Inductive transition of niobium and tantalum in the 10-MHz range. II. The peak in the inductive skin depth for T just less than T_c †

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In the region of temperature just below the superconducting transition temperature T_c, where the inductive skin depth δ_i is less than the dc penetration depth, a peak was observed in δ_i as a function of temperature in some Ta and Nb specimens. The Mattis-Bardeen theory of the surface impedance of superconductors is used to investigate the occurrence of such peaks. In order to explain quantitatively the experimental results, it is necessary to take into account the effect of impurities near the sample surface. This is done by changing the boundary condition on the order parameter at the sample surface. The necessary change is similar in magnitude to that needed to explain the penetration-depth measurements presented in a previous paper.

I. INTRODUCTION

In this paper we discuss in detail the peak in the inductive skin depth δ_i as a function of the temperature T which occurred just below T_c in some samples of Ta and Nb. This peak was mentioned in Paper I[1] with regard to the precise determination of T_c for the samples. The theory of Mattis and Bardeen[2] of the surface impedance of superconductors is used to investigate the occurrence of such peaks, and the measurements are compared with the predictions of this theory. In order or explain the experimental results it is necessary to allow for the presence of impuritites near the sample surface. This is accomplished by taking the boundary condition on the Ginzburg-Landau order parameter ψ at the sample surface to be

$$\frac{d\psi}{dx}=\frac{\psi}{b}, \qquad (1)$$

where b, the extrapolation length, is a parameter which characterizes the surface. The value of b which is needed to explain the results is similar to that used in Paper I to explain the dc-penetration-depth measurements. A peak in δ_i has been observed previously at microwave frequencies, and the situation has been reviewed by Waldram.[3] Although the measurements considered by Waldram were in the GHz frequency range, we believe that essentially the same phenomenon was involved.

II. EXPERIMENTAL

Details of the experimental arrangement are given in Paper I. It suffices here to mention that the cylindrical sample was at the center of a coil which formed part of the LC section of a tunnel-diode oscillator with a resonant frequency of about 14 MHz. Changes in the inductive skin depth δ_i of the sample produce changes in the inductance of the coil and hence changes in the resonant frequency which can be measured, and δ_i can therefore be determined. The purification and annealing of the samples under ultrahigh-vacuum conditions is discussed in Paper I. Detailed attention was given for the surface treatment of the samples.

III. THEORY

At a frequency ω the inductive skin depth δ_i is related to the reactive part X of the surface impedance of a sample by

$$X=\mu_0\omega\delta_i \; . \qquad (2)$$

To calculate the surface impedance we ignore for the moment the spatial variation of the order parameter ψ produced by the impurities near the sample surface and assume that ψ is spatially constant and equal to its bulk thermal equilibrium value. For convenience we normalize ψ so that in this spatially homogeneous case it is equal to the energy gap $\epsilon_0(T)$ in the single-particle excitation spectrum. The theory of Mattis and Bardeen is directly applicable to this case. In this theory, first-order perturbation theory is used to calculate the current produced by a transverse-magnetic vector potential $\vec{A}(\vec{r})e^{i\omega t}$. The response is nonlocal and is given by

$$\vec{j}(\vec{r},t)=e^{i\omega t}\frac{e^2N(0)v_F}{2\pi^2\hbar}\int\frac{\vec{R}[\vec{R}\cdot\vec{A}(\vec{r}\,')]}{R^4}\times I(\omega,R,T)e^{-R/l}\,d\vec{r}\,',$$

where $\vec{R}=\vec{r}\,'-\vec{r}$, l is the electronic mean free path, v_F is the Fermi velocity, and $N(0)$ is the density of states at the Fermi surface (per unit volume for one spin direction). The kernel $I(\omega,R,T)$ is given by Mattis and Bardeen.

In the case under consideration here it is more convenient to work with the Fourier components of the vector potential and current and to consider the kernel

$$K(q,\omega)=-\frac{\mu_0e^2N(0)v_F}{2\pi\hbar}\int_0^\infty dR\int_{-1}^1 du\times e^{iqRu}e^{-R/l}(1-u^2)I(\omega,R,T).$$

