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Anomalous spin-flip lifetime near the Heisenberg-ferromagnet critical point*

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The inverse lifetime, $1/\tau_s$, of spin-flip processes for the conduction electrons in a Heisenberg ferromagnet is calculated in the first Born approximation. It is found that the temperature derivative of $1/\tau_s$ varies as $|T - T_c|^{2\nu-\gamma-1}$ and is negative above T_c and positive below T_c. Comparison is made with the behavior of the electrical resistivity near T_c. The impact upon the superconducting transition temperature is examined.

I. INTRODUCTION

Conduction electrons are scattered in magnetic materials by thermal fluctuations of the localized spins. This process yields an anomalous contribution to the spin-flip scattering cross section of the conduction electrons near the critical temperature T_c. The anomaly arises from the critical-temperature dependence of the spin-spin correlation function which appears in the expression for the differential scattering cross section. In this paper, we study the critical fluctuation singularity in the inverse lifetime $1/\tau_s$ of the spin-flip processes for the conduction electrons. Such a term is important, for example, in the determination of T_c for superconductors either containing magnetic impurities,[1] or involved in proximity-effect sandwiches.[2]

The expression for $1/\tau_s$ in first Born approximation contains the spin-spin correlation function. Near T_c the dominant contribution comes from long-range fluctuations, i.e., small values of momentum transfer. This is because the Fourier transform of the correlation function $g_{\vec{k}}$ appears in the expression for $1/\tau_s$, but is multiplied by k^{-1} as compared to the average of $g_{\vec{k}}$. As a result, we shall show that $d(1/\tau_s)/dT$ varies as $\mp|\Delta T|^{2\nu-\gamma-1}$ $[\Delta T=(T-T_c)/T_c]$ for $T \gtrless T_c$, where above T_c it is negative and below positive. The appearance of the spontaneous magnetization below T_c does not change the singularity in $d(1/\tau_s)/dT$ as one passes through T_c. This is because below T_c, the incoherent scattering caused by the presence of the spontaneous magnetization gives a contribution to $d(1/\tau_s)/dT$ which varies as $|\Delta T|^{2\beta-1}$. This basically short-range behavior is less divergent than the long-range critical fluctuations, the latter varying as $|\Delta T|^{2\nu-\gamma-1}$.

The anomaly in the electrical resistivity ρ due to critical fluctuations was first discussed by de Gennes and Friedel.[3] They argued that the dominant contribution came from the long-range critical fluctuations. However, it was shown by Fisher and Langer[4] that above T_c the short-range fluctuations are dominant. This is because $g_{\vec{k}}$ in the expression for ρ is multiplied by k as compared to its average value (and by k^2 as compared to $1/\tau_s$). For $T>T_c$, ρ has a temperature dependence similar to the magnetic internal energy.[4–6] Thus the leading singularity in $d\rho/dT$ varies as $|\Delta T|^{-\alpha}$. Contrary to the temperature dependence of $d(1/\tau_s)/dT$, $d\rho/dT$ is found to behave differently below T_c than above. Thus, Fisher and Langer[4] point out that the contribution of the incoherent scattering to $d\rho/dT$ caused by the square of the spontaneous magnetization varies as $|\Delta T|^{2\beta-1}$. In the case of $d\rho/dT$, this is the dominant term for $T<T_c$.

The purpose of this paper is to apply modern scaling theory to the problem of spin-flip scattering near the critical point of ferromagnets. The results will be similar to the early conclusions of de Gennes and Friedel[3] (who calculated ρ), because their treatment assumed the Ornstein-Zernike form for the correlation function for all wave vectors. This form heavily weights the small-wave-vector region. However Fisher and Langer[4] demonstrated that large-$\vec{k}$ contributions are most important for ρ as a consequence of the $1-\cos\theta$ factor in the transport integral (proportional to k^2) in combination with the correct large-$\vec{k}$ behavior for $g_{\vec{k}}$. The de Gennes and Friedel approach for ρ is formally correct, but they make use of the Ornstein-Zernike result outside of its region of validity (i.e., they use it for all $\vec{k}$) leading to an incorrect conclusion. In the case of $1/\tau_s$ however, the absence of the $1-\cos\theta$ transport weighting factor means that the small-$\vec{k}$ region is dominant and use of an Ornstein-Zernike-like function is allowable. For this reason our results look approximately the same as those of de Gennes and Friedel. We use scaling theory, however, to obtain the precise behavior in the vicinity of T_c.

