Fermi-liquid theory of a two-dimensional electron liquid: Magnetoplasma waves*

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The Landau-Silin theory of an electron liquid is applied to a two-dimensional system like that occurring in the inversion layer of a metal-insulator-semiconductor structure. The frequency- and wave-vector-dependent electric and magnetic susceptibilities in the presence of a dc magnetic field are studied. The magnetoplasma modes of the system are investigated for both very short and very long wavelengths.

Under the application of a sufficiently strong electric field normal to the surface, the electrons in the inversion layer of a metal-insulator-semiconductor (MIS) structure form an essentially two-dimensional electron liquid.[1] One very useful property of this system is that the electron concentration can be experimentally varied over a wide range. This property makes the two-dimensional electron liquid a useful testing ground for many-body theories, since the effective interparticle interaction depends upon the electron concentration. One very successful many-body theory is the phenomenological Landau-Silin theory[2] of a degenerate electron liquid as applied to simple normal metals.[3] Although we expect the quasiparticle picture and the Landau theory to remain valid for the two-dimensional electron liquid, the system is sufficiently different from a simple three-dimensional metal[4] that interesting new effects might be anticipated. In this paper we apply the Landau-Silin approach to the two-dimensional electron liquid associated with an MIS inversion layer. The kinetic equations for both the spin-dependent and spin-independent parts of the distribution function in the presence of a dc magnetic field are studied. The electric polarizability and the magnetic spin susceptibility are considered for long-wavelength disturbances. The magnetoplasma modes, including the cyclotron harmonic waves, are investigated.

We consider a two-dimensional electron liquid confined to the plane $z=0$ in the presence of a dc magnetic field $\vec{B}=(0,0,B)$. In the presence of an ac electromagnetic disturbance with electric field $\vec{E}$ and magnetic induction $\vec{b}$ proportional to $e^{i\omega t-iqy}$, the distortion $\delta n(\vec{p},\vec{r},t)$ of the electron distribution function from its equilibrium value $n_0(\vec{p})$ can be separated into a spin-independent and a spin-dependent part $\delta n=\delta f+\vec{\delta}g\cdot\vec{\sigma}$, where σ_x, σ_y, and σ_z are the Pauli matrices. The spin-independent kinetic equation can be written[3]

$$i\omega f(\phi)+\left(-iqv_y+\tau^{-1}+\omega_c\frac{\partial}{\partial\phi}\right)[f(\phi)+\delta\epsilon_1(\phi)] = e\vec{E}\cdot\vec{v}+\sum_l \tau_l^{-1}f_l e^{il\phi}. \quad (1)$$

Here ϕ is angle defining the direction of the wave vector $\vec{k}$ and $f(\phi)$ is defined by $\delta f(\vec{k})=(-\partial n_0/\partial\epsilon)f(\phi)$. The function $\delta\epsilon_1(\phi)$ is the value on the Fermi surface, i.e., at $|\vec{k}|=k_F$, of

$$\delta\epsilon_1(\vec{k})=\frac{2}{(2\pi)^2}\int d^2k'\,\Phi(\vec{k},\vec{k}')\delta f(\vec{k}'), \quad (2)$$

where $\Phi(\vec{k},\vec{k}')$ is the spin-independent part of the Landau interaction function. The parameters $\tau^{-1}=2m^*W_0$ and $\tau_l^{-1}=2m^*\alpha_l W_l$ are proportional to Fourier components of the spin-independent part of the transition probability, $2\pi W(\vec{k}-\vec{k}')\delta(\epsilon_k-\epsilon_{k'})$, for scattering a quasiparticle from state $\vec{k}$ to $\vec{k}'$ due to collision with an impurity. The functions $f(\phi)$ and $\delta\epsilon_1(\phi)$ are periodic functions of ϕ and can be expanded in Fourier series. In fact, f_l, appearing on the right-hand side of Eq. (1), is simply the lth Fourier coefficient of $f(\phi)$, i.e., $f_l=(2\pi)^{-1}\times\int d\phi\, f(\phi)e^{-il\phi}$. The interaction function $\Phi(\vec{k},\vec{k}')$ is periodic in $\phi-\phi'$ and can be expressed as $\Phi(\vec{k},\vec{k}')=\pi m^{*-1}\sum_l A_l e^{il(\phi-\phi')}$, where m^* is the effective mass of a quasiparticle on the Fermi surface (defined by $\delta\epsilon_p=m^{*-1}p_F\delta p$). The parameter α_l appearing in the definition of τ_l^{-1} is simply $1+A_l$. With these definitions the spin-independent kinetic equation can be rewritten