The current response $\vec{j}(\vec{q})e^{i(\vec{q}\cdot\vec{r}+\omega t)}$ to a transverse vector potential $\vec{A}(\vec{q})e^{i(\vec{q}\cdot\vec{r}+\omega t)}$ is then given by

$$\vec{j}(\vec{q})=-[K(q,\omega)/\mu_0]\vec{A}(\vec{q}).$$

The range of variation of the vector potential and current is δ_i, and therefore the important q values are of order $(\delta_i)^{-1}$. We identify the currents due to superconducting and normal electrons as arising from the real and imaginary parts of $K(q,\omega)$, respectively. Using the notation of Miller,[4] generalized to the case of finite mean free path, we can write $K(q,\omega)$

$$K(q,\omega)=\frac{2\mu_0e^2N(0)v_F}{\hbar q}\bigg(\int_{\epsilon_0-\hbar\omega}^{\epsilon_0}dE[1-2f(E+\hbar\omega)]\,[g(E)G(\beta,\alpha+\gamma)+F(\beta,\alpha+\gamma)]$$
$$-\int_{\epsilon_0}^{\infty}dE[1-f(E)-f(E+\hbar\omega)][g(E)-1]F(\alpha+\beta,\gamma)$$
$$+i\int_{\epsilon_0}^{\infty}dE[f(E)-f(E+\hbar\omega)]\{(g(E)-1)G(\alpha+\beta,\gamma)+[g(E)+1]G(0,\gamma)\}\bigg),\qquad(3)$$

where

$$\alpha=\epsilon_1/\hbar v_Fq,\quad \beta=\epsilon_2/\hbar v_Fq,\quad \gamma=1/ql,$$

$$\epsilon_1=|E^2-\epsilon_0^2|^{1/2},\quad \epsilon_2=[(E+\hbar\omega)^2-\epsilon_0^2]^{1/2},$$

$$g(E)=(E^2+\hbar\omega E+\epsilon_0^2)/\epsilon_1\epsilon_2,\quad f(E)=1/(e^{E/K_BT}+1)\text{ is the Fermi function,}$$

$$F(a,b)=\int_0^\infty dx\,e^{-bx}\frac{\sin ax}{x^3}(\sin x-x\cos x)=\frac{a}{2}+\frac{1+b^2-a^2}{8}\ln\left(\frac{b^2+(1+a)^2}{b^2+(1-a)^2}\right)-\frac{ab}{2}\left(\tan^{-1}\frac{1+a}{b}+\tan^{-1}\frac{1-a}{b}\right)$$

$$G(a,b)=\int_0^\infty dx\,e^{-bx}\frac{\cos ax}{x^3}(\sin x-x\cos x)=-\frac{b}{2}+\frac{1+b^2-a^2}{4}\left(\tan^{-1}\frac{1-a}{b}+\tan^{-1}\frac{1+a}{b}\right)+\frac{ab}{4}\ln\left(\frac{b^2+(1+a)^2}{b^2+(1-a)^2}\right).$$

In writing $K(q,\omega)$ in this form we have made two assumptions. First, we have ignored pair breaking which can only occur if $\hbar\omega>2\epsilon_0$. To observe pair breaking in Ta and Nb at 14 MHz would involve working within 1 μK of T_c, whereas the phenomena discussed in this paper occur at about 1 mk below T_c. The second assumption is that retardation effects may be ignored, which requires $(\hbar\omega\epsilon_0)^{1/2}\delta_i/(\hbar v_F)\ll 1$. This inequality is well satisfied for the results considered here. Calculations including retardation confirm that the effects are negligible. Both these assumptions are also valid for the experiments discussed by Waldram.[3]

The surface impedance Z, which gives the boundary condition for electromagnetic fields at the specimen surface, is defined by

$$Z=E(0)/H(0),$$

where $E(0)$ and $H(0)$ are the complex amplitudes at the specimen surface of the electric and magnetic fields, respectively. In terms of the kernel $K(q,\omega)$, the surface impedance is given by

$$Z=i\omega\mu_0\pi\Big/\int_0^\infty\ln\left(1+\frac{K(q,\omega)}{q^2}\right)dq\qquad(4)$$

for diffuse scattering of electrons at the specimen surface. The calculation of Z for a specular surface was also performed and led to results for the peak in δ_i which were very little different from those for a diffuse surface. The major contribution to the integral over q in Eq. (4) comes from the vicinity of $q=(\delta_i)^{-1}$. The kernel $K(q,\omega)$ was obtained by the numerical integration of (3) and inserted into (4) to obtain Z. The inductive skin depth was then obtained by using (2). The numerical calculations were accurate to about 1 part in 10^3.

Before comparing the experimental results with the calculations, we will discuss the conditions under which a peak in δ_i might be expected to occur. We note that there are two special limits in which the calculation of the surface impedance is relatively straightforward, and we now consider these:

A. Dirty limit $l \ll \delta_i, \xi_0$ [ξ_0 is the BCS coherence length, $hv_F/\pi\epsilon_0(0)$]

In this limit the normal metal exhibits the classical skin effect with a surface impedance

$$Z_N=(1+i)(\omega\mu_0/2\sigma_N)^{1/2}\propto\sigma_N^{-1/2}, \tag{5}$$

where σ_N is the electrical conductivity in the normal state. The electrodynamics of the superconducting state is also local in this limit, and the surface impedance Z_S is given by an equation similar to (5) in which the conductivity σ_N is replaced by an effective conductivity σ_S of the superconducting state. Z_S is therefore given by

$$Z_S/Z_N=(\sigma_S/\sigma_N)^{-1/2}. \tag{6}$$

σ_S is in general complex and is related to the zero-q value of $K(q,\omega)$ by

$$\sigma_S=\sigma_1-i\sigma_2=K(0,\omega)/i\omega\mu_0, \tag{7}$$

where σ_1 and σ_2 are the real and imaginary parts of σ_S and are given by

$$\frac{\sigma_1}{\sigma_N}=\frac{2}{\hbar\omega}\int_{\epsilon_0}^{\infty}[f(E)-f(E+\hbar\omega)]g(E)dE,$$

$$\frac{\sigma_2}{\sigma_N}=\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega}^{\epsilon_0}[1-2f(E+\hbar\omega)]g(E)dE$$

$$\left(\cong\frac{\pi\epsilon_0}{\hbar\omega}\tanh\frac{\epsilon_0}{2K_BT}\quad\text{for } \hbar\omega\ll\epsilon_0\right).$$

At low frequencies $[\hbar\omega\ll\epsilon_0(0)]$ as the temperature is lowered through T_c both σ_1/σ_N and σ_2/σ_N increase as ϵ_0 increases. As discussed by Bardeen and Schrieffer,[5] the increase in σ_1/σ_N from unity is due to the very large density of states for excitations just above the energy gap in a BCS superconductor. This increase would correspond to an increasing density of normal electrons in any simple two-fluid model. The increase of σ_2/σ_N from zero corresponds to the presence of an increasing density of superconducting electrons. The locality of the electrodynamics enables σ_2 for $\hbar\omega\ll\epsilon_0$ to be related simply to the dc penetration depth λ:

$$\sigma_2\lambda^2=1/\omega\mu_0=2\sigma_N\delta_{iN}^2, \tag{8}$$

where δ_{iN} is the inductive skin depth in the normal state and we have used (2) and (5) to obtain the last equality. Figure 1 shows the rise in σ_1/σ_N and σ_2/σ_N with increasing ϵ_0 just below T_c for a BCS superconductor with $\hbar\omega=0.00015K_BT_c$, which corresponds to the case of Ta at 14 MHz. Bearing in mind the difference between the scales used for σ_1/σ_N and σ_2/σ_N, it can be seen that the rise in σ_2/σ_N is very much bigger than that in σ_1/σ_N in this case.

The sign of the change $\Delta\delta_i$ in δ_i on decreasing T from T_c will determine whether there is a peak in δ_i. From (6) we can easily show that

$$\Delta\delta_i=\frac{\delta_{iN}}{2}\left[\frac{\sigma_2}{\sigma_N}-\Delta\left(\frac{\sigma_1}{\sigma_N}\right)\right]\quad\text{for}\quad\left|\frac{\sigma_S-\sigma_N}{\sigma_N}\right|\ll1, \tag{9}$$

where $\Delta(\sigma_1/\sigma_N)$ is the change in σ_1/σ_N. The condition for a peak is then that the increase in σ_2/σ_N should dominate that in σ_1/σ_N, and from Fig. 1 this is seen to be the case at low frequencies $[\hbar\omega\ll\epsilon_0(0)]$. As Waldram[3] points out, σ_1/σ_N initially rises as ϵ_0 for small ϵ_0, whereas σ_2/σ_N rises as ϵ_0^2, as can be seen from Fig. 1. For sufficiently small ϵ_0 therefore the change in σ_1/σ_N always dominates. To see this effect, however, would involve working with values of ϵ_0 which are much smaller than in the present work. We see from (9) that it

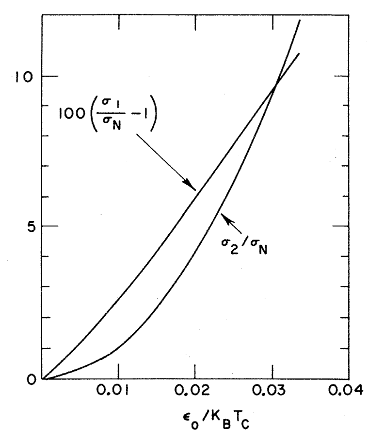

FIG. 1. Conductivity ratios σ_1/σ_N and σ_2/σ_N as functions of ϵ_0/K_BT_c near T_c for a superconductor with $\hbar\omega/K_BT_c=0.00015$. (Note the difference between the scales for σ_1/σ_N and σ_2/σ_N.)

