Critical dynamics of kinetic Ising models in four dimensions*

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The effective kinetic coefficient of a kinetic Ising model with a nonconserved order parameter coupled to a conserved energy field in four dimensions varies as $|\ln(T-T_c)|^\sigma$ with $-\frac{2}{3} \leq \sigma \leq 0$. These results should apply to dipolar Ising systems in three dimensions whose dynamics are purely relaxational, although it is very difficult to find the exact value of σ.

Recent work in dynamical critical phenomena has dealt with kinetic Ising models in $4-\epsilon$ dimensions extrapolated to $\epsilon=1$.[1] Larkin and Khmel'nitskii and Aharony have shown that near T_c the free energy of a dipolar Ising system should behave as a four-dimensional Ising system with conventional short-range interactions.[2] There is also recent experimental evidence for such behavior.[3] The critical properties of conventional kinetic Ising models in four dimensions are potentially interesting. The results of Ref. 2 carry over immediately to kinetic Ising dynamics.

The two models of Ref. 1 in which there is a nontrivial renormalization of the transport coefficient are (A) Order parameter not conserved; (C) order parameter not conserved, but coupled to a conserved energy field. In four dimensions one expects the Van Hove theory to be valid except for logarithms. We will follow the procedure of Wegner and Riedel to calculate the corrections to scaling.[4]

The recursion formula for the kinetic coefficient Γ in model A was analyzed in Ref. 5 for an arbitrary dimension. The value of the coupling constant after the lth iteration is $\sim l^{-1}$ so we may integrate on l to find the leading singularity of Γ near $t=(T-T_c)/T_c=0$,

$$\Gamma=\Gamma_0[1+(\tfrac{2}{9}\, ln\tfrac{3}{4})/|\ln t|\,].$$

There are additional terms such as $|\ln t|^{-n}$, $n>1$, and $\ln|\ln t|$ which will not in general have universal coefficients.

Model C is more complex since an energy field ϵ, with transport coefficient λ is statically coupled to the order parameter S:

$$F(S\epsilon)=F_{\mathrm{GLW}}(S)+\int \gamma S^2\epsilon+\tfrac{1}{2}C^{-1}\epsilon^2,$$

where F_{GLW} is the conventional Ginsburg-Landau-Wilson functional. The recursion formulas for the transport coefficients are[1]

$$\frac{\partial \ln\Gamma_l}{\partial l}=\frac{-4(C\gamma^2)_l B\ln b}{1+\mu_l},$$

$$\frac{\partial\lambda_l}{\partial l}=0,$$

$$\mu_l\equiv\frac{\lambda_l}{C_l\Gamma_l},\quad B\ln b=\int_{1/b}^{1}\frac{d^4p/(2\pi)^4}{p^4},$$

and for the statics

$$(C\gamma^2)_l=\frac{1/B\ln b}{6(l+l_0)+c_1(l+l_0)^{2/3}}.$$

The constants c_1 and l_0 depend on the initial values of u and $C\gamma^2$. For large l we find

$$\mu_l/(1-\mu_l)^2=l^{1/3}\mu_0/(1-\mu_0)^2.$$

The singularity of Γ at $t=0$ is complicated by the marginal operator μ_l which decays slowly to its fixed point value 1. We may distinguish three regimes according to the initial value of μ_l, although only the second is truly asymptotic

$$\frac{\Gamma}{\Gamma_0}\sim\begin{cases}|\ln t|^{-2/3}, & \mu_0|\ln t|^{1/3}\ll 1,\\ |\ln t|^{-1/3}, & |\ln t|^{1/3}\gg\mu_0+1/\mu_0,\\ 1-(c_2/\mu_0)|\ln t|^{1/3}, & \mu_0/|\ln t|^{1/3}\gg 1.\end{cases}$$

In the last line c_2 is a positive nonuniversal constant. Experimentally, one could not hope to see any crossover effects. The effective exponent σ varies from zero to $-\frac{2}{3}$ with μ_0. The characteristic frequency involves the susceptibility which contains additional correction terms.[2,4]

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