We outline the calculation of $1/\tau_s$ in Sec. II, then compare our results with the resistivity in Sec. III. Our concluding remarks in Sec. IV include a discussion of the implications of our results.

II. CALCULATION OF $1/\tau_s$

We assume an s-d or s-f exchange model for the interaction between the conduction electrons and the localized spins, and an isotropic Heisenberg interaction between the localized spins. The Hamiltonian of the system is

$$\mathcal{H}=\mathcal{H}_s+\mathcal{H}_d+\mathcal{H}_{s-d}\ , \tag{1}$$

where

$$\mathcal{H}_s=\sum_{\vec{q}\sigma}\epsilon_{\vec{q}}a^\dagger_{\vec{q}\sigma}a_{\vec{q}\sigma}\quad(\epsilon_{\vec{q}}=\hbar^2q^2/2m)\ ,$$

$$\mathcal{H}_d=-\frac{1}{N}\sum_{\vec{k}}J_{\vec{k}}\vec{S}_{\vec{k}}\cdot\vec{S}_{-\vec{k}}\quad\left(\vec{S}_{\vec{k}}=\sum_j e^{i\vec{k}\cdot\vec{R}_j}\vec{S}_j\right)\ ,$$

$$\mathcal{H}_{s-d}=-\frac{1}{N}\sum_{\vec{q}\vec{q}'}I_{\vec{q}'-\vec{q}}[(a^\dagger_{\vec{q}'\uparrow}a_{\vec{q}\uparrow}-a^\dagger_{\vec{q}'\downarrow}a_{\vec{q}\downarrow})S^z_{\vec{q}-\vec{q}'}$$

$$+a^\dagger_{\vec{q}'\uparrow}a_{\vec{q}\downarrow}S^+_{\vec{q}-\vec{q}'}+a^\dagger_{\vec{q}'\downarrow}a_{\vec{q}\uparrow}S^-_{\vec{q}-\vec{q}'}]\ .$$

Here, $\mathcal{H}_s$ is the conduction electron's Hamiltonian, and $a^\dagger_{\vec{q}\sigma}$ and $a_{\vec{q}\sigma}$ are, respectively, the creation and annihilation operators of conduction electrons with a wave vector $\vec{q}$ and a spin σ. $\mathcal{H}_d$ is the Heisenberg Hamiltonian for the localized spins $\vec{S}_{\vec{k}}$ with wave vector $\vec{k}$, and $\mathcal{H}_{s-d}$ describes the interaction between the conduction electrons and the localized spins, where $I_{\vec{q}'-\vec{q}}$ is the interaction strength.

In lowest order in time-dependent perturbation theory, the transition probably per unit time for the conduction electrons's spin to flip, $1/\tau_s$, is given by

$$\frac{1}{\tau_s}=\frac{4}{\hbar^2N^2}\sum_{\vec{k}\vec{q}}|I_{\vec{k}}|^2f_{\vec{q}}(1-f_{\vec{k}+\vec{q}})$$

$$\times\int_0^\infty dt\,\exp[(i/\hbar)(\epsilon_{\vec{q}}-\epsilon_{\vec{k}+\vec{q}})t]g_{\vec{k}}(t)\ . \tag{2}$$