is the superelectrons which cause the peak in δ_i, and this seems to be true generally. If we ignore the small normal electron contribution then (9) is essentially the first term in the expansion of $\Delta\delta_i/\delta_{iN}$ as a power series in $(\delta_{iN}/\lambda)^2$, which is a small quantity just below T_c because λ diverges as $T\to T_c$. Using (8) we can write (9)

$$\frac{\Delta\delta_i}{\delta_{iN}}=\frac{\delta_{iN}^2}{\lambda^2} \quad \text{or} \quad \frac{d\delta_i}{dT}=-\frac{2\delta_{iN}^3}{\lambda^3}\frac{d\lambda}{dT} . \tag{10}$$

Recalling that δ_{iN} is half the classical skin depth, we find that this last result is Eq. (17) of Paper I, from which it is easy to see that the decrease in λ as T falls through T_c implies an increase in δ_i. That δ_i must go through a maximum and eventually decrease as T falls still further can easily be seen because at low temperatures $\sigma_2 \gg \sigma_1$ and, as noted in Paper I, δ_i becomes very close to the dc penetration depth $\lambda(T)$ which is less than δ_{iN}. Indeed as $T\to 0$ we have in the dirty limit

$$\frac{\lambda(0)}{\delta_{iN}}=\left(\frac{2}{\pi}\frac{\hbar\omega}{\epsilon_0(0)}\right)^{1/2} \ll 1 \quad \text{at low frequencies.} \tag{11}$$

B. Limit $\delta_i \ll l, \xi_0$

In this limit the normal metal exhibits the extreme anomalous skin effect with a surface impedance

$$Z_N=(1+\sqrt{3}i)\left(\frac{\sqrt{3}}{16\pi}\frac{\omega^2\mu_0^2 l}{\sigma_N}\right)^{1/3} \propto \left(\frac{\sigma_N}{l}\right)^{-1/3} .$$

This expression is obtained if we assume that for all q of importance in the integral of Eq. (4) we have $ql \gg 1$. We can then approximate $K(q,\omega)$ by its limiting form at high ql, which is

$$\lim_{ql\to\infty} K_N(q,\omega)=3\pi i\mu_0\omega\sigma_N/4ql . \tag{12}$$

In the superconducting state, the large-q form of $K(q,\omega)$ differs only in that σ_N is replaced by σ_S, where σ_S is the effective conductivity in the superconducting state defined by (7). The surface impedance in the superconducting state is therefore given by

$$Z_S/Z_N=(\sigma_S/\sigma_N)^{-1/3} .$$

This result bears such a strong resemblance to the corresponding result [Eq. (6)] in the dirty limit that the rest of the results for limit B are also very similar. Corresponding to Eq. (9) we have

$$\Delta\delta_i=\frac{\delta_{iN}}{3}\left[\frac{1}{\sqrt{3}}\frac{\sigma_2}{\sigma_N}-\Delta\left(\frac{\sigma_1}{\sigma_N}\right)\right] \quad \text{for} \left|\frac{\sigma_S-\sigma_N}{\sigma_N}\right| \ll 1;$$

so the condition for a rise in δ_i below T_c is that $(1/\sqrt{3})\sigma_2/\sigma_N > \Delta(\sigma_1/\sigma_N)$, which is seen to be true from Fig. 1 for a superconductor at low frequencies.

Corresponding to Eq. (8) we have

$$\frac{\sigma_2}{l}\lambda^3=\frac{\sqrt{3}}{2\pi\omega\mu_0}=\frac{8}{3\sqrt{3}}\frac{\sigma_N}{l}\delta_{iN}^3 ,$$

and corresponding to Eq. (10) we find

$$\frac{\Delta\delta_i}{\delta_{iN}}=\frac{8}{27}\frac{\delta_{iN}^3}{\lambda^3} \quad \text{or} \quad \frac{d\delta_i}{dT}=-\frac{8}{9}\frac{\delta_{iN}^4}{\lambda^4}\frac{d\lambda}{dT} .$$

As in the dirty limit, δ_i approaches λ at low temperatures and must therefore drop below δ_{iN}. As $T\to 0$, corresponding to Eq. (11), we find

$$\frac{\lambda(0)}{\delta_{iN}}=\left(\frac{8\hbar\omega}{\sqrt{3}\pi\epsilon_0(0)}\right)^{1/3} \ll 1 \quad \text{at low frequencies.}$$

That there should be a peak in δ_i in the two very different limits A and B discussed above suggests that a peak might be observed in any limit. That this is not so can be seen by considering a third limit.

