Oscillatory component of the magnetoconductivity tensor for nondegenerate semiconductor thin films*

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(Received 4 November 1974)

The isothermal magnetoconductivity of a thin nondegenerate semiconducting film in the presence of a magnetic field perpendicular to the plane of the film is shown to exhibit oscillations as a function of magnetic field at constant temperature and oscillations as a function of temperature at constant magnetic field. Comparison of experiment to theory should allow a determination of the effective mass in thin semiconducting films.

INTRODUCTION

Metallic samples having a dimension comparable to the carrier mean free path have resistivities which, at low temperatures, have been observed to be oscillatory with respect to an externally applied magnetic field. Since Sondheimer's prediction[1] of such behavior, on the basis of a solution of Boltzmann's equation, the oscillations have been observed in sodium,[2] aluminum,[3,4] cadmium[5-8] (see Ref. 8 for a comprehensive set of references), gallium,[9] and zinc.[10,11]

The theoretical treatment of Sondheimer contains quantum effects only through the use of the Fermi-Dirac distribution function. The final results for the conductivity components depend on Planck's constant only through the Fermi momentum, so that a purely classical theory might also predict oscillations characterized by a temperature-dependent momentum.

In this paper modifications of the Sondheimer theory appropriate to electrical conduction in a thin nondegenerate semiconductor are explained and results of the calculation are presented. They show a damped, nonsinusoidal behavior as a function of magnetic field at constant temperature. Similarly, for constant magnetic field, oscillations appear as a function of temperature. To our knowledge these size-effect oscillations have not been observed, but in a recent paper, Wieder[12] gives room-temperature data for the mobility of InSb which show thickness dependence for thickness of a few microns. In the last section numerical estimates are made that indicate the oscillations should be detectable.

THEORY

Sondheimer[1] gave the first solution to the problem of the conductivity of a thin film in the presence of a magnetic field normal to the film. Assuming a spherical, quadratic dispersion law, diffuse scattering at the bounding surfaces $z=0, a$ $(H\|z)$, and applying the Fuchs[13] boundary conditions, he found by numerical integration that there is an oscillatory term in the conductivity, periodic in the magnetic field H. Grenier[7] has explicity demonstrated this periodic content by expanding Sondheimer's result for large magnetic field. He finds

$$\begin{aligned}&\tilde{\sigma}_{11}+\tilde{\sigma}_{12}=3nec(H_0^3/H^4)e^{-a/\lambda}(\cos\beta-i\sin\beta),\\&H_0=mv_Fc/ea,\quad \beta=H/H_0\gg 1,\\&\lambda=v_F\tau_F,\quad n=8\pi(mv_F)^3/3h^3.\end{aligned}\tag{1}$$

The tilde in $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{12}$ refers to the oscillatory part of σ_{11} and σ_{12}, the transverse magnetoconductivity and Hall conductivity. The subscript F on v_F and τ_F implies evaluation on the Fermi sphere. This occurs because of the assumption of a highly degenerate electron gas, and thus Eq. (1) is applicable to the case of metals and semimetals at low temperature. Specifically, $\partial f_0/\partial\epsilon=-\delta(\epsilon-\epsilon_F)$ was used in carrying out an integration over energy.

In order to treat the case of a nondegenerate semiconductor one may start with Eq. (1), dropping the F subscript and reinserting the factor $-\partial f_0/\partial\epsilon$. Then using Boltzmann statistics such that

$$\frac{-\partial f_0}{\partial\epsilon}=(kT)^{-1}e^{(\epsilon_F-\epsilon)/kT},\tag{2}$$

the integration over energy may be carried out. One then obtains

$$\begin{aligned}\tilde{\sigma}_{11}=&\frac{8\pi ec}{h^3H^4}\left(\frac{c}{ea}\right)^3\frac{m^6e^{-a/\lambda}e^{\epsilon_F/kT}}{kT}\\&\times\int_0^\infty e^{-\epsilon/kT}v^6\cos\left(\frac{Hea}{mvc}\right)d\epsilon,\end{aligned}\tag{3}$$

where $\tau(\epsilon)$ has been assumed appropriate for acoustic phonon scattering such that $\lambda=v\tau$ is dependent only on temperature. In the above, ϵ_F is

