$$A_n(k_z) = \left| \int_0^{2\pi} \frac{d\phi'}{2\pi} Q(\phi') e^{in\phi'} \right|^2 \geq 0, \tag{A8b}$$

which is the desired result. We will demonstrate below that $\lim_{\omega \to \omega_R(0)+0^+} F_2(\omega) = -\infty$ [i.e., $F_2(\omega)$ approaches an infinite negative value as ω approaches the edge of the $m=0$ cut from the high-frequency side] and also that $\lim_{\omega \to \omega_L(1)-0^+} F_2(\omega) = +\infty$. It is then clear that the two functions $F_1(\omega)$ and $F_2(\omega)$ must cross at some value ω_0^+ such that $\omega_R(0) < \omega_0^+ < \omega_L(1)$. For $B < 0$, there is a collective mode at ω_1^-, $\omega_R(0) < \omega_1^- < \omega_L(1)$, which is associated with the singular behavior of $F_2(\omega)$ at $\omega_L(1)$ and may be interpreted as a collective excitation derived from a combined quasiparticle spin flip and cyclotron-resonance absorption; collective CESR lies at $\omega_0^- < \omega_L(0)$. A graphical solution of (A5) which illustrates the above ideas is shown in Fig. 2.

In order to complete our work we must prove that (a) $\lim_{\omega \to \omega_R(0)+0^+} F_2(\omega) = -\infty$ and (b) $\lim_{\omega \to \omega_L(1)-0^+} F_2(\omega) = +\infty$. In order to prove (a) we must consider the function $\Delta g_0(k_z)$ appearing in the definition of $F_2(\omega)$ [see (A6b)]. As k_z ranges over the Fermi surface say for $-k_M \leq k_z \leq k_M$, $\Delta g_0(k_z)$ has a spread in values, say $-\Delta g_- \leq \Delta g_0(k_z) \leq \Delta g_+$. Two alternatives must be considered: (1a) $\Delta g_0(k_z)$ has its maximum value at some point k_0 such that $-k_M < k_0 < k_M$ or (2a) $\Delta g_0(k_z)$ has its maximum value at an end point, say at k_M. First we consider case (1a). Then we can write

$$\Delta g_0(k_z) = \Delta g_+ - b_0(k_z - k_0)^2 \quad \text{for } k_z \approx k_0, \tag{A9}$$

for $b_0 > 0$ a constant. If we substitute (A9) into (A6b) and keep only the $m=0$ term [the terms $m \neq 0$ are nonsingular as $\omega \to \omega_R(0) + 0^+$] we find

$$F_2(\omega) = \frac{1}{\nu} \frac{m_c(k_0) A_0(k_0)}{2\pi^2} \int_{k_0(1-\epsilon)}^{k_0(1+\epsilon)} \frac{dk_z}{\Delta\omega - b_0 \mu_B H_0 (k_z - k_0)^2} + (\text{terms with } m \neq 0), \tag{A10}$$

where in arriving at (A10) we have written the k space volume element $d^3k = m_c(k_z) d\mathcal{E}\, d\phi\, dk_z$ in terms of standard coordinates[15] and have used (A7). We have also defined the quantity

$$\Delta\omega \equiv \frac{\langle g \rangle \mu_B H_0}{1 + B} + \Delta g_+ \mu_B H_0 - \omega. \tag{A11}$$

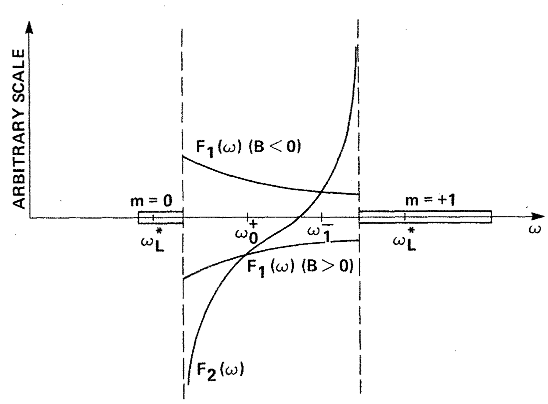

FIG. 2. Schematic (not drawn to scale) graphical solution of Eq. (A5) for $\omega_R(0) < \omega < \omega_L(1)$. The frequency $\omega_R(0)$ [$\omega_L(1)$] is the right- (left-) hand branch point of the branch cut corresponding to $m=0$ ($m=+1$) in (A4). The functions $F_1(\omega)$ and $F_2(\omega)$ are defined in (A6). For $B > 0$ the collective CESR is at ω_0^+. The collective mode at $\omega_1^- > \omega_0^+$ is derived from a combined quasiparticle spin-flip and cyclotron-resonance absorption. The broken lines are at $\omega_R(0)$ and $\omega_L(1)$. $\omega_L^* = \langle g \rangle \mu_B H_0/(1+B)$. For $\omega_R(-1) < \omega < \omega_L(0)$ the collective CESR exists for $B < 0$ and the graphical solution of (A5) (although not shown in Fig. 2) is similar to that shown.