C. Limit $\xi_0 \ll \delta_i$

This limit is a good approximation for Ta and Nb at 14 MHz. In this limit the electrodynamics of the superconducting electrons is local so the real part of $K(q,\omega)$ at low frequencies is just $1/\lambda^2$, as in limit A. If the electronic mean free path is very short, then the electrodynamics of the normal electrons is also local and limit A is simultaneously obtained. As the electronic mean free path increases the normal electrons become nonlocal and limit A no longer applies. In both limits A and B the normal electrons behave essentially as in the normal state but with a conductivity σ_1 rather than σ_N. We assume the same is true in limit C, and $K(q,\omega)$ is given by

$$K(q,\omega)=(\sigma_1/\sigma_N)K_N(q,\omega)+1/\lambda^2 , \tag{13}$$

where $K_N(q,\omega)$ is the kernel in the normal state. This form of $K(q,\omega)$ may be inserted into Eq. (4) to obtain the surface impedance and hence δ_i. Near T_c, λ is large; so the real part of $K(q,\omega)$ is much smaller than the imaginary part. $\Delta\delta_i/\delta_{iN}$ can then be obtained as a power series in δ_{iN}^2/λ^2, and (10) gives the first term in this series in the dirty limit. In the limit $ql\to\infty$, $K_N(q,\omega)$ in Eq. (13) takes such a simple form [see Eq. (12)] that the integral in Eq. (4) may be performed exactly, without recourse to a power series. The solution is, however, complicated, and the power series is more useful from our point of view. As the electronic mean free path is increased the coefficient of the term in δ_{iN}^2/λ^2 decreases and vanishes for $l \gg \delta_{iN}$. The leading term in the power series is then in δ_{iN}^4/λ^4 and we find

$$\Delta\delta_i/\delta_{iN}=-0.50\delta_{iN}^4/\lambda^4-\tfrac{1}{3}\Delta(\sigma_1/\sigma_N) ,$$

where the second term which is due to the normal electrons is sufficiently small that our assumption as to the behavior of the normal electrons is not crucial. In this limit of $\xi_0 \ll \delta_{iN} \ll l$, therefore, $\Delta\delta_i$ is negative and there is no peak. This is confirmed by the computer calculations and explains why no peak was observed in the purest Ta specimens and also in some single-crystal specimens of Sn and In which were also studied. Presumably this also explains why Waldram observed a peak at 3 GHz in dirty Sn specimens but no peak in clean Sn specimens; but note that in clean Sn at 3 GHz we have $\delta_{iN} \sim \xi_0$; so limit C is not a good approximation in this case.

The conclusions to be drawn from the consideration of the three special limits discussed above are that in a superconductor for which the extreme anomalous skin depth in the normal state $\delta_{N\infty}$ is less than ξ_0 a peak in δ_i is likely to be observed whatever the value of l. Such a material would pass from limit B to limit A as l is decreased. On the other hand, superconductors for which $\delta_{N\infty} > \xi_0$ such as Ta and Nb at 14 MHz are unlikely to exhibit a peak in δ_i for large l but should exhibit a peak as l is decreased.

IV. COMPARISON OF THEORY AND EXPERIMENT

The phenomenon is illustrated by the transition for a Ta specimen shown in Fig. 2. Figure 3 shows a more expanded version of the peak in two Ta specimens of resistance ratios (RR) 270 and 410. (We define RR to be the ratio of the resistivity at 300 °K to that at T_c.) Figure 4 shows the peak in two Nb specimens of RR 170 and ~1000 (estimated). We note that the peak height decreases as l increases, in agreement with the conclusions of Sec. III. If we ignore for the moment the spatial variation of the order parameter ψ that is produced by the impurities near the sample surface and assume that ψ is spatially constant and equal to its BCS

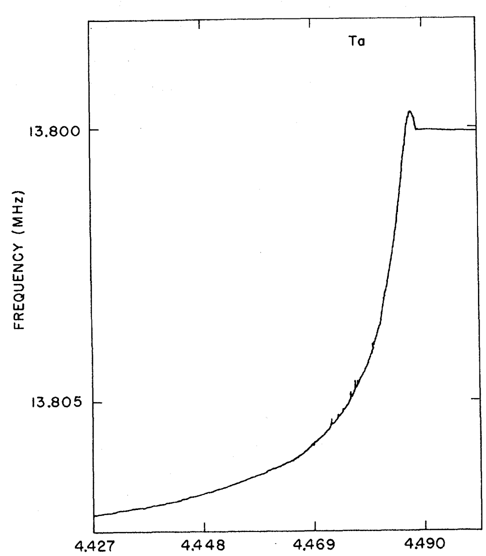

FIG. 2. Experimental data indicating variation of δ_i with T for a Ta specimen with a resistance ratio of 600.

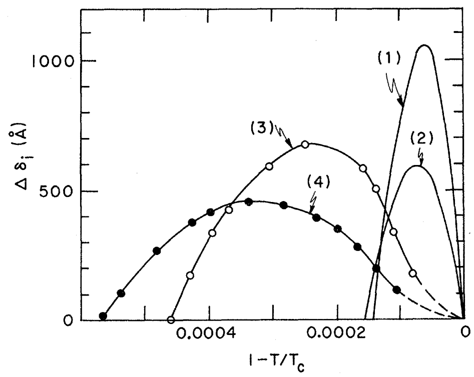

FIG. 3. Experimental results of $\Delta\delta_i$ as a function of $1-T/T_c$ for two Ta specimens of RR 270 and 410. Also shown are the theoretical results of the theory of Mattis and Bardeen if no allowance is made for the variation in ψ near the sample surface. Curves 1 and 3 are theory and experiment, respectively, for RR 270, and curves 2 and 4 are theory and experiment, respectively, for RR 410.