Here $f_{\vec{q}}$ is the Fermi distribution and $g_{\vec{k}}(t)$ is the time-dependent spin-spin correlation function for the localized spins

$$g_{\vec{k}}(t)=\langle\vec{S}_{\vec{k}}(t)\cdot\vec{S}_{-\vec{k}}\rangle-\langle\vec{S}_{\vec{k}}\rangle\cdot\langle\vec{S}_{-\vec{k}}\rangle\ . \tag{3}$$

We now perform the time integral and the summation over $\vec{q}$. By Fourier transforming $g_{\vec{k}}(t)$

$$g_{\vec{k}}(t)=\int\frac{d\omega}{2\pi}g_{\vec{k}}(\omega)\,e^{-i\omega t}\ , \tag{4}$$

the time integral yields a δ function so that (2) can be rewritten in the form

$$\frac{1}{\tau_s}=\frac{4\pi}{\hbar^2N}\left(\frac{v_c}{(2\pi)^2}\right)\sum_{\vec{k}}|I_{\vec{k}}|^2\frac{m}{\hbar k}\int\frac{d\omega}{2\pi}g_{\vec{k}}(\omega)\int dq\,q$$

$$\times\int_{(\hbar/2m)(k^2-2kq)}^{(\hbar/2m)(k^2+2kq)}dx\,f(\hbar^2q^2/2m)$$

$$\times[1-f(\hbar^2q^2/2m+\hbar x)]\,\delta(\omega+x)\ , \tag{5}$$

where $v_c=V/N=3\pi^2/k_F^3$.

It can be seen from Eq. (5) that $\hbar\omega$ is at most of the order of k_BT_c near the transition temperature. We shall show below that small wave vectors dominate the integrand of (5) near T_c. The principal frequencies $\hbar\omega\ll k_BT_c$, and we can therefore expand $f(\epsilon_{\vec{q}}-\hbar\omega)$ in a power series in $\hbar\omega$. Using the fact that $(-\partial f/\partial\epsilon_{\vec{q}})$ behaves like a δ function of width k_BT_c around $\epsilon_{\vec{q}}=\epsilon_F$, we obtain

$$\frac{1}{\tau_s}\simeq\frac{v_c}{\pi}\frac{m^2}{\hbar^5}\frac{1}{N}\sum_{\vec{k}}|I_{\vec{k}}|^2\frac{1}{k}$$

$$\times\int\frac{d\omega}{2\pi}g_{\vec{k}}(\omega)[k_BT_c-\hbar\omega f(\epsilon_F)]\ , \tag{6}$$

where we have kept only the first-order term in the expansion of $f(\epsilon_{\vec{q}}-\hbar\omega)$.

When the limits of the ω integral are taken to $\pm\infty$, the first integral in Eq. (6) gives $g_{\vec{k}}(t=0)$, i.e., the equal-time spin-spin correlation function. The ω integral for the second term gives

$$\int\frac{d\omega}{2\pi}\omega g_{\vec{k}}(\omega)=i\,\partial g_{\vec{k}}(t)/\partial t\,|_{t=0}=-i\nu_{\vec{k}}g_{\vec{k}}(t=0)\ , \tag{7}$$

where $\nu_{\vec{k}}$ is the initial decay rate of the correlation function.[7] From scaling theory, $\nu_k\propto\omega_k$, where ω_k is the characteristic frequency of the spin-spin correlation function, and has the scaling form[7]

$$\omega_k=k^z\left(\frac{k\xi_0}{|\Delta T|^\nu}\right)\ . \tag{8}$$

Here, $\xi=\xi_0|\Delta T|^\nu$ is the correlation length. The critical index $z\simeq2$,[7] so that at the critical temperature and as $k\to0$, the characteristic frequency vanishes (critical slowing down).