$$(i\omega-\tau_l^{-1}+\alpha_l\tau^{-1}+il\alpha_l\omega_c)f_l-\tfrac{1}{2}qv_F(\alpha_{l-1}f_{l-1}-\alpha_{l+1}f_{l+1}) = \tfrac{1}{2}ev_F(E_+\delta_{l,-1}+E_-\delta_{l,+1}). \quad (3)$$

In this equation v_F is the Fermi velocity and $E_\pm=E_x\pm iE_y$.

For the spin-dependent part of the distribution function, we define $\vec{g}(\phi)=(-\partial n_0/\partial\epsilon_p)\vec{\delta}g(p)$ and introduce circularly polarized coordinates $g^{(\pm)}=g_x\pm ig_y$. The resulting kinetic equation for $g^{(\pm)}$ can be written

$$(i\omega - S_l^{-1} + \beta_l \tau^{-1} + il\beta_l\omega_c \pm i\Omega_0\beta_l)\, g_l^{(\pm)} - \tfrac{1}{2} qv_F(\beta_{l-1} g_{l-1}^{(\pm)} - \beta_{l+1} g_{l+1}^{(\pm)}) = \gamma_0 b^{(\pm)} [\tfrac{1}{2} qv_F(\delta_{l,-1} - \delta_{l,+1}) \pm i\Omega_0 \delta_{l0}]. \quad (4)$$

In this equation g_l is the lth Fourier component of $g(\phi)$; $\Omega_0 = 2\gamma B$, where γ is the renormalized gyromagnetic ratio (γ_0 is the unrenormalized gyromagnetic ratio). The parameters S_l and $\beta_l = 1 + B_l$ are the spin-dependent analogs of τ_l and $\alpha_l = 1 + A_l$. The equation for $g_l^{(z)}$ can be obtained from Eq. (4) by simply setting Ω_0 equal to zero and replacing $b^{(\pm)}$ by $b^{(z)}$.

To determine the electrical polarizability $\overleftrightarrow{\chi}$ and the magnetic susceptibility $\overleftrightarrow{\alpha}$, we must evaluate the current density $\vec{j}$ ($i\omega\overleftrightarrow{\chi}\cdot\vec{E} = \vec{j}$) and the magnetization $\vec{M}$ ($\overleftrightarrow{\alpha}\cdot\vec{b} = \vec{M}$). The electrical current density depends only on the Fourier components of $f(\phi)$ with $l = \pm 1$; $j_x(q\omega) = (\alpha_1 m^* v_F e/2\pi)(f_1 + f_{-1})$ and $j_y(q,\omega) = (i\alpha_1 m^* v_F e/2\pi)(f_1 - f_{-1})$. The magnetization depends only on the $l = 0$ component of $g(\phi)$; $\vec{M} = \pi^{-1} m^* \gamma_0 \vec{g}_0$. For small values of the parameter $X = qv_F/\omega_c$, Eqs. (3) and (4) can be solved for $f_{\pm 1}$ and $\vec{g}_0$ to low orders in X by simple iteration. We have evaluated $\overleftrightarrow{\chi}$ and $\overleftrightarrow{\alpha}$ to order X^2; the results, which agree with the semiclassical results for a noninteracting electron gas when the Fermi-liquid interaction parameters are set equal to zero, are not displayed here since they can easily be evaluated by the interested reader.

The dispersion relation for magnetoplasma oscillations of a two-dimensional electron gas embedded in a material of dielectric constant ϵ_0 has been given by Chiu and Quinn[5]:

$$(\chi_{xx} - \beta c^2/2\pi\omega^2)(\chi_{yy} + \epsilon_0/2\pi\beta) = \chi_{xy}\chi_{yx}. \quad (5)$$

We have already discussed the evaluation of $\overleftrightarrow{\chi}$ within the framework of the Fermi-liquid theory; the result could be substituted into Eq. (5) to investigate the effect of Fermi-liquid interactions on the magnetoplasma modes. However, this method is not the most convenient, so we use a different approach.