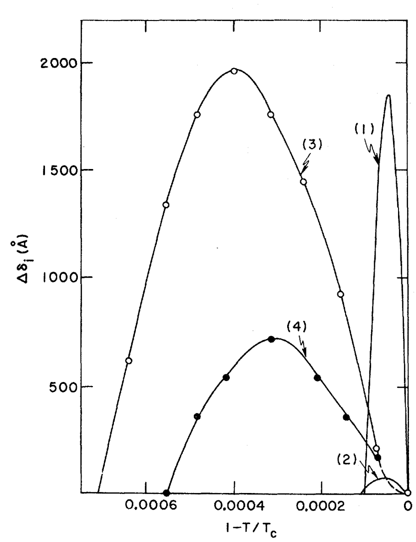

FIG. 4. Experimental results of $\Delta\delta_i$ as a function of $1-T/T_c$ for two Nb specimens of RR 170 and ~1000. Also shown are the theoretical results of the theory of Mattis and Bardeen if no allowance is made for the variation in ψ near the sample surface. Curves 1 and 3 are theory and experiment, respectively, for RR 170, and curves 2 and 4 are theory and experiment, respectively, for RR ~ 1000.

equilibrium value $\epsilon_0(T)$, we may use the theory of Mattis and Bardeen which is described at the beginning of Sec. III to calculate the theoretical curves for δ_i as a function of T. These are also plotted in Figs. 3 and 4. The theoretical curves display the expected qualitative dependence on l. For the Ta specimens the theoretical peaks are slightly higher than the experimental ones. For the Nb specimen with a RR of 170 the heights of the theoretical and experimental peaks are very similar but the height of the theoretical peak for a specimen of RR 1000 is very much less than the experimental peak for the specimen of RR ~ 1000. For both Nb and Ta the theoretical peaks occur much closer to T_c than the experimental peaks. We believe that these discrepancies are due to two factors: an uncertainty in the values of $N(0)$, v_F, and l to be used in the theoretical calculation and the spatial variation of ψ produced by the impurities near the sample surface.

The values of $N(0)$ and v_F which were used in the theoretical calculations were obtained from the measured values of σ/l $[=\frac{2}{3}e^2N(0)v_F]$ and $\lambda_L^2(0)\{=3/[2\mu_0e^2N(0)v_F^2]\}$ for Ta and Nb. Obtaining $N(0)$ and v_F in this way ensures that the theory correctly predicts for pure Ta and Nb the normal-state skin depth for T just above T_c, which depends only on σ/l, and the superconducting penetration depth for T just below T_c, which depends only on $\lambda_L(0)$. l was obtained from the RR of the specimen and the measured value of σ/l. The resistivities of Ta and Nb at room temperature were assumed to be 13.1 and 14.5 $\mu\Omega$ cm, respectively.[6] σ/l was obtained from measurements of the normal-state inductive skin depth δ_{iN} in different specimens. Figure 5 shows a plot of $1/\delta_{iN}^2$ against the conductivity for the specimens. The theoretical curves are obtained from the theory of the anomalous skin effect[7] by using values of σ/l of 1.4, 1.7, and 2.0×10^{15} $\Omega^{-1}\text{m}^{-2}$. We deduce that the value of σ/l for both Ta and Nb is of order 1.7×10^{15} $\Omega^{-1}\text{m}^{-2}$. This value of σ/l for Ta agrees with that used by Auer and Ullmaier.[8] For Nb, this value of σ/l is intermediate between that used by Auer and Ullmaier (2.14×10^{15} $\Omega^{-1}\text{m}^{-2}$) and that used by Hopkins and Finnemore[9] (0.92×10^{15} $\Omega^{-1}\text{m}^{-2}$). $\lambda_L(0)$ values for Ta and Nb were taken from Paper I to be 350 and 315 Å, respectively.

