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Theory of electrical instabilities of mixed electronic and thermal origin. II. Switching as a nucleation process

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The dynamics of the switching process in amorphous semiconductors is analyzed using a previously described electrothermal model. Utilizing recent preswitching noise data as input, we calculate the switching delay time as a function of current in the low-overvoltage regime in the nucleation approximation. It is found that the present, basically macroscopic theory is incapable of explaining the observed phenomena. A full explanation of switching in amorphous semiconductors requires, therefore, a more detailed understanding of the underlying microscopic electronic processes.

INTRODUCTION

Several theories of switching in two terminal chalcogenide glass devices have been introduced recently. On the one hand, it is argued that an electrothermal theory based solely on the bulk electronic and thermal properties of the semiconducting material is sufficient to explain the primary transient and steady-state behavior of these devices. On the other hand, it is held that nonequilibrium electronic processes such as carrier avalanche, double injection, or high-field tunneling must be invoked, and that any heating is of only secondary importance.[1] Because most of these theories give qualitatively the primary characteristics which are observed, it has been difficult to decide what effects are truly basic to the switching process.

For this reason, we have extended our previous calculations on the mixed electronic and thermal (MET)[2] model to include fluctuations. Using recent preswitching noise measurement data as input, we calculate the switching delay time as a function of current (or voltage) in the low-overvoltage regime in the nucleation approximation. It is shown that for the noise levels observed switching in this model does not occur in realistic times until one is very near the previously determined local instability point of the off-state characteristic. For all relevant device geometries this is on the negative-differential-resistance portion of the characteristic, in disagreement with the observations of Shaw[3] and Henisch and co-workers,[4,5] who found switching as much as 20% below turnaround voltage for thin-film devices of the geometry we consider. At such switching voltages, the noise level required within the present theory for switching to occur in the observed times is almost four orders of magnitude greater than the highest noise levels observed by Shaw.[3] Furthermore, the results obtained are essentially independent of whether the glass is of memory or threshold composition. Experimentally this is apparently not so. In contrast with the observations of Shaw and Henisch on threshold composition switches, Buckley and Holmberg[6] report switching after turnaround for devices of a similar thin-film geometry constructed of $Ge_{17}Te_{79}Sb_2S_2$, a memory material. The MET model, basically a macroscopic model, is thus incapable of accounting for these and other central features of the switching process. The underlying microscopic electronic processes must therefore be considered in more detail.

MODEL

Briefly, the electrothermal theories attempt to explain the observed switching characteristic in terms of an instability of a certain class of solutions of the local equilibrium conservation and rate equations of the bulk macroscopic system. The basic equations of the theory are simply Maxwell's equations and the conservation of energy, together with the thermodynamic constitutive relations. The problem is completely defined once we specify the thermal and electrical conductivities as functions of temperature and field, and select proper boundary conditions (BC) for T and $\vec{E}$, where $\vec{E}$ is the electric field. Nevertheless, without certain further simplifying assumptions the task of determining the properties of solutions of this system is prohibitively difficult. For this reason our calculations are based in general on somewhat simplified models which, if carefully chosen, can be shown to predict current-voltage characteristics in good agreement with those determined by more extensive numerical solutions of the full system.

For thin-film geometries of cylindrical symmetry, we have previously shown (I) that one gets good results by considering a model in which the axial dependence of the temperature and field are suppressed and axial heat loss is approximated by an effective cooling term proportional to the difference of $T(r)$ from ambient. If we further ignore displacement-current effects, the system of equations describing the model reduces to[7]

$$C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\frac{8K}{d^2}(T-T_0)+\sigma(T,\,E)E^2,$$

$$V_{\rm ex}=Ed+2\pi R\int_0^a r\,dr\,\sigma(T,\,E)E\,,\quad \left.\frac{\partial T}{\partial r}\right|_a=0\,, \tag{1}$$

where C is the specific heat (~ 1 J/cm^3 °K), K the thermal conductivity, a the device radius, d the device thickness, $V_{\rm ex}$ the externally applied voltage, and R the external load resistance. For calculations with this model we have taken

$$\sigma(T,\,E)=\sigma(T)e^{|E|/E_0},\quad K(T,\,E)=K_l+L\sigma T,$$

where $\sigma(T)$ is given in Fig. 2 of I, $E_0=3.7\times10^4$ V/cm, $K_l=4$ mW/cm °K, $L=2.45\times10^{-8}$ W Ω/°K. Detailed justification of these choices is given in I.