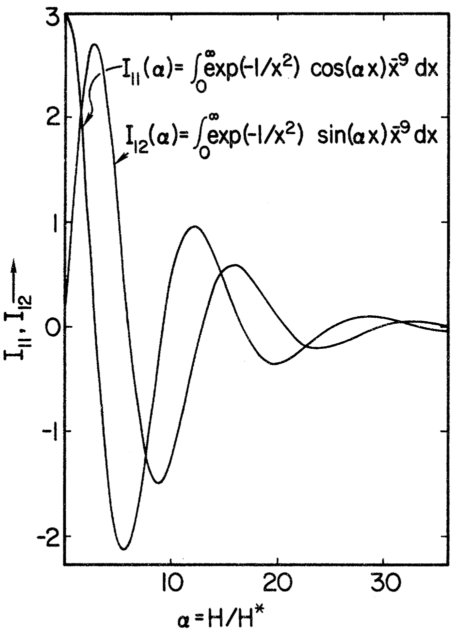

FIG. 1. The oscillatory behavior of $I_{11}(\alpha)$ and $I_{12}(\alpha)$ as determined by numerical integration.

measured from the conduction-band edge, $\epsilon=0$, such that $\epsilon_F \ll 0$.

Define n, x, α, H^* and I_{11} by the following relations:

$$n=2(mkT/2\pi\hbar^2)^{3/2}e^{\epsilon_F/kT}, \tag{4a}$$

$$x=(2kT/m)^{1/2}/v, \tag{4b}$$

$$\alpha=eaH/c(2mkT)^{1/2}=H/H^*, \tag{4c}$$

$$I_{11}(\alpha)=\int_0^\infty e^{-1/x^2}x^{-9}\cos(\alpha x)dx. \tag{4d}$$

In terms of these quantities Eq. (2) becomes

$$\tilde{\sigma}_{11}=(8/\sqrt{\pi})\text{nec}(H^{*3}/H^4)e^{-a/\lambda}I_{11}(\alpha). \tag{5}$$

Now replacing $\cos(\alpha x)$ in Eq. (4c) by $\sin(\alpha x)$ one may define $I_{12}(\alpha)$ as

$$I_{12}(\alpha)=\int_0^\infty e^{-1/x^2}x^{-9}\sin(\alpha x)dx, \tag{6}$$

so that according to Eq. (1)

$$\tilde{\sigma}_{12}=-(8/\sqrt{\pi})\text{nec}(H^{*3}/H^4)e^{-a/\lambda}I_{12}(\alpha). \tag{7}$$

The magnetic field dependence of $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{12}$ is due to the factor H^{-4} and the integrals I_{11} and I_{12}. Figure 1 exhibits these integrals as functions of $\alpha=H/H^*$ as determined by numerical integration. An examination of $I_{11}(\alpha)$ and $I_{12}(\alpha)$ shows that the oscillations are not damped sinusoids.

The similarity of Eqs. (5) and (7) to Eq. (1) is surprising. Other than replacing $\cos(H/H_0)$ and $\sin(H/H_0)$ by $I_{11}(\alpha)$ and $I_{12}(\alpha)$ respectively, one need only replace H_0 by H^* and the factor 3 by $8/\sqrt{\pi}$ in Eq. (1) to obtain the present results. Noting that H_0 and H^* differ by mv_F and $(2mkT)^{1/2}$, one sees that the Fermi radius in momentum space is replaced by the most probable momentum in the corresponding Boltzmann problem.

RELEVANCE TO EXPERIMENT

The treatment above shows that there does exist an oscillatory component of the magnetoconductivity tensor for nondegenerate semiconductors. The pertinent question is whether in fact the relative magnitudes of quantities appearing in Eqs. (5) and (7) predict an effect of significant magnitude. Wieder[12] has studied the zero-magnetic-field size effect in n-InSb films at room temperature. He obtains a value of $\lambda=0.6$ μm and has grown films of this order of thickness by the electron beam method. His specimens of this thickness range have $n\simeq 6\times 10^{16}$ cm^{-3} and conductivity $\sigma_0\simeq 50$ $(\Omega\,\text{cm})^{-1}$. Using $m=0.013m_0$ one obtains $H^*\simeq 1000$ G. Figure 1 shows that for $H=10$ kG such that $\alpha\simeq 10$, $|I_{11}|$ and $|I_{12}|\simeq 1$. Inserting the above values into Eqs. (5) and (7) yields $|\tilde{\sigma}_{11}|\simeq|\tilde{\sigma}_{12}|\simeq 1$ $(\Omega\,\text{cm})^{-1}$ which is approximately 2% of the zero-field value σ_0. Therefore it appears that the effect is significant at room temperature and that by comparison of data to the above theory one may extract a value for the effective electronic mass in thin nondegenerate semiconducting films.

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*Work supported by the North Texas State University Faculty Research Fund.

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