For self-sustaining oscillations of the electron liquid, the total current is electronic. We already know the electronic current $\vec{j}(q,\omega)$ in terms of the $l = \pm 1$ Fourier components of $f(\phi)$. But Maxwell's equations relate the total current to the electric field

$$\vec{j}(q,\omega) = (-i\omega\epsilon_0/2\pi\beta)(-\beta^2 c^2\omega^{-2}\epsilon_0^{-1} E_x,\ E_y). \quad (6)$$

We use these results to eliminate $\vec{E}$ from the spin-independent kinetic equation and obtain the infinite set of equations

$$(i\omega + \alpha_l\tau^{-1} - \tau_l^{-1} + il\alpha_l\omega_c) f_l - \tfrac{1}{2} qv_f(\alpha_{l-1} f_{l-1} - \alpha_{l+1} f_{l+1}) = -i\mu e^2 \alpha_1 \omega \beta^{-1} c^{-2} [(1 - \epsilon_0^{-1}\omega^{-2}\beta^2 c^2)(f_1\delta_{l,1} + f_{-1}\delta_{l,-1}) + (1 + \epsilon_0^{-1}\omega^{-2}\beta^2c^2)(f_{-1}\delta_{l,1} + f_1\delta_{l,-1})]. \quad (7)$$

Here $\mu = \frac{1}{2} m^* v_F^2$. We use the equation of continuity to replace f_0 by $(i\alpha_1 qv_f/2\omega)(f_1 - f_{-1})$, and define $\lambda_l = \omega_c^{-1}(i\omega + il\alpha_l\omega_c + \alpha_l\tau^{-1} - \tau_l^{-1})$ and $\gamma_\pm = \beta\epsilon_0^{-1}\omega^{-1} \times (\mp 1 + \epsilon_0\omega^2/\beta^2c^2)$. The nontrivial solutions of the resulting equation are obtained by requiring that the determinant of the matrix M_{nl} multiplying the column vector $f_l = (\ldots f_{-n}, f_{-n+1}, \ldots f_{-1}, f_1, f_2, \ldots f_n, \ldots)$ vanish. Fortunately most of the elements of M_{nl} are zero. The nonvanishing elements of $\underline{M}$ are the diagonal elements $M_{nn} = \lambda_n + (\delta_{n,1} + \delta_{n,-1}) P_+$, and the two elements immediately adjacent to the diagonal in each row (or column) $M_{n,\pm 1} = \pm\frac{1}{2} X\alpha_{n\pm 1}$, with the exception $M_{1,-1} = M_{-1,1} = P_-$. We have introduced the symbols $X = qv_F/\omega_c$ and $P_\pm = (i\alpha_1 a\gamma_\pm/2\omega_c) \pm (\frac{1}{2}X)^2\lambda_0^{-1}\alpha_0\alpha_1$, where $a = 2\pi Ne^2/m^*$, with N equal to the electron concentration. If the parameter X is very small, the diagonal elements for $|l| \geq 2$ lead to a series of modes beginning for $X = 0$ at $\omega = l\alpha_l\omega_c$, together with a mode described (up to terms linear in X) by the equation

$$(\lambda_1 + P_+)(\lambda_{-1} + P_+) = P_-^2. \quad (8)$$

In the low-magnetic-field limit ($a \gg c\omega_c$) Eq. (8) reduces to $q^2 = (\epsilon_0\omega^2/c^2) + (\epsilon_0\omega^2/a\alpha_1)^2$, which is exactly the plasmon dispersion relation given by Stern.[6] Keeping terms of order X^2 in the determinantal equation gives for $|l| > 2$

$$\omega = \alpha_l l\omega_c + \delta_l X^2, \quad (9)$$

where

$$\delta_l = -\frac{\alpha_l\omega_c}{4}\left(\frac{\alpha_{l+1}}{(l+1)\alpha_{l+1} - l\alpha_l} - \frac{\alpha_{l-1}}{l\alpha_l - (l-1)\alpha_{l-1}}\right). \quad (10)$$