The properties of solutions of this system for $d=1$ μm, $a=10$ μm are summarized in Fig. 1. There were found to be two distinct classes of solutions $T(r)$ which, for certain ranges of BC lead to stable branches of the I-V characteristic. The first are radially independent solutions corresponding to a uniform flow of current across the device. This is the preswitching branch of the characteristic. The second are radially dependent solutions for which $T(0)>T(r)$, $r>0$. This class was found to bifurcate from the radially uniform class at point 1, just above turnaround. We found that the branch determined by radially uniform solutions was unstable with respect to constant current fluctuations above the first bifurcation point, and that the branch determined by solutions of the second class was stable with respect to such fluctuations above about 2×10^{-4} A, for the present set of device and material parameters. This branch is again predicted to become unstable at very high currents, on the order of those at which it intersects the radially uniform branch. We should reemphasize that in the neighborhood of 1, the illustrated bifurcation point, these two branches lie very close together. At 14 V (over $1\frac{1}{2}$ V below the voltage maximum) the difference in current is only 2%.

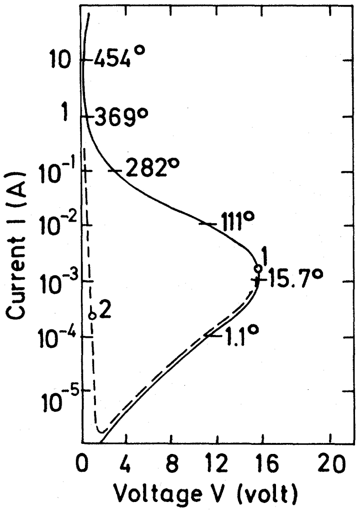

FIG. 1. Current-voltage characteristic obtained by numerical solution of Eq. (1). Solid curve: radially uniform branch; dashed curve: branch arising from channelized current flow. Temperatures listed are °K above ambient, $T_0=295$ °K. Branch arising from channelized current flow stable with respect to constant current fluctuations above point 2.

The above results were obtained by analyzing the MET theory in a specific device geometry for a particular set of material parameters relevant to the chalcogenide glasses, as specified above. In general, the specific form of the characteristic depends upon the device geometry and heat-flow boundary conditions, as well as the dependence of σ and K on temperature and field. The particular results outlined above are critically dependent on the field dependence of σ, the rapid increase and subsequent saturation of σ with increasing temperature, the enhancement of K at high temperatures, and the thin-film geometry in which $a\gg d$ so that the electrodes provide very strong coupling to a heat sink. A more detailed discussion of the dependence of the characteristic on all these parameters is presented in I.

The above model also explained the fact that the switching event in chalcogenide-glass threshold switches is qualitatively different for large and small overvoltages. Near threshold, when a device is repeatedly addressed with constant voltage pulses, it is found that the switching delay time is not well defined, but rather is subject to substantial fluctuations. At threshold, the observed delay times may vary by as much as a factor of 10.[5] For higher voltage pulses, the width of the delay-time distribution decreases until at approximately 20% above threshold a rather abrupt transition occurs to a region in which the switching event loses its statistical character. In the low-overvoltage statistical regime, the current remains constant during the delay time, while at higher voltages it is found to increase noticeably prior to switching. Figure 2 clearly illustrates these two regions.

Since it may be shown that the observed statistical distribution of switching voltages or delay times in the low-overvoltage region is consistent with nucleation with a time-independent, but voltage-dependent, probability,[8] we were lead to the following interpretation of the switching event. For voltages not too far below turnaround, two branches of the static I-V characteristic determined by the MET theory lie very close together. One of these, determined by radially uniform solutions of Eq. (1), corresponds to the preswitching state. The other, unstable branch, is determined by radially dependent solutions for which the temperature and current are

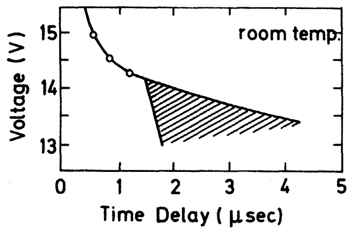

FIG. 2. Relationship between applied voltage and switching delay time t_D for 1-μm-thick $Te_{40}Si_{18}Ge_7AS_{35}$ threshold switch. (See Ref. 4.)

peaked at $r=0$. Temperature and current fluctuations of sufficient size to carry the system along the load line across this unstable branch will cause switching to occur.

