Spin-wave relaxation and phenomenological damping in ferromagnetic resonance

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Relaxation rates for the uniform precession mode in ferromagnetic resonance, with general elliptical polarization, have been calculated for several microscopic scattering processes using the spin-wave formalism. These results are compared with the widely used phenomenological formulations for ferromagnetic resonance. The results demonstrate in relatively general terms the specific features of the Landau-Lifshitz and Gilbert phenomenological formulations on the one hand, and of what may be called "intrinsic" confluence processes in the microscopic formulation. These formulations are consistent with the assumption of an intrinsic damping parameter describing the motion of the magnetization vector under sufficiently general conditions. The two-magnon process and the Bloch-Bloembergen phenomenological description of damping in ferromagnetic resonance are not consistent with such an assumption.

I. INTRODUCTION

The form of the ferromagnetic equation of motion,

$$\frac{d\vec{M}}{dt} = -|\gamma|(\vec{M}\times\vec{H}) - \vec{R}, \tag{1a}$$

where $\vec{M}$ is the vector magnetization, γ is the electron gyromagnetic ratio, $\vec{H}$ is the total internal field, and $\vec{R}$ is a phenomenological relaxation term, has often been discussed from a geometric viewpoint involving the exact direction of $\vec{R}$ and the physical difference between processes conserving M_0, the absolute value of $\vec{M}$, or M_z, the component of $\vec{M}$ along the field.[1-3] These two types of relaxation are usually described by,

$$\vec{R}^{LL} = (\alpha|\gamma|/M_0)\vec{M}\times(\vec{M}\times\vec{H}), \tag{1b}$$

$$\vec{R}^{BB} = (\nu/H_S^2)\vec{H}_S\times(\vec{M}\times\vec{H}). \tag{1c}$$

The superscripts LL and BB refer to the Landau-Lifshitz[4] and the original Bloch-Bloembergen[5] formulations of ferromagnetic relaxation; α and ν denote phenomenological relaxation parameters. In the BB form, $\vec{H}_S$ denotes the static internal field. A modified form of $\vec{R}^{BB}$, in which the relaxation is toward the instantaneous internal field $\vec{H}$, avoids the problem of negative loss for anti-Larmor excitations[6,7]:

$$\vec{R}^{MB} = (\nu/H^2)\vec{H}\times(\vec{M}\times\vec{H}). \tag{1d}$$

For completeness, it is also important to mention the Gilbert formulation,[8]

$$\vec{R}^{G} = (\alpha/M_0)\vec{M}\times\frac{d\vec{M}}{dt}, \tag{1e}$$

which yields results identical to $\vec{R}^{LL}$ for $\alpha^2 \ll 1$.[11] For physical reasons, the Gilbert formulation appears preferrable.[9]

Apart from the differences in the behavior of M_z for these different relaxation forms and the related implications concerning nonlinear effects, the relaxation terms yield quite different expressions for the ferromagnetic resonance linewidths, which are obtained from the susceptibility solution of Eq. (1a), linearized in the transverse components of the time-dependent precessional magnetization. For an ellipsoidal sample with demagnetizing factors (N_x, N_y, N_z), and the static external field along the z-directed principal axis, the frequency-swept half-linewidths (in angular frequency units) for the uniform precession (UP) mode are:

$$(\tfrac{1}{2}\Delta\omega)^{LL} = \alpha\omega_0 P_A, \tag{2a}$$

$$(\tfrac{1}{2}\Delta\omega)^{BB} = \nu, \tag{2b}$$

$$(\tfrac{1}{2}\Delta\omega)^{MB} = (\nu/\omega_H)\omega_0 P_A. \tag{2c}$$

Here, ω_0 is the Kittel resonance frequency,

$$\omega_0 = (\omega_S^2 - \omega_A^2)^{1/2}, \tag{3a}$$

$$\omega_{S,A} = \tfrac{1}{2}[(\omega_H + N_x\omega_M) \pm (\omega_H + N_y\omega_M)], \tag{3b}$$

$$\omega_H = |\gamma|(H_0 - 4\pi N_z M_0), \tag{3c}$$

with $\omega_M = |\gamma| 4\pi M_0$. The quantity P_A is an ellipticity factor,

$$P_A = \frac{\partial\omega_0}{\partial\omega_H} = [1 - (\omega_A/\omega_S)^2]^{-1/2}. \tag{4}$$

The conventional field swept linewidth ΔH is related to $\Delta\omega$ according to

$$|\gamma|(\tfrac{1}{2}\Delta H) = (\tfrac{1}{2}\Delta\omega)/P_A. \tag{5}$$

The demagnetizing factors satisfy $N_x + N_y + N_z = 1$.

Equations (2) define one-half of the usual half-power linewidth in angular frequency units; this convention will make possible a direct comparison with relaxation-rate expressions.