In (A10) $0<\epsilon\ll 1$ is arbitrary and can be chosen sufficiently small so that (A9) is valid. The cyclotron mass $m_c(k_z)$ and $A_0(k_z)$ are assumed to be constant over this small interval. The integral in (A10) is trivial and we find as $\omega\to\omega_R(0)+0^+$ (i.e., as $|\Delta\omega|\to 0$)

$$F_2(\omega)\approx -C_0/(|\Delta\omega|)^{1/2}\to -\infty\,, \qquad \text{(A12)}$$

where $C_0>0$ is a constant. Now we consider case (2a). Then we can write

$$\Delta g_0(k_z)=\Delta g_+ - S_0(k_M-k_z) \quad \text{for } k_z\approx k_M\,, \qquad \text{(A13)}$$

where $S_0>0$ is a constant. Following the same procedure as for case (1a) we find that for $\omega\to\omega_R(0)+0^+$

$$F_2(\omega)\approx K_0\ln|\Delta\omega|\to -\infty\,, \qquad \text{(A14)}$$

where $K_0>0$ is a constant. In order to prove case (b), namely, that $\lim_{\omega\to\omega_L(1)-0^+}F_2(\omega)\to+\infty$, it is convenient to define the function $\Omega_1(k_z)\equiv\Delta g_0(k_z)\times\mu_B H_0+\omega_c(k_z)$. As k_z ranges over the Fermi surface the function $\Omega_1(k_z)$ has a spread in values, say $\Omega_1^-\le\Omega_1(k_z)\le\Omega_1^+$. The analysis for case (b) is essentially identical to that for case (a) except that $\omega_L(1)=\langle g\rangle\mu_B H_0/(1+B)+\Omega_1^-$ so that we will be interested in the behavior of $\Omega_1(k_z)$ near its minimum. For case (1b) we can write

$$\Omega_1(k_z)=\Omega_1^- + b_1(k_z-k_0)^2 \quad \text{for } k_z\approx k_0\,, \qquad \text{(A15)}$$

where $b_1>0$ is a constant. Proceeding exactly as in case (1a) we find that as $\omega\to\omega_L(1)-0^+$ [i.e., as $\Delta\omega\to 0^+$, here $\Delta\omega\equiv\langle g\rangle\mu_B H_0/(1+B)+\Omega_1^- - \omega$]

$$F_2(\omega)\approx C_1/(\Delta\omega)^{1/2}\to+\infty\,, \qquad \text{(A16)}$$

where $C_1>0$ is a constant. In arriving at (A16) we have only retained the $m=+1$ term in (A6b) since this term is the only singular term for $\omega\to\omega_L(1)-0^+$. We have also used (A7) for $n=1$ and have assumed that $A_1(k_z)$ can be taken as constant over the small region of integration near $k_z\approx k_0$. For case (2b) we write

$$\Omega_1(k_z)=\Omega_1^- + S_1(k_M-k_z) \quad \text{for } k_z\approx k_M\,, \qquad \text{(A17)}$$

for $S_1>0$ a constant. Following the same procedure as above we find for $\omega\to\omega_L(1)-0^+$ (i.e., for $\Delta\omega\to 0^+$)

$$F_2(\omega)\approx -K_1\ln\Delta\omega\to+\infty \qquad \text{(A18)}$$

for $K_1>0$ a constant. This completes our proof.

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[1]D. R. Fredkin and R. Freedman, Phys. Rev. Lett. 29, 1390 (1972). In our letter the equations on p. 1391 for $\delta E_{\alpha\beta}(k,r)$ and $\chi_-(\omega)$ contained misprints. These were corrected in an erratum published in Phys. Rev. Lett. 30, 37 (1973). The notation of our letter differs slightly from that of the present paper.

[2]George Feher and A. F. Kip, Phys. Rev. 98, 337 (1955); Freeman J. Dyson, Phys. Rev. 98, 349 (1955).

[3]In general the electron g value in an anisotropic medium is a second-rank tensor $g_{ij}(\vec{k})$. To our knowledge there are no first-principles calculations for metals of $g_{ij}(\vec{k})$ in the literature. Professor W. Kohn and Mr. W. Lamb are currently attempting such a calculation for Al (private communication).