For $V< V_t$, where V_t is the turnaround voltage, the switching event is statistical in character. In this region switching occurs by the nucleation and growth of a hot spot in the device interior. Since the separatrix lies very near the metastable solution in I-V space, the current will appear constant until the critical fluctuation has formed; at this point, the system is unstable, and the energy stored in the circuit discharges through the embryonic channel, heating the material and switching the device. This last process can take place very rapidly, on the order of 10^{-10} sec.[9]

For $V>V_t$, the current-temperature distribution in the device is unstable with respect to infinitesimal perturbations, and the switching event becomes deterministic. In this region switching is envisioned to occur by the normal field-assisted thermal runaway mechanism, and the current should begin to increase continuously until switching occurs.

In this paper we shall be primarily interested in a further application of the MET theory to the dynamical problem in the low-overvoltage statistical regime. We introduce phenomenological fluctuation terms in Eq. (1) and, using recent preswitching noise data as input, determine within the nucleation approximation the switching delay time as a function of voltage.

In general, fluctuations of both E and T must be considered. But for a general temperature dependence of σ and load resistance R, this leads to considerable difficulties even for Markoffian noise sources since it may not be possible to integrate the functional Fokker-Planck equation for the temperature-field distribution to obtain the potential function for the nucleation process.[10] However, for specific forms of $\sigma(T)$, namely $e^{\beta T}$, for which it is possible to include field fluctuations if one considers the limit $R\rightarrow\infty$, i.e., constant total current fluctuations, we found that the delay times calculated both for constant current and constant voltage fluctuations were essentially identical. For this reason, although we shall limit ourselves to the simpler problem of considering only constant-E fluctuations (limit $R\rightarrow 0$), we feel that the results obtained should apply equally well to the more general problem of finite load resistance.

Finally, because we shall be interested only in phenomena whose time scales are very slow, on the order of the thermal relaxation times for these devices (~ 10^{-6} sec for a 1-μm device), we shall ignore displacement-current effects in subsequent calculations.

FLUCTUATIONS AND NUCLEATION IN THE LOW-OVERVOLTAGE REGION

In order to quantify the above ideas we introduce an external-random-force term $\xi(r,t)$ on the right-hand side of Eq. (1):

$$C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\frac{8K}{d^2}(T-T_0)+\sigma^0(T,E)E^2+\xi(r,t),\qquad \left.\frac{dT}{dr}\right|_a=0. \tag{2}$$

Microscopically, such noise terms are the resultant effect of all the projected-out nonhydrodynamic microscopic variables of the system. In true equilibrium these are the normal thermodynamic fluctuations and are related to the system's transport coefficients by the fluctuation-dissipation theorem. More generally, excess nonequilibrium sources may be present for which the noise is non-Markoffian.

Such excess non-Markoffian current noise is indeed observed whenever sufficient current is flowing in a low-conductivity material. Rather generally, in uniform samples, it is found that

$$\left\langle\left(\frac{\Delta I}{I}\right)^2\right\rangle=\left\langle\left(\frac{\Delta\sigma}{\sigma}\right)^2\right\rangle=\frac{\beta}{N}\frac{\Delta f}{f}, \tag{3}$$

where N is the total number of free carriers and β is a constant approximately equal to 2×10^{-3} for hole or electron conduction.[11] As indicated in Eq. (3), it appears that the fluctuations are in the conductivity. In fact, it has been argued by Hooge that bulk $1/f$ noise is due to fluctuations in the mobility of free carriers.[11] We shall make use of this result later.

Early measurements of current noise in the chalcogenide glasses $As_2Te_3Tl_2Se$ and As_2SeTe_2 by Main and Owen[12] gave results in general agreement with Eq. (3). More recently, however, Shaw[3] has performed excess-current-noise measurements on 0.2-μm-thick threshold switches of $Te_{40}As_{35}Ge_7S_{18}$ composition and found

$$\langle(\Delta I/I)^2\rangle\propto(I^\gamma/f^\delta)\Delta f, \tag{4}$$

with $\gamma=2.8\pm1.5$ and $\delta=1.2\pm0.1$ in the immediate preswitching regime. In contrast with Eq. (3), the noise level was found to increase faster than qua-