In the present work, we are concerned only with these low-power resonance linewidths, and their variation with the parameters characterizing a particular experiment on a given material, such as frequency, field, and sample geometry.

The importance of the ellipticity factor P_A has been pointed out previously in connection with linewidth data for Ni-Fe films.[10] It has also been noted[10,11] that the Bloch-Bloembergen damping leads to a dependence of the linewidth on sample geometry different from that described by Eq. (2a). On the other hand, Callen[1] has proposed that the phenomenological damping parameters themselves should depend on sample geometry; his argument was based on a comparison with theoretical spin-wave transition rates. However, recent calculations of the damping in metals[12-14] indicate (in agreement with experiment) that the physical relaxation is consistent with the assumption of an intrinsic damping parameter α, which is constant except for a possible temperature dependence. Part of the argument was based directly on comparison of the theory with Eq. (2a), which indicated that the factor P_A should appear even in microscopic calculations.[12]

In the present study, the uniform precession decay rate has been examined as a function of the ellipticity of the UP polarization for several simple scattering models, using standard second-order-perturbation-theory methods in calculating transition probabilities.[1,3] It is explicitly assumed that the scattering exhibits, on the average, at least rotational symmetry.

The results show that: (i) in general, *both* the UP decay rate and the corresponding phenomenological parameters depend on the sample geometry, the UP polarization, and the frequency. An example of this behavior is the well-known two-magnon process, where the decay rate is not consistent with the assumption of a constant (i.e., intrinsic) damping parameter in any of the above formulations. (ii) In the case of the general "confluence" process, the UP decay rate depends on an *intrinsic* (material) parameter α and on the field and frequency in exact accord with Eq. (2a), provided that the participant excitations are unaffected by the magnetic field and that ω_0 is small compared with their characteristic frequencies. This simple result reflects the qualitative features of the known damping mechanisms involving electron scattering, such as those studied in metals,[12-14] the fast-relaxing impurity mechanism in ferrites,[15] and, in part, the multi-particle magnon-phonon processes.[16] It is not rigorously applicable to three-magnon confluence relaxation.

II. MICROSCOPIC RELAXATION CALCULATION

The formulation of the relaxation problem follows essentially the approach of Sparks.[3] The interaction of the UP with other excitations is assumed to be of the form,

$$\mathcal{H}_I = D^\dagger b_0 + D b_0^\dagger \tag{6}$$

where the $b_0^\dagger$ and b_0 are creation and destruction operators of circularly polarized magnons, and the $D^\dagger, D$ are composed of operators of other excitations. The actual uniform precession eigenmodes in an ellipsoidal sample are represented by the operators $c_0^\dagger$ and c_0 obtained from diagonalization of the UP Hamiltonian (in frequency units),

$$\mathcal{H}_0 = \omega_S b_0^\dagger b_0 + \tfrac{1}{2}\omega_A(b_0^\dagger b_0^\dagger + b_0 b_0)\,, \tag{7}$$

by the transformation[3,17]

$$c_0 = \rho b_0 + \sigma b_0^\dagger\,, \tag{8}$$

where ρ and σ are real coefficients satisfying,

$$\rho^2 - \sigma^2 = 1\,, \tag{9a}$$

$$\rho^2 + \sigma^2 = P_A\,. \tag{9b}$$

The parameters $\omega_{S,A}$ and the ellipticity factor P_A are the same as given above.

The UP decay rate is calculated from the transition-rate equation,

$$\frac{dn_0}{dt} = p^+ - p^- \tag{10}$$

where the "up" and "down" (p^+ and p^-, respectively) rates are;

$$p^\pm = 2\pi \sum_{i,f} |\langle n_0 \pm 1, n_f|\mathcal{H}_I|n_0, n_i\rangle|^2\, \delta(\omega_f - \omega_i \pm \omega_0)\,. \tag{11}$$

Here (i,f), (ω_i, ω_f), and (n_i, n_f) denote the initial and final eigenstates, frequencies, and occupation numbers of the system, n_0 is the occupation number of the uniform precession eigenmode, and $\mathcal{H}_I$ is written in terms of the $c_0^\dagger$ and c_0.

For a general confluence process as shown in Fig. 1, the interaction D is given by,

$$D = \sum_{k,k'} f_{kk'}\, a_{k'}^\dagger a_k\,. \tag{12}$$

The $a_k^\dagger$ and a_k are either fermion or boson operators for the participant excitations with wave number k and frequency ω_k. The matrix elements $f_{kk'}$ are assumed known, even though their actual calculation is nontrivial and generally involves indirect processes,[12,14] so that the values are

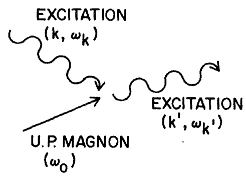

FIG. 1. Schematic illustration of three-particle confluence with one incident UP magnon and two other general excitations.

temperature dependent. The matrix $\{f_{kk'}\}$ need not be Hermitian.