As we shall argue, the dominant contribution to the sum over $\vec{k}$ in Eq. (6) arises from the region of small wave vectors. As one approaches T_c, the ν_k term is negligible compared to k_BT_c. Indeed, when the sum over $\vec{k}$ is transformed to an integral the k_BT_c term gives $kg_{\vec{k}}(t=0)$ which at $T=T_c$ is proportional to $k^{\eta-1}$. In the region of small wave vectors, this is much larger than the ν_k term which varies as $k^{\eta+z-1}$ near $T=T_c$. By neglecting the second term in Eq. (6) we omit inelastic scattering processes for the conduction electrons. This approximation is plausible at low wave vectors where the critical fluctuations are so slow that the amount of energy exchanged with the conduction electrons in a spin-flip process is negligible compared to k_BT_c.

Neglecting the second term in Eq. (6), the temperature derivative of $1/\tau_s$ is

$$d(1/\tau_s)/dT\simeq\tfrac{1}{2}\pi\left(\frac{3}{2\epsilon_F}\right)^2\frac{k_BT_c}{\hbar k_F^2}\int dk\,|I_{\vec{k}}|^2kg'_{\vec{k}}\ , \tag{9}$$

where $g'_{\vec{k}}=dg_{\vec{k}}(t=0)/dT$. In order to investigate the behavior of $d(1/\tau_s)/dT$ close to T_c, we shall assume that $|I_{\vec{k}}|^2$ is a smooth function of $\vec{k}$ which does not change much over the region of integration. Thus $d(1/\tau_s)/dT\propto\int dk\,kg'_{\vec{k}}$.

The behavior of $d(1/\tau_s)/dT$ for $T>T_c$ is then obtained as follows: Above T_c, $g_{\vec{k}}$ has a finite maximum at some fixed k.[4,6] Therefore $g'_{\vec{k}}$ changes its sign at some value k_0 (see Fig. 1). Because of the sum rule[6] for $g_{\vec{k}}(t=0)$,

$$\frac{1}{N}\sum_{\vec{k}} g_{\vec{k}}(t=0) = NS(S+1), \quad T > T_c . \tag{10}$$

$\int dk\, k^2 g'_{\vec{k}} = 0$ and thus $d(1/\tau_s)/dT \propto \int dk\, k g'_{\vec{k}} < 0$. This follows because the latter integrand is more heavily weighted near $\vec{k}=0$ than the former, and $g'_{\vec{k}}$ is negative near $\vec{k}=0$. The leading singular term is found by using the scaling function for $g_{\vec{k}}(t=0)$ for $k\xi \ll 1$[8]:

$$\int dk\, k g'_{\vec{k}} \simeq \frac{d}{dT}\left[\Delta T^{-\gamma} \int dk\, kD\left(\left(\frac{k\xi_0}{\Delta T^{\nu}}\right)^2\right)\right]$$

$$= -\Delta T^{-\gamma-1+2\nu}\left[\int dx\, x\left(\gamma D(x^2) + 2\nu x^2 \frac{dD(x^2)}{dx^2}\right)\right], \tag{11}$$

where the terms in the brackets are temperature independent. For $x^2 \ll 1$,

$$D^{-1}(x^2) = C^{-1}(1 + x^2 - \Sigma_4 x^4 + \Sigma_6 x^6 + \ldots) ,$$

where C is the amplitude of the static susceptibility χ near T_c, $\chi \simeq C|\Delta T|^{-\gamma}$, and the coefficients Σ_{2n} are calculated in the ϵ expansion.[8] Using the scaling law $\gamma = \nu(2-\eta)$ we find that $d(1/\tau_s)/dT$ diverges with the index $\eta\nu - 1$.