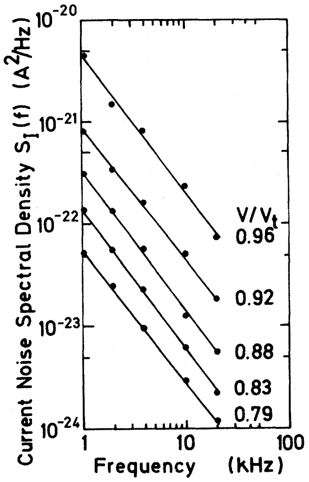

FIG. 3. Current-noise spectral density $S_I(f)$ for a 0.2-μm thick $Te_{40}Si_{18}Ge_7As_{35}$ threshold switch. Threshold voltage $V_t \approx 12$ V. Threshold current $\sim 10^{-5}$ A. (See Ref. 3.)

dratically with current. For slow-rise-time pulses, switching was always observed to occur on the positive differential resistance portion of the I-V characteristic. At room temperature, the differential resistance of the device at threshold was about 1.5×10^5 Ω. The noise levels observed as functions of voltage and frequency are shown in Fig. 3.

Assuming then that the noise is generated uniformly in the bulk and that it indeed arises from conductivity fluctuations, we obtain the following explicit form of Eq. (2):

$$C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\frac{8K}{d^2}(T-T_0)$$
$$+\sigma^0(T,\ E)E^2+f(r,\ t)E^2, \tag{5}$$
$$\langle f(r,\ t)\rangle=0,\quad \langle f(r,\ t)f(r',\ t+\tau)\rangle=\delta(r-r')k(\tau).$$

Lacking more detailed information concerning the microscopic noise statistics, we have made the simplest possible assumptions, taking $f(r,\ t)$ δ correlated in space and $k(\tau)$ Gaussian. We further assume that the process described by $k(\tau)$ is stationary, i.e., $k(\tau)$ has a Fourier transform in the normal sense. This last requirement will become clearer later.

Since analysis of the corresponding Markoffian problem is much simpler, we will also investigate the properties of the system for $k(\tau)$ δ correlated in time. In fact, we shall see that for the present purposes comparison between the Markoffian and long-correlation-time results will be instructive.

A. Markoffian noise

In general, the $\sigma^0(T,\ E)$ in Eqs. (2) and (5) is not equal to $\sigma(T,\ E)$ in Eq. (1). Upon taking the ensemble average of Eq. (2), we obtain

$$C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\frac{8K}{d^2}(T-T_0)+\langle\sigma^0(T,\ E)\rangle E^2.$$

In general, since $\langle G(x)\rangle\neq G(\langle x\rangle)$ if G is nonlinear, one would be forced to attempt to find a σ^0 such that $\langle\sigma^0(T)\rangle=\sigma(\langle T\rangle)$. This renormalization effect is very similar to the problem of transport-coefficient renormalization in critical dynamics. But if we limit ourselves to the nucleation regime in which the mean square size of fluctuations is small, this renormalization effect is negligible, and we may identify σ^0 with σ. The consistency of this identification can be checked *a posteriori*.

Starting from Eq. (5) then, with $\sigma^0=\sigma$, it is straightforward to obtain an expression for the mean switching delay time from the metastable "off" state, in the nucleation approximation, given a Gaussian, Markoffian noise source. Since we will only look at constant-E fluctuations, we may immediately write down a functional Fokker-Planck equation for the process. If $P(\{T(r)\},\ \tau|\{T_i(r)\},\ \tau_0)$ is the transition probability between a state defined by $T_i(r)$ at time τ_0 and a state $T(r)$ at time τ we have[13,15]

$$\frac{\partial P}{\partial t}=\int_\Omega d\vec{r}\,\frac{\delta}{\delta T(r)}\left(\frac{\delta\Psi}{\delta T(r)}P\right)+L^0\int_\Omega d\vec{r}\,\frac{\delta^2 P}{\delta T(r)\delta T(r)}, \tag{6}$$

where L^0 is such that

$$(E^4/C^2)\langle f(r,\ t)f(r',\ t+\tau)\rangle=2L^0\delta(r-r')\delta(\tau)$$

and

$$\Psi=\frac{1}{C}\int_\Omega d\vec{r}\left[\frac{K}{2}|\nabla T|^2+\frac{8K}{d^2}\left(\frac{T^2}{2}-TT_0\right)\right.$$
$$\left.-\int^{T(r)}\sigma(u(r),\ E)E^2\delta u\right].$$

Integration is over the volume of the active material, Ω.