From $\lambda_L(0)$ and σ/l we obtain $N(0)=2.6$ and 2.1×10^{47} $\text{J}^{-1}\text{m}^{-3}$ for Ta and Nb, respectively, to be compared with the higher values obtained from specific-heat measurements[10] of 4.2 and 5.6×10^{47} $\text{J}^{-1}\text{m}^{-3}$. We obtain $v_F=3.8$ and 4.7×10^5 m sec^{-1} for Ta and Nb, respectively. From v_F and the measured value of T_c it is possible to calculate ξ_0 and hence the Ginzburg-Landau parameter κ_0 of the pure metal. This gives $\kappa_0=0.29$ and 0.44 for

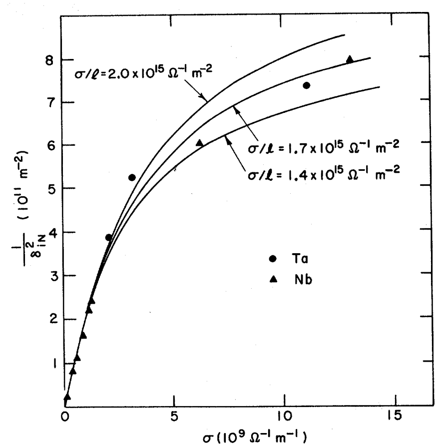

FIG. 5. Experimental results of normal-state skin depth at T_c as a function of the electrical conductivity for a variety of Ta and Nb specimens. The three theoretical curves are from the theory of the anomalous skin effect and correspond to values of σ/l of 1.4, 1.7, and $2.0\times10^{15}\Omega^{-1}\text{m}^{-2}$.

Ta and Nb, respectively, lower than the experimental values of 0.35[8,11] and 0.76.[8,12] Such discrepancies as these are always likely when, as here, a pseudo-free-electron model of a metal is used in which the normal state is characterized by just two parameters $N(0)$ and v_F. These discrepancies can only be overcome by using a more complicated theory which includes the detailed band structure of the metal and many-body effects such as the electron-phonon interaction.

The uncertainty in the values to be used for $N(0)$, v_F, and l introduces a large uncertainty into the height of the peak in δ_i. To illustrate this we show in Fig. 6 how the theoretical peak for Nb is affected by changing l but keeping $N(0)$ and v_F constant.

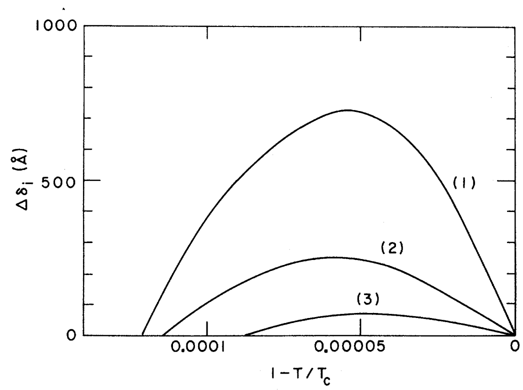

FIG. 6. Theoretical curves for $\Delta\delta_i$ for Nb as a function of $1-T/T_c$ for values of RR of 400, curve 1; RR of 700, curve 2; and RR of 1000, curve 3. No allowance is made for the variation of ψ near the sample surface.

This is equivalent to changing the RR, and Fig. 6 shows curves for values of RR of 400, 700, and 1000. It can be seen that the height of the peak is dramatically changed but that the temperature at which it occurs is only slightly affected. We therefore tentatively associate much of the discrepancy between the heights of the experimental and theoretical peaks with the large uncertainties in the parameters discussed above, in particular the uncertainty in σ/l for Nb. We must, however, search for another explanation for the discrepancies in the widths of the theoretical and experimental peaks.

This explanation is easily found. In Paper I, in order to explain dc-penetration-depth measurements in the same Ta and Nb specimens, it was necessary to include the effects of the impurities near the sample surface by using the boundary condition (1) for the Ginzburg-Landau order parameter ψ at the sample surface. On solving the Ginzburg-Landau equation for ψ with the boundary condition (1), we find that ψ is depressed near the surface below its value $\epsilon_0(T)$ deep in the bulk. Note that the peak in δ_i occurs so close to T_c that the use of the Ginzburg-Landau equation gives only a small error. The value ψ_0 at the surface is given by

$$\psi_0 = \tfrac{1}{2}\epsilon_0(T)[(c^2+4)^{1/2} - c] ,$$

where $c = \sqrt{2}\,\xi(T)/b$ and $\xi(T)$ is the Ginzburg-Landau coherence length, which in the pure material is

$$\xi(T) = 0.74\,\xi_0(1 - T/T_c)^{-1/2} .$$

If we assume that b is only weakly temperature dependent then as $T \to T_c$, $\xi(T)$ diverges, c becomes large, and ψ_0 is depressed considerably below $\epsilon_0(T)$. For $\psi_0 \ll \epsilon_0(T)$ we may write

$$\psi_0/\epsilon_0(0) = 1.23(1 - T/T_c)b/\xi(0) . \qquad (14)$$

Figure 7 shows a plot of $\psi_0/\epsilon_0(0)$ against $1 - T/T_c$ for various values of $\xi(0)/b$. These curves are valid for any BCS superconductor. Also plotted in Fig. 7 is $\epsilon_0(T)/\epsilon_0(0)$, and it can be seen that as T falls below T_c, ψ_0 rises more slowly than $\epsilon_0(T)$. We might expect therefore the experimental peaks to occur at lower temperatures than the theoretical ones, and Figs. 3 and 4 show this to be the case.