[4]For an excellent review of the role of the spin-orbit interaction in solids, see Y. Yafet, Solid State Phys. 14, 1 (1963). Also see R. J. Elliott, Phys. Rev. 96, 266 (1954).

[5]Motional narrowing of the g distribution is expected to occur for $\sigma_g\mu_B H_0\tau\ll 1$, where σ_g is the rms width of the distribution of g values, μ_B is the Bohr magneton, τ is the mean free time for electron collisions, and H_0 is the dc magnetic field.

[6]V. P. Silin, Zh. Eksp. Teor. Fiz. 35, 1243 (1958) [Sov. Phys-JETP 35, 870 (1959)]; P. M. Platzman and P. A. Wolff, Phys. Rev. Lett. 18, 280 (1967); S. Schultz and G. Dunifer, Phys. Rev. Lett. 18, 283 (1967); Andrew Wilson and D. R. Fredkin, Phys. Rev. B 2, 4656 (1972); Gerald L. Dunifer, Daniel Pinkel, and Sheldon Schultz, Phys. Rev. B 10, 3159 (1974).

[7]D. Lubzens, M. R. Shanabarger, and S. Schultz, Phys. Rev. Lett. 29, 1387 (1972). Since this work was reported D. Lubzens and S. Schultz have increased their experimental capabilities and currently have spectrometers operating at 9.2, 35, and 1.25 GHz. The additional frequency (1.25 GHz) will enable them to make a more careful analysis of the frequency dependence of the CESR linewidth and position (unpublished).

[8]D. Landau, Zh. Eksp. Teor. Fiz. 30, 1058 (1956) [Sov. Phys.-JETP 3, 920 (1957)]; V. P. Silin, Zh. Eksp. Teor. Fiz. 33, 495 (1957) [Sov. Phys.-JETP 6, 387 (1958)].

[9]M. B. Walker has considered the effects on CESR of a momentum-dependent isotropic g tensor for a system having a spherical Fermi surface. See M. B. Walker, Phys. Rev. Lett. 33, 406 (1974).

[10]In the usual Landau Fermi-liquid theory of isotropic systems one writes $\delta\mathcal{E}=\mathrm{tr}'\sum_{\vec{k}'}\phi(\vec{k},\vec{\sigma},\vec{k}',\vec{\sigma}')\delta n(\vec{k}',\vec{\sigma}',\vec{r},t)$. For a system having time-reversal invariance, inversion symmetry, and for which there is no spin-orbit coupling the most general form for the Landau interaction function is $\phi(\vec{k}\vec{\sigma},\vec{k}',\vec{\sigma}')=a(\hat{k}\cdot\hat{k}')+b(\hat{k}\cdot\hat{k}')\vec{\sigma}\cdot\vec{\sigma}'$. One then expands the functions $\nu a(\hat{k}\cdot\hat{k}')$ and $\nu b(\hat{k}\cdot\hat{k}')$ into a series of Legendre polynomials with expansion coefficients A_l and B_l, respectively.

[11]In isotropic metals the Landau parameters A_l for $l\geq 2$ are known to have observable effects on the dispersion relations of the so-called high-frequency waves (also called cyclotron waves). See P. Platzman,

W. M. Walsh Jr., and E-Ni Foo, Phys. Rev. 172, 689 (1968).

[12]P. M. Platzman and P. A. Wolff, Solid State Phys. Suppl. 13, 232 (1973). A more detailed reference for which the momentum dependence of $g(\vec{k})$ is considered in the derivation of the transport equation can be found in R. Freedman, Ph.D. thesis (University of California, San Diego, 1972) (unpublished).

[13]D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966), p. 24.

[14]The observed signal, for example, in transmission CESR is given by $\chi'' = \mathrm{Im}\chi$, but there are standard corrections which are applied to the observed signal to make it comparable to χ''. See Gerald L. Dunifer, Daniel Pinkel, and Sheldon Schultz (Ref. 6).

[15]D. R. Fredkin and R. Freedman, Phys. Rev. B 9, 360 (1974).

[16]J. I. Kaplan and M. L. Glasser, Phys. Rev. 183, 408 (1969).

[17]I. M. Lifshits, M. Ya. Azbel', and M. I. Kaganov, *Electron Theory of Metals* (Consultants Bureau, New York, 1973), p. 179. For a useful compilation of the topological properties of metallic Fermi surfaces see Appendix III, p. 299.

[18]For a discussion of several model Fermi surfaces having open orbits see p. 54 of Ref. 12 (R. Freedman).