Explicitly, the nucleation approximation is valid when $\Delta\Psi=\bar{\Psi}-\Psi^0$, where $\bar{\Psi}$ is evaluated at the unstable saddle-point solution and Ψ^0 at the metastable "off-state" solution, is much larger than L^0. In this limit, the mean time for a system originally in the radially uniform state to nucleate the transition (the mean first passage time) is[14]

$$\langle t\rangle\approx t_p e^{\Delta\Psi/L^0},$$

where t_p is expressible in terms of the eigenvalues of the linearized form of Eq. (1) (with $R\to 0$, so E remains constant) at the separatrix and metastable minimum. Denoting the eigenvalues on the separatrix and metastable minimum by $\bar{\lambda}_i$ and λ_i^0, $i=0,\ \ldots,\infty$, respectively, we have

$$t_p=\frac{\pi}{|\bar{\lambda}_0|}\prod_{j=0}^{\infty}\left(\frac{\bar{\lambda}_j}{\lambda_j^0}\right)^{1/2}.$$

TABLE I. Values of the nucleation potentail $\Delta\Psi$ and the rate coefficient t_p for 1- and 2-μm-thick $Ge_{15}Te_{81}X_4$ model devices. Turnaround voltage for 1-μm device: 15.79 V; for 2-μm device: 24.87 V. Device radius: 10 μm.

1-μm device			2-μm device		
V(V)	$\Delta\Psi$ (cm^3 °K^2/sec)	t_p (sec)	V (V)	$\Delta\Psi$ (cm^3 °K^2/sec)	t_p (sec)
14.6	4.3×10^{-3}	4.85×10^{-7}	22	1.1×10^{-2}	1.4×10^{-6}
14.7	3.9×10^{-3}	5.3×10^{-7}	23	7.3×10^{-3}	2.3×10^{-6}
14.8	3.5×10^{-3}	5.9×10^{-7}	23.5	5.3×10^{-3}	3.2×10^{-6}
14.9	3.2×10^{-3}	6.6×10^{-7}	24	3.4×10^{-3}	5×10^{-6}
15	2.8×10^{-3}	7.5×10^{-7}	24.25	2.4×10^{-3}	6.8×10^{-6}
15.1	2.5×10^{-3}	8.0×10^{-7}	24.5	1.5×10^{-3}	1.1×10^{-5}
15.2	2.1×10^{-3}	1.0×10^{-6}	24.7	6.9×10^{-4}	1.9×10^{-5}
15.3	1.8×10^{-3}	1.2×10^{-6}			
15.4	1.4×10^{-3}	1.5×10^{-6}			
15.6	7.2×10^{-4}	2.7×10^{-6}			
15.7	3.6×10^{-4}	4.8×10^{-6}			
15.75	1.7×10^{-4}	8×10^{-6}			

Values of t_p and $\Delta\Psi$ in the neighborhood of turnaround are given in Table I for model devices 20 μm in diameter and 1 and 2 μm thick. Table II contains values of $\Delta\Psi$ for a range of device thicknesses at various points in the neighborhood of turnaround.

In order to obtain a feeling for the actual size of the values of $\Delta\Psi$, it is worth noting that for Johnson noise, i.e., when $\zeta(r, t)$ in Eq. (2) is given by

$$\langle\zeta(r, t)\rangle = 0,$$

$$\langle\zeta(r, t)\zeta(r', t+\tau)\rangle = 2\sigma k_B T E^2 \delta(r-r')\delta(\tau),$$

L^0 near turnaround, for a device with I-V characteristic illustrated in Fig. 1, is on the order of 2×10^{-13}, in units used in the table. Equilibrium thermodynamic noise clearly could not cause switching in any but an infinitesimal neighborhood of the local instability point.

B. Non-Markoffian noise

For noise sources with a long correlation time, it is in general a very difficult task to derive a kinetic equation for the transition probability P starting from Eq. (5). It is possible to set up a formal perturbation scheme for doing this, but it has the disadvantage of requiring summation to infinite order to give convergence over reasonable time scales. Attempts along this line have therefore not yet been successful.