Below T_c, $g'_{\vec{k}}$ is presumed positive[4] (see Fig. 1) and therefore $d(1/\tau_s)/dT > 0$. In fact, this (strong) assumption (for all $\vec{k}$) is not necessary. The scaling hypothesis is alone sufficient to generate this change in sign because we are interested only in the small-$\vec{k}$ regime. Below T_c, the sum rule for the correlation function is

$$\frac{1}{N}\sum_{\vec{k}} g_{\vec{k}}(t=0) = NS(S+1) - NM^2 , \tag{12}$$

where NM is the spontaneous magnetization, and therefore $\int dk\, k^2 g'_{\vec{k}} \propto |\Delta T|^{2\beta-1}$. The leading singular term in $\int dk\, k g'_{\vec{k}}$ again comes from the low-$\vec{k}$ region and therefore again varies as $|\Delta T|^{2\nu-\gamma-1}$ according to scaling theory. Now, however, the coefficient in front of it is positive (see Fig. 2).

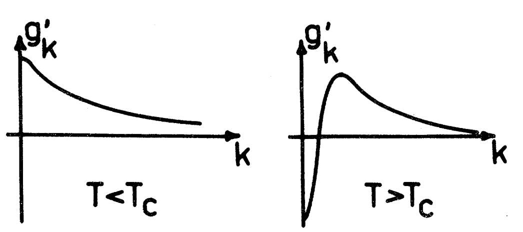

FIG. 1. Schematic plots of $g'_{\vec{k}} \equiv dg_{\vec{k}}(t=0)/dT$ for $T \gtrless T_c$. The peaks at $k=0$ are proportional to $|\Delta T|^{-\gamma-1}$ for $T \gtrless T_c$. Above T_c, $\int dk\, k^2 g'_{\vec{k}} = 0$ and below T_c, $\int dk\, k^2 g'_{\vec{k}} \propto |\Delta T|^{2\beta-1}$.

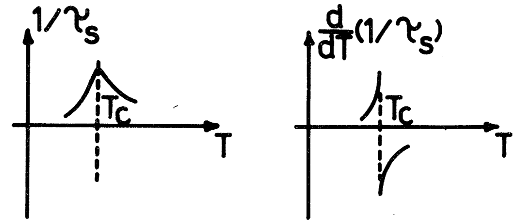

FIG. 2. Schematic plots of $1/\tau_s$ and $d(1/\tau_s)/dT$ for $T \to T_c$. At $T=T_c$ the leading singular term in $1/\tau_s$ tends to zero as $|\Delta T|^{2\nu-\gamma}$. The leading singular term in $d(1/\tau_s)/dT$ diverges at T_c with the critical index $2\nu-\gamma-1 \simeq -0.986$ (Ref. 8).

The term in the spin-spin correlation function which is proportional to the square of the spontaneous magnetization, and yields the incoherent scattering below T_c,[4] has a weaker anomaly than $|\Delta T|^{2\nu-\gamma}$. We write $1/\tau_s$ as follows:

$$\frac{1}{\tau_s} \propto \frac{1}{k_F^2}\int dk\, k g_{\vec{k}} = \sum_j \frac{1}{k_F^2}\int dk\, k \times e^{i\vec{k}\cdot\vec{R}_j}(\langle\vec{S}_j \cdot \vec{S}_0\rangle - \langle\vec{S}_j\rangle \cdot \langle\vec{S}_0\rangle) , \tag{13}$$

and approximate the sum by an integral. This gives

$$\frac{1}{\tau_s} \propto \int dR\, R^2(\langle\vec{S}_R \cdot \vec{S}_0\rangle - \langle\vec{S}_R\rangle \cdot \langle\vec{S}_0\rangle)F(R) , \tag{14}$$

where $F(R)$ is a decaying oscillatory function

$$F(R) = [1/(k_F R)^2](1 - \cos 2k_F R) . \tag{15}$$

Since $F(0)=2$, the term $R=0$ (incoherent scattering) in Eq. (14) yields

$$2(\langle\vec{S}_0 \cdot \vec{S}_0\rangle - \langle\vec{S}_0\rangle^2) = 2[S(S+1) - M^2] . \tag{16}$$