In addition to these modes, we find modes described by

$$\lambda_{-2}\lambda_2\left(1 + (\tfrac{1}{2}X)^2 {\sum}' \lambda_n^{-1}\lambda_{n+1}^{-1}\alpha_n\alpha_{n+1}\right)[(\lambda_{-1} + P_+)(\lambda_1 + P_+) - P_-^2] + (\tfrac{1}{2}X)^2\alpha_1\alpha_2[(\lambda_{-1} + P_+)\lambda_{-2} + (\lambda_1 + P_+)\lambda_2] = 0, \quad (11)$$

where the prime on the summation denotes the fact that the terms with $n = 2, -1, 0,$ and 1 are to be omitted from the infinite sum. Equation (11) clearly represents coupled-plasmon $|l| = 2$ cyclotron waves. For $X \to 0$, Eq. (11) reduces to the plasmon described by Eq. (8) and the cyclotron harmonic described by $\lambda_{\pm 2} = 0$. These modes are coupled by the terms proportional to X^2. The higher cyclotron harmonics described by Eq. (9) in the long-wavelength limit will also be coupled to the plasmon mode, but the coupling term is of higher order than quadratic in X. Despite the weakness of this coupling, the cyclotron harmonic-plasmon interaction cannot be neglected in the vicinity of the crossing point (where both modes have the same velocity).

Equation (7) is not in the most convenient form for investigating magnetoplasma modes of short wavelength (i.e., with $X > 1$). To investigate the shorter wavelengths we find it convenient to introduce the position vector of a quasiparticle on the Fermi surface $\vec{R}(\phi) = \omega_c^{-1} \int \vec{v}(\phi)\, d\phi$, where $\vec{v}(\phi) = v_F(\cos\phi, \sin\phi)$ is its velocity. Since we have chosen the wave vector q to lie in the y direction, $e^{-i\vec{q}\cdot\vec{R}(\phi)} = e^{iX\cos\phi}$. The functions $f(\phi)\, e^{-i\vec{q}\cdot\vec{R}(\phi)}$, $\delta\epsilon_1(\phi)\, e^{-i\vec{q}\cdot\vec{R}(\phi)}$, and $\vec{v}(\phi)\, e^{-i\vec{q}\cdot\vec{R}(\phi)}$ are all periodic functions of ϕ and can be expanded in Fourier series. The Fourier coefficients are F_l, ϵ_l, and $\vec{v}_l$, respectively. By substituting into Eq. (1) we obtain

$$i\omega F_l + (\tau^{-1} + il\omega_c)(F_l + \epsilon_l) = e\vec{E}\cdot\vec{v}_l + \sum_{l'} \tau_{l'}^{-1} f_{l'}\, i^{l-l'} J_{l-l'}(X). \qquad (12)$$

Here J_n is the Bessel function of order n, and we have used the result $e^{-i\vec{q}\cdot\vec{R}} = \sum_n i^n J_n(X) e^{in\phi}$. The Fourier coefficients ϵ_l are related to the Fourier coefficients f_l of the distribution function $f(\phi)$ by the relation $\epsilon_l = \sum_n i^{l-n} J_{l-n}(X) A_n F_n$, where A_n is the nth Fermi-liquid interaction parameter. Using this result in Eq. (12) we obtain

$$F_l = \sum_n i^{l-n} A_n \left(-1 + \frac{i\omega + A_n^{-1}\tau_n^{-1}}{i\omega + \tau^{-1} + il\omega_c}\right) J_{l-n}(X) f_n + e\vec{E}\cdot\vec{v}_l (i\omega + \tau^{-1} + il\omega_c)^{-1}. \qquad (13)$$

To obtain the magnetoplasma wave dispersion relation we again use Eq. (12) and the expression for the current density in terms of f_1 and f_{-1} in order to eliminate $\vec{E}$ from Eq. (13). It is straightforward to show that F_l and f_n are related by the equation $f_n = \sum_l (-i)^{l-n} J_{l-n}(X) F_l$. Therefore, after eliminating $\vec{E}$ from Eq. (13), we multiply the equation by $(-i)^{l-n} \times J_{l-n}(X)$ and sum over all l. This gives the result

$$f_n = \sum_{ll'} (-i)^{l'-n} A_{l'} \left(-1 + \frac{i\omega + \tau_{l'}^{-1} A_{l'}^{-1}}{i\omega + \tau^{-1} + il\omega_c}\right) J_{l-n} J_{l-l'} + i^n \left(\frac{\mu e^2 \alpha_1}{\beta\epsilon_0\omega}\right) \sum_l (i\omega + \tau^{-1} + il\omega_c)^{-1} J_{l-n}[q^2 J_{l+1} + (\beta^2 - \epsilon_0\omega^2 c^{-2}) J_{l-1}] f_1$$
$$- i^n \left(\frac{\mu e^2 \alpha_1}{\beta\epsilon_0\omega}\right) \sum_l (i\omega + \tau^{-1} + il\omega_c)^{-1} J_{l-n}[q^2 J_{l-1} + (\beta^2 - \epsilon_0\omega^2 c^{-2}) J_{l+1}] f_{-1}. \qquad (14)$$