It is possible, though, to do the full linear analysis of fluctuations for an arbitrary stationary random-noise source. Because the response of any system is essentially determined by its own relaxation time, the very-low-frequency components of the external source have little effect upon the general response of the system, and one may obtain a reasonably good idea of the effectiveness of longer-correlation-time noise sources in the nucleation regime by comparing the results of linear analyses of the non-Markoffian and Markoffian problems.

Linear variations in temperature, $x(r, t) = T(r, t) - T_u$, about the radially uniform metastable solution satisfy

$$C\frac{\partial x}{\partial t} = \frac{K}{r}\frac{\partial}{\partial r}\left(r\frac{\partial x}{\partial r}\right) - \left(\frac{8K}{d^2} - \frac{\partial\sigma}{\partial T}E^2\right)x + f(r, t)E^2,$$

$$\left.\frac{dx}{dr}\right|_a = 0.$$

Expanding $x(r, t)$ in the complete set of basis functions $\{1, J_0(k_n r)\}$, $k_n \ni J_1(k_n a) = 0$,

$$x(r, t) = x_0(t) + \sum_{n=1}^{\infty} x_n(t) J_0(k_n r),$$

it is not difficult to show that we have the following Fokker-Planck equation for the transition probability $P(\{x_n(t)\} \mid \{x_n(0)\})$ for a process switched on at $t=0$, given a general Gaussian, stationary noise source $f(r, t)E^2$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x_n}(\lambda_n^0 x_n P) + E^4 \frac{\partial^2}{\partial x_n^2}\left(P\int_0^t e^{-\lambda_n^0(t-s)} k_n(t-s)\,ds\right), \tag{7}$$

TABLE II. Nucleation potential $\Delta\Psi$ as a function of device thickness d and voltage V for model $Ge_{15}Te_{81}X_4$ devices. T_u is the uniform temperature solution in °K above ambient at same voltage V, and V_T is turnaround voltage. Device radius: 10μm.

Device thickness d (cm)	V (V)	T_u (°K)	V_T (V)	$\Delta\Psi$ (cm^3 °K^2/sec)
2×10^{-5}	4.71	7.32	4.89	6.5×10^{-4}
2.5×10^{-5}	5.58	7.32	5.8	7.16×10^{-4}
5×10^{-5}	9.28	7.32	9.7	1.4×10^{-3}
7.5×10^{-5}	12.34	7.32	12.95	2.1×10^{-3}
1×10^{-4}	15.0	7.32	15.79	2.83×10^{-3}
2×10^{-4}	23.4	7.32	24.87	5.67×10^{-3}
3.5×10^{-4}	32.6	7.55	34.81	9.4×10^{-3}

where

$$\langle f(r,\ t)f(r',\ t+\tau)\rangle = \delta(r-r')k(\tau),$$

$$k_n(\tau) = k(\tau)/\Omega C^2 J_0^2(k_n a),$$

and

$$\lambda_n^0 = \frac{1}{C}\left(Kk_n^2 + \frac{8K}{d^2} - \frac{\partial\sigma}{\partial T}E^2\right),$$

as above. Summation over n from zero to infinity is implied, and, for $n=0$, $k_0(\tau) = k(\tau)/\Omega C^2$ and $\lambda_0^0 = (1/C)\,[8K/d^2 - (\partial\sigma/\partial T)E^2]$.

For a δ-correlated noise source with

$$k(\tau) = 2(L^0C^2/E^4)\delta(\tau)$$

this reduces to

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x_n}(\lambda_n^0 x_n P) + \frac{L^0}{\Omega J_0^2(k_n a)}\frac{\partial^2}{\partial x_n^2}P, \qquad (8)$$

the linearized, Fourier-transformed form of Eq. (6).

In order to derive more explicit results for $1/f$ noise, we shall choose the following form for $k(\tau)$[11]:

$$k(\tau) = \alpha\int_{\tau_1}^{\tau_2}\frac{e^{-\tau/x}}{x}\,dx, \qquad (9)$$

where α is a constant chosen to fit the measured noise data. This correlation function leads to a spectral density which varies as $1/f$ for $1/\tau_1 \gg 2\pi f \gg 1/\tau_2$, is independent of frequency for $2\pi f \ll 1/\tau_2$, and varies as $1/f^2$ at sufficiently high frequencies. The assumption of stationarity implies that any $k(\tau)$ chosen must be such that its Fourier transform varies more slowly than $1/f$ at low frequencies and decreases more rapidly than $1/f$ at high frequencies. The lack of data at sufficiently high and low frequencies makes it impossible to distinguish between various $k(\tau)$ which give a $1/f$ spectrum over the measured range. As we shall see, this leads to only a slight ambiguity in the response function for $T(r,\ t)$.