It is possible to use the discrepancies in the widths of the experimental and theoretical peaks to derive values of $\xi(0)/b$ for the specimens of Figs. 3 and 4 as follows. Provided that $\xi(T) \gg \delta_i$, which is true in the relevant temperature range, we can assume that the value of ψ which determines the surface impedance is ψ_0. This quantity is the effective energy gap in the specimen. In the region very close to T_c, the only parameter in

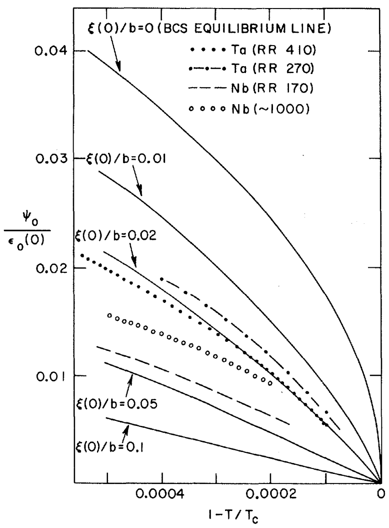

FIG. 7. Continuous lines are theoretical plots of $\psi_0/\epsilon_0(0)$ as functions of $1 - T/T_c$ for various values of $\xi(0)/b$, where ψ_0 is the order parameter at the surface of the specimen and b is the extrapolation length which appears in Eq. (1). The discontinuous curves are for the specimens indicated and were obtained from the experimental results of Fig. 3 and 4 in the manner described in the text.

the theory of Mattis and Bardeen which is varying significantly with temperature is the energy gap ϵ_0. The inductive skin depth in this temperature region is then a function only of the energy gap in the specimen. A comparison of the experimental and theoretical curves of Figs. 3 and 4 therefore enables us to deduce ψ_0 as a function of T for each specimen. To overcome the previously discussed problem of the discrepancy between the height of the theoretical and experimental peaks, we normalized the theoretical peaks to make the heights equal.

The results obtained for $\psi_0/\epsilon_0(0)$ are shown on Fig. 7. For the two Ta specimens the results are near to the theoretical curve for $\xi(0)/b = 0.02$, which was the value of $\xi(0)/b$ used in Paper I to explain the dc-penetration-depth measurements in pure Ta. For the pure Nb specimen a value of $\xi(0)/b$ of about 0.03 is indicated. A large renormalization of the theoretical results was required for this specimen in order to make the heights of the experimental and theoretical peaks equal. This casts some doubt upon the results for this specimen. However, the value of $\xi(0)/b$ is in reasonable agreement with the value of 0.02 from Paper I for pure Nb specimens and also with the value of 0.02 obtained by Hopkins and Finnemore[9] in a pure Nb specimen. This latter agreement is perhaps surprising in that b might be expected to depend on the detailed nature of the specimen surface and therefore to vary from one specimen to another, particularly if the two specimens are prepared by different workers. For our dirtier Nb specimen we obtain $\xi(0)/b \cong 0.04$, which is apparently less than the value for this specimen from Paper I. However, this disagreement is to be expected since the values of $\xi(0)/b$ obtained from dc-penetration-depth measurements are likely to be in error for the dirtier Nb specimens for reasons which involve problems of surface roughness and inhomogeneities, as discussed in Paper I. In these present considerations where $\lambda > \delta_i$ and both quantities are probably greater than the distance which characterizes surface roughness or inhomogeneities more ideal values of $\xi(0)/b$ are likely to be obtained. For a Nb specimen of RR 150 Hopkins and Finnemore obtained $\xi(0)/b = 0.029$, which is much closer to the value obtained in our specimen from Fig. 7. The results shown in Fig. 7 are a striking verification of the need to take into account the effect of the sample surface on the measurement of the superconducting properties that are restricted to the surface such as the ac skin depth.

From Eq. (14) we see that ψ_0 rises from zero with a finite slope for $T < T_c$, unlike $\epsilon_0(T)$ which rises with infinite slope. It follows from this and the theory outlined in Sec. III that δ_i as a function of T should have zero slope at T_c. This leads to an uncertainty in the determination of T_c for each specimen of about 0.2 mK. The corresponding uncertainty in the experimental values of $\Delta\delta_i$ for T near T_c is indicated in Figs. 3 and 4 by the use of dotted lines. In practice T_c was adjusted to obtain as good a fit as possible between experiment and theory in Fig. 7. This uncertainty in T_c is sufficiently small that no significant error was involved in using in Paper I the value of T_c determined by the method described there.

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