Below T_c this contributes to $1/\tau_s$ a term proportional to $-(T_c - T)^{2\beta}$. This results in a positive contribution to $d(1/\tau_s)/dT$, proportional to $|\Delta T|^{2\beta-1}$. This term is less singular than $|\Delta T|^{2\nu-\gamma-1}$. The temperature derivative of $1/\tau_s$ is therefore dominated by the latter, and diverges in the same manner for T approaching T_c either from above or from below (see Fig. 2). The leading singular term in $1/\tau_s$ vanishes at T_c as $|\Delta T|^{2\nu-\gamma}$. The critical index $2\nu - \gamma = \eta\nu$ is very small, $\eta\nu \simeq 0.014$[8] in second order in the ϵ expansion.

III. COMPARISON WITH THE ELECTRICAL RESISTIVITY

The behavior of $1/\tau_s$, $d(1/\tau_s)/dT$ near T_c is different from that of ρ, $d\rho/dT$ (Figs. 2 and 3). The dominant contribution to $1/\tau_s$ arises from long-range fluctuations. However, ρ contains the transport quantity $(1-\cos\theta)$ so that the divergent long-range fluctuation contribution is quenched and most

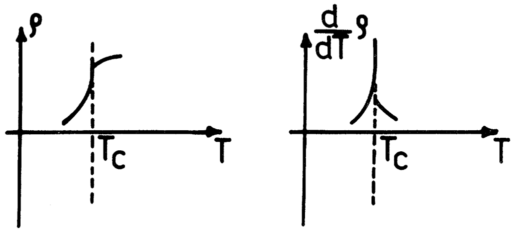

FIG. 3. Schematic plots of the resistivity ρ and $d\rho/dT$ for $T \to T_c$. As $T \to T_c$ from below the leading singular term in $d\rho/dT$ diverges as $|\Delta T|^{2\beta-1}$. As $T \to T_c$ from above the leading anomalous term in $d\rho/dT$ varies as $|\Delta T|^{-\alpha}$ and does not diverge for $\alpha < 0$.

of the contributions arise from short-range fluctuations. The temperature derivative of $1/\tau_s$ diverges with the same index as T approaches T_c from below or above, and the appearance of a spontaneous magnetization does not influence the leading singular term. The leading singular term in $d\rho/dT$, however, varies as $|\Delta T|^{-\alpha}$ above T_c, being proportional to the magnetic specific heat. In the case of an isotropic Heisenberg ferromagnet, where $\alpha < 0$, it will therefore not diverge at T_c. Below T_c, the appearance of a spontaneous magnetization causes a divergent term $\sim |\Delta T|^{2\beta-1}$ in $d\rho/dT$ (Ref. 4) (see Fig. 3).

The dominance of the large-$\vec{k}$ fluctuation contribution to ρ and $d\rho/dT$ does not allow for the direct application of scaling theory. This is the cause for the asymmetry of behavior above and below T_c. The opposite is true for $1/\tau_s$.

It is amusing to use the sum rule [Eq. (10)] to compare the temperature derivatives of $1/\tau_s$ and ρ immediately above T_c. From Eq. (10),

$$\int k^2 dk \frac{d}{dT} \left[g_{\vec{k}}(t=0)\right] = 0 . \qquad (17)$$

The expression for $d(1/\tau_s)/dT$, Eq. (9), is proportional to

$$\int k\, dk \frac{d}{dT}\left[g_{\vec{k}}(t=0)\right] , \qquad (18)$$

while that for $d\rho/dT$ is proportional to

$$\int k^3\, dk \frac{d}{dT}\left[g_{\vec{k}}(t=0)\right] . \qquad (19)$$

Now, from Fig. 1 we see that $dg_{\vec{k}}/dT$ is large and negative for small $\vec{k}$ near T_c, while it is positive and reasonably well behaved for large $\vec{k}$.[4,8] For Eq. (17) to be valid, the area under the positive peak must cancel that under the negative (small-$\vec{k}$) peak. But Eqs. (18) and (19) weight $dg_{\vec{k}}/dT$ at opposite ends of the $\vec{k}$ scale compared to Eq. (17) [small $\vec{k}$ for Eq. (18); large $\vec{k}$ for Eq. (19)]. Hence,

$$d(1/\tau_s)/dT < 0, \quad T > T_c$$

and

$$\frac{d}{dT}(\rho) > 0, \quad T > T_c$$

for T close to T_c. Hence, our remark that the two quantities behave oppositely above T_c.