If we write $\langle\delta I^2(f)\rangle = (b/f)\delta f$ for the spectral density of current fluctuations, where b will in general be a function of the current and device volume, the coefficient α in Eq. (9) is given by

$$\alpha = bd^2/\Omega E^2, \qquad (10)$$

where d is the device thickness, Ω its volume, and E the electric field. In obtaining (10) we have used the standard definition of the spectral density[15]

$$S(f) = 2\int_{-\infty}^{\infty} d\tau\, k(\tau)e^{2\pi i f\tau}.$$

Using (7), (9), and (10), the following result is obtained for the mean-square size of fluctuations of $x(r,\ t)$ for $t \gtrsim 1/\lambda_0^0$:

$$\langle x_n^2(t)\rangle \approx bE^2\frac{d^2}{\Omega^2}\left(\frac{\ln(\lambda_0^0\tau_2)}{(\lambda_0^0)^2} + \sum_{n=1}^{\infty}\frac{\ln(\lambda_n^0\tau_2)}{(\lambda_n^0)^2}\,\frac{J_0^2(k_n r)}{J_0^2(k_n a)}\right). \qquad (11)$$

τ_2 is the low-frequency cutoff of the $1/f$ spectrum, and is an additional parameter which has not yet been measured. Luckily it appears only logarithmically, and its precise value is not critical.

For a white noise source with the spectral density $\langle\delta I^2(f)\rangle = b_M\delta f$ over the region of interest, we have, again for $t \gtrsim 1/\lambda_0^0$,

$$\langle x_n^2(t)\rangle \approx \frac{1}{4}\frac{d^2}{\Omega^2}b_M E^2\left(\frac{1}{\lambda_0^0} + \sum_{n=1}^{\infty}\frac{1}{\lambda_n^0}\,\frac{J_0^2(k_n r)}{J_0^2(k_n a)}\right). \qquad (12)$$

From Eqs. (11) and (12), we see that for

$$b_M \approx 4b[\ln(\lambda_0^0\tau_2)]/\lambda_0^0 \qquad (13)$$

the two spectral densities are comparable. In fact, this choice of b_M would lead to an overestimate of the mean-square fluctuation size given by Eq. (11), particularly at small radii r. Both equations (11) and (12) show that the low-frequency components of any noise source have very little effect on the system's response. Only the noise at those frequencies on the order of the inverse relaxation time is sampled. In situations where the metastable minimum is relatively deep, so that it is only very rare large fluctuations which nucleate the transition, the analogy between the response of the system to Markoffian and non-Markoffian noise sources remains valid. In this case, the mean-square size of fluctuations, for times beyond the local relaxation time of the system, will be given by functions similar to Eqs. (11) and (12), the only difference being a small renormalization of the eigenvalues λ_i^0 due to the nonlinearities. The functional probability distribution functions for the temperature will be of very similar form in both cases. Since it is the attempt frequency, mean temperature fluctuation size, and barrier height which are most important in calculating the nucleation rates, we expect the results to be quite similar in both cases. For large fluctuations, or where the potential barrier is smaller, the renormalization and non-Markoffian effects become more important, and we would no longer expect the above arguments to hold.

We have therefore estimated the mean switching rates due to $1/f$ noise by calculating the rates for a Markoffian processes with a b_M determined using Eq. (13). If the calculated transition rates are very small so that a slightly renormalized linear theory correctly describes the mean response, we can be confident that we have not overestimated the switching times.

For the glass $Ge_{15}Te_{81}X_4$ (where X is a multicomponent additive), using the material and device parameters described above, we calculate a turnaround voltage of 4.89 V for a 0.2-μm-thick device in the MET theory. At $V=4.71$ V, in this case, $\Delta\Psi = 6.5\times10^{-4}$ from Table II. Assuming the same relative noise level $\langle\delta I^2(f)/I^2\rangle$ as measured by

Shaw at the highest voltages attainable just before switching (Fig. 3), and taking $\ln(\lambda_0^0\tau_2) = 25$,[16] we calculate an L^0 for the corresponding Markoffian process of only 5.7×10^{-7}. At this voltage the differential resistance in our model system is much smaller than that measured by Shaw at threshold, probably indicating that the voltage corresponds to a point nearer turnaround than he was able to attain. Nevertheless, such a low noise level can clearly never cause switching in observable times.