Substituting (6) and (8) into (10) and (11), with the usual assumptions that the occupation numbers $(n_k, n_{k'})$ are equal to their thermal values $(\bar{n}_k, \bar{n}_{k'})$, and that $dn_0/dt = 0$ when n_0 is equal to its thermal value, one obtains

$$\frac{dn_0}{dt} = -(n_0 - \bar{n}_0)(1/\tau_0)\,, \tag{13}$$

where $1/\tau_0$ generally depends on ρ and σ. We assume that the scattering has at least rotational symmetry, i.e., dn_0/dt is invariant under rotational transformations about the field axis. This allows retention of only terms proportional to ρ^2 and σ^2; those involving $\rho\sigma$ crossterms are not invariant. Then, one obtains

$$1/\tau_0 = 2\pi \sum_{k,k'} (\rho^2 |f_{k'k}|^2 + \sigma^2 |f_{kk'}|^2) \times (\bar{n}_k - \bar{n}_{k'})\delta(\omega_{k'} - \omega_k - \omega_0)\,. \tag{14}$$

The factor $\bar{n}_k - \bar{n}_{k'}$ may be expanded in the difference $\omega_k - \omega_{k'}$, at least for $\omega_0 \ll kT$; owing to the δ function it is then equal to $-\omega_0\, d\bar{n}_k/d\omega_k$. Further, if ω_0 is small compared to the ω_k and $\omega_{k'}$ over which the matrix elements and density-of-states factors implied by the summation in Eq. (14) vary appreciably, the sum may be evaluated using $\delta(\omega_{k'} - \omega_k)$. With these assumptions, the result is

$$1/\tau_0 = \alpha_c\, \omega_0 P_A, \tag{15}$$

$$\alpha_c = 2\pi \sum_k \left(-\frac{\partial \bar{n}_k}{\partial k}\right)_k \sum_{k'} |f_{kk'}|^2\, \delta(\omega_{k'} - \omega_k)\,. \tag{16}$$

This is in exact accordance with the phenomenological expression (2a) based on the Landau-Lifshitz (or Gilbert) equation; $1/\tau_0$ is proportional to the frequency ω_0, the ellipticity factor P_A, and a scattering summation α_c, which is an intrinsic parameter under the above assumptions and the additional provision that the excitations (k, ω_k) and the $f_{kk'}$ are not appreciably affected by the magnetic field.

The above conditions are satisfied for electron scattering relaxation,[12-14] fast-relaxing impurity processes,[15] and certain classes of magnon-phonon relaxation. The key ingredient for the extraction of a linear frequency factor is that $\omega_k, \omega_{k'} \gg \omega_0$ is satisfied. The key ingredient for the P_A factor is that the participant excitations *not* be appreciably field affected. Consider the case of magnon confluence where neither is satisfied. The $(a_k, a_k^\dagger)$ then represent circularly polarized magnon operators, which must be transformed to elliptical magnons as are the $(b_0, b_0^\dagger)$. The resulting relaxation-time expression is complicated considerably by this modification, and a simple P_A factor is not obtained. Further, $k \lesssim 10^6$ cm^{-1} for typical magnon confluence (in ferrites, for example), so that $\omega_k \gg \omega_0$ is not satisfied.

As an example of processes where such consistency is not found for any of the damping formulations cited above, consider the magnon-boson process where, generally,

$$D = \sum_k (F_k b_k + G_k b_k^\dagger)\,. \tag{17}$$

In the case that the $(b_k^\dagger, b_k)$ represents magnons (other than UP), they are also elliptically polarized due to dipolar effects. However, for sufficiently short-wavelength spin waves the corresponding Holstein-Primakoff transformation does not involve the effect of sample shape except for a static-field shift in energy; the matrix elements F_k and G_k thus depend on intrinsic properties and on the total static internal field. If the Holstein-Primakoff transformation is done explicitly [expressing D in terms of the elliptically polarized spin wave operators $(c_k, c_k^\dagger)$], an alternative demonstration [to that preceding Eq. (14)] of the disappearance of terms in $\rho\sigma$ in Eq. (18) derived below can be obtained. The source of the G_k terms in Eq. (17) is twofold: (1) in the anisotropy of individual scattering centers (or events),[18] (2) in the above-mentioned transformation for the secondary magnons.[3,17]

Proceeding in the same way as in the preceding case, one obtains the UP decay rate in the form of Eq. (13) with,

$$1/\tau_0 = 2\pi \sum_k (\rho^2 |F_k|^2 + \sigma^2 |G_k|^2)\delta(\omega_k - \omega_0)\,. \tag{18}$$