Below T_c the presence of the magnetization complicates the comparison, [e.g., Eq. (17) is no longer true] and one must return to the detailed discussion of Sec. II. Nevertheless, as we show there, scaling allows us to predict the change in sign of $d(1/\tau_s)/dT$ below T_c, and to demonstrate that the strength of the divergence is the same as it is immediately above T_c. Detailed knowledge of $dg_{\vec{k}}/dT$ is not necessary [e.g., that it is positive for all $\vec{k}$ (if indeed it is)]. The validity of the scaling hypothesis at the small-wave-vector limit, applicable as it is in our calculation, is all that is needed. This is not the case for ρ, where assumptions about the explicit form of $dg_{\vec{k}}/dT$ at large $\vec{k}$ are needed.

Another reason why the calculation of $1/\tau_s$ in the critical region is more satisfactory than for ρ has to do with the assumption of elasticity in the scattering process. Scaling theory is applicable at small $\vec{k}$, and one can explicitly demonstrate the validity of the static (elastic scattering) approximation [see the discussion after Eq. (6)]. This is not necessarily the case for ρ. Quoting Fisher and Langer[4]: "...all the relevant properties... may be described by an equal-time spin-spin correlation function (elastic scattering). (Owing to the thermodynamic slowing down of critical fluctuations, this is plausible for low wave numbers $\vec{k}$, but it may bear further investigation for the higher values of $\vec{k}$ which we claim are also important.)" Thus, one may rely more comfortably on the predicted behavior for $1/\tau_s$ than for ρ, giving impetus to a detailed experimental study of the former.

IV. CONCLUSIONS

We have demonstrated that the spin-flip rate $1/\tau_s$ for conduction electrons in a ferromagnet near T_c is symmetrical above and below T_c. The derivative $d(1/\tau_s)/dT$ is discontinuous at T_c, implying a cusp in $1/\tau_s$ at T_c. This behavior has implications for any physical property which depends on $1/\tau_s$ and not on the transport weighted value $\langle 1/\tau_s(1-\cos\theta)\rangle$.

Such a property is superconductivity, where Abrikosov and Gorkov[9] have shown that $1/\tau_s$ strongly supresses the superconducting critical temperature T_s and can lead to gaplessness. Thus, consider a superconductor containing magnetic impurities where, above T_c, superconductivity is not destroyed ($T_s > T_c$). Then, as the temperature is lowered, the increase in $1/\tau_s$ at T_c can quench the superconductivity and the material will go "normal." As T is further reduced two eventualities can occur. If the spin-orbit relaxation rate of the conduction electrons is sufficiently short,[1] the appearance of the spontaneous magnetization will not have undue influence and the material will go superconducting again. If not, the material will remain normal till zero temperature.

A more flexible example is that of proximity-effect sandwiches where T_c can be easily controlled in the normal magnetic film. By appropriate alloying, T_c can be made less than the T_s of the superconducting film, and following Werthamer's calculation in Ref. 2, the superconductivity can again be destroyed in the vicinity of T_c. This may have already been seen in the work of Sato[10] on thin (Pd : Ni) : Pb sandwiches.

In summation, the cusplike behavior of $1/\tau_s$ near T_c allows one to observe the impact of τ_s on other measurable quantities. A careful analysis of experimental results (e.g., re-entrant superconductivity) may allow one to probe the long-range behavior of the spin-spin correlation function directly.

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