Unfortunately, we have data for only one device thickness, and it is not clear how the relative noise scales with thickness. Nevertheless, the above results may be used to estimate the current noise levels required to give switching in agreement with experiment. In Fig. 2 it may be seen that the stochastic switching region is approximately $1-1\frac{1}{2}$ V wide for a 1-μm-thick device constructed of $Te_{40}Si_{18}Ge_7As_{35}$. Within the framework of the present theory, switching within this region is interpreted as occurring by the nucleation and growth of a hot spot, below turnaround.

At 1 V below turnaround (14.7 V), for the present set of material parameters (see Fig. 1), it is seen from Table I that $\Delta\Psi = 3.9\times 10^{-3}$, while L^0 for this geometry, at this voltage, scales as $1.4k$, where $k = \langle \delta I^2(f)/I^2\rangle f/\delta f$. Thus for switching to be possible we would require $k \approx 10^{-4}$. This is almost four orders of magnitude greater than the highest relative noise levels observed by Shaw. Even at 15.75 V (turnaround voltage is 15.79 volts) we have $\Delta\Psi = 1.7\times 10^{-4}$. At this voltage L^0 scales as $37k$, requiring k on the order of 10^{-6} before there is an appreciable probability of switching in short times. Even this is nearly two orders of magnitude larger than the highest noise levels observed. For noise levels an order of magnitude less than those quoted above as required to cause switching in observably short times, nonlinear corrections to a calculation of the temperature-fluctuation spectra are negligible. We can be reasonably confident therefore that the order-of-magnitude estimates of the noise levels required for switching within this theory are correct.

DISCUSSION AND CONCLUSIONS

Within the framework of the MET theory (I) we have investigated the possibility of switching by macroscopic fluctuations from points on the metastable positive differential resistance portion of the off-state characteristic. The calculations were performed using $Ge_{15}Te_{81}X_4$ material parameters, but are believed to be relevant for a fairly wide range of glass compositions. In particular, we would not expect qualitatively different behavior for glasses of more stable threshold switching composition. It was found that current noise far in excess of that reported by Shaw is required to cause switching in any but an infinitesimal neighborhood of the point of local instability.

These results are contradicted by experiment in two ways. First, in switches constructed of threshold materials, switching is observed well before turnaround, perhaps as much as 20% below turnaround voltage for a 1-μm-thick device. Second, there does appear to be a qualitative difference between threshold and memory materials in this respect. Buckley and Holmberg have recently reported extensive measurements on virgin devices constructed of the memory composition $Ge_{14}Te_{81}Sb_2S_2$ in which they rarely find switching before turnaround when slow current ramps are applied to devices as little as 0.5 μm thick.[6]

Furthermore, the recent measurements of Buckley and Holmberg point quite clearly to the existence of a critical switching field under isothermal conditions. They found that as larger and larger overvoltages were applied to a resistor-switching device circuit, the voltage across the device increased until a critical switching voltage was attained. For higher applied voltages the characteristic turned vertical and the voltage across the device remained constant while the current increased rapidly until switching occurred. The device was unable to sustain voltages above some critical value. Such behavior is also not predicted by the present theory.

Therefore, although most aspects of the static properties of thin film chalcogenide glass switches are well explained by the MET theory in its present form, the theory is incapable of explaining several of the observed dynamical phenomena, in particular the low-overvoltage statistical regime present at least in threshold switches, and the critical-field effects in the high-overvoltage regime.

A fuller explanation of these points would require a more detailed microscopic understanding of the field dependence of the conductivity and the associated nonequilibrium carrier redistribution. It is now clear that such purely electronic phenomena are central to the switching mechanism.

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[16] This choice corresponds to $\tau_2 \approx 10^5$, approximately the inverse of the lowest frequency at which $1/f$ noise has been observed in any material (see Ref. 11). In effect, τ_2 is a disposable parameter whose exact value depends upon details of the noise spectra not yet measured. Fortunately, it enters only logarithmically and its exact value is not critical.