Obviously, there is no general "ellipticity factor" to be factored out, and no frequency factor. Even in concrete simple models, such as the isotropic approximation to the pseudo-dipolar two-magnon process[18] where it is possible to do the sums over $|F_k|^2$ and $|G_k|^2$ explicitly, the result is not proportional to $P_A = \rho^2 + \sigma^2$ since the two sums are not equal. Moreover, a significant and rather complicated variation of $1/\tau_0$ with frequency and sample geometry is known to be caused by the variation in the density of degenerate magnon states participating in the k sums as these parameters are changed. Equations (17) and (18) apply as well, at least formally, to a magnon-phonon process. As long as the scattering amplitudes are considered proportional to the deformation, $|F_k| = |G_k|$ is satisfied[19] so that $\rho^2 + \sigma^2 = P_A$ does factor out of Eq. (18).

III. DISCUSSION

The observed consistency (or lack thereof) between phenomenological and microscopic results reveals criteria for the definition of relaxation parameters related to different physical processes which may be used in phenomenological formulations for practical calculations. The point of interest is whether any such definition may correctly characterize the general magnetization response, irrespective of the particular experimental parameters. This point has been examined by Callen,[1] who observed that the phenomenological parameters could not be constant if the quantum-mechanical relaxation rates were constant. However, Callen did not investigate the conditions for invariance of $1/\tau_0$. The present results show that one class of microscopic processes and one form of the phenomenological damping, among the simple cases considered, are explicitly consistent even though $1/\tau_0$ is a function of external parameters in the microscopic result.

Concerning the negative results with the Bloch forms of damping on the one hand, and relaxations involving magnetic excitations on the other, one does not intuitively expect to obtain relaxation parameters which are field independent in such cases. On the phenomenological side, the ν parameter in the unmodified Bloch-Bloembergen equation is not expected to be invariant because of known thermodynamic deficiencies in the formulation (such as negative loss for anti-Larmor modes). As to the modified form, the lack of agreement simply shows that this particular modification is not sufficiently sophisticated. Intuitively, the internal field to which $\vec{M}$ relaxes must involve not only $\vec{H}$ but the molecular-field characteristic of the ferromagnetic interaction as well. [Note Gilbert's observation quoted by Wangsness,[20] that the replacement of $\vec{H}$ in Eq. (1c) by the sum of $\vec{H}$ plus the molecular field leads to the Landau-Lifshitz form of Eq. (1b).]

The positive result concerning the LL form and "nonmagnetic" confluence is less trivial. The assumptions which allow the extraction of the frequency and ellipticity factors, in the simple derivation presented above, are also made implicitly in Gilbert's derivation.[8,21] The use of a Rayleigh dissipation function which is not strictly allowed,[22] is approximate and valid only in the low-frequency and low-field limit. Moreover, the particle-hole character of the secondary excitations assumed in the present derivation is a feature common to other viscous phenomena such as electrical conductivity. Similar implications appear in recent work concerning the damping in metals.[13,14]

It is to be emphasized that the variation of M_z is not considered in the present analysis, done in the limit of low power and small excitations. Only the transverse relaxation is considered. From this perspective, the equivalence of longitudinal and transverse relaxation rates implied in the LL formulation is of no direct interest. In fact, the $(a_k^\dagger, a_k)$ excitations in the confluence theory may even represent nonuniform spin excitations, in which case M_z would be conserved in the second-order kinetics. The only point is that the secondary excitations must not be substantially affected by the static field $\vec{H}$. Thus, low-frequency magnons are excluded; others, e.g., Stoner excitations in metals, are not excluded.

IV. CONCLUSION

The above results demonstrate in relatively general terms the specific features of the Landau-Lifshitz and Gilbert phenomenological formulations on the one hand, and of what may be called "intrinsic" confluence processes in the microscopic formulation. These formulations are consistent with the assumption of an intrinsic damping parameter describing the motion of the magnetization vector under sufficiently general conditions. The two-magnon process and the Bloch-Bloembergen phenomenological description of damping in ferromagnetic resonance are not consistent with such an assumption.

Similar implications may be inferred from calculations of the two circular susceptibilities (of the Larmor and anti-Larmor modes) using the methods of linear response theory[7,12-14,18,19] and comparing again with the phenomenological equations. We have chosen to treat the elliptically polarized eigenmodes because the calculations are in a very familiar form, and elliptical-mode resonances are probably the simplest situation in which the role of the anti-Larmor components can be assessed experimentally. Microscopic calculations concerning elliptically polarized eigenmodes have obvious practical implications in that linewidth data may be used to distinguish between various scattering mechanisms.

Finally, it is recognized that the consistency of the "viscous" damping proposed by Gilbert with the "confluence" mechanism is not incidental and on the other hand, other processes involving scattered magnons exhibit features similar to those observed with the two-magnon process. These points will be further developed in separate papers.

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