Here $\mu = \frac{1}{2} m^* v_f^2$ and $\beta = (q^2 - \epsilon_0\omega^2 c^{-2})^{1/2}$. We may eliminate f_0 from this infinite set of equations by using the equation of continuity $f_0 = (i\alpha_1 q v_f / 2\omega) \times (f_1 - f_{-1})$. To obtain nontrivial solutions, the determinant of the matrix multiplying the infinite column vector f_l must vanish. This determinantal equation can be written $|a_{ml} - \delta_{ml}| = 0$, where

$$a_{ml} = -A_m \delta_{m,l} + i^{m-l}(i\omega A_l + \tau_l^{-1}) T_{ml} \qquad (15)$$

for $l \neq \pm 1$. The symbol T_{ml} stands for

$$T_{ml} = \sum_n (i\omega + \tau^{-1} + in\omega_c)^{-1} J_{n-m}(X) J_{n-l}(X). \qquad (16)$$

For $l = \pm 1$ we have

$$a_{m,\pm 1} = [i^{m\mp 1}(i\omega A_1 + \tau_1^{-1}) \pm i^m \mu e^2 \alpha_1 \gamma_+] \times T_{m,\pm 1} + i^m \mu e^2 \alpha_1 \gamma_- T_{m,\mp 1} + i^{m+l}(\alpha_1 q v_f / 2\omega) \times (i\omega A_0 + \tau^{-1}) T_{m,0} - A_m \delta_{m,\pm 1}. \qquad (17)$$

Here $\gamma_+ = -(\beta\epsilon_0\omega)^{-1}(\beta^2 - \epsilon_0\omega^2 c^{-2})$ and $\gamma_- = (\beta\epsilon_0\omega)^{-1} \times (\beta^2 + \epsilon_0\omega^2 c^{-2}) = q^2/\beta\epsilon_0\omega$.

If we assume that $\omega\tau_l \gg 1$ for all values of l and set $A_l = 0$ for all l, the determinantal equation becomes

$$\begin{vmatrix} a_{-1,-1} - 1 & a_{-1,1} \\ a_{1,-1} & a_{1,1} - 1 \end{vmatrix} = 0. \qquad (18)$$

This is identical to the dispersion relation for a noninteracting electron gas given by Chiu and Quinn.[5] In the limit of small X the Bessel functions appearing in Eq. (16) can be expanded in power series, and it is not difficult to show that the dispersion relation obtained from $|a_{ml} - \delta_{ml}| = 0$ is identical to order X^2 to Eqs. (10) and (11). Because all the a_{ml} except a_{m1} and a_{m-1} are proportional to A_l, we can obtain a reasonable approximation to the dispersion relation at short wavelengths by retaining terms of only zero and first order in the coefficients A_l. The resulting equation is

$$K\left(1 - \sum_{|l|>1} a_{ll}\right) - \sum_{|m|>1} (a_{m,-1} a_{-1,m} - a_{m,1} a_{1,m}) = 0, \qquad (19)$$

where $K = (a_{-1,-1} - 1)(a_{1,1} - 1) - a_{1,-1} a_{-1,1}$. In the short-wavelength limit where $X \gg 1$ we use the asymptotic form of the Bessel function $J_l(X) = (2/\pi X)^{1/2}[\cos(X - \frac{1}{2}l\pi - \frac{1}{4}\pi) + O(X^{-1})]$. For reasonable values of the electron concentration N and the cyclotron frequency ω_c, Eq. (19) reduces to

$$1 - \Omega X^{-1}(\cot\pi\Omega + \csc\pi\Omega \sin 2X) = 0. \qquad (20)$$

In this equation $\Omega = \omega/\omega_c$ and only through the m^* appearing in ω_c do Fermi-liquid effects appear. The interaction coefficient A_l appearing in the a_{lm} of Eq. (19) drops out of the equation when the $X \gg 1$ approximation is made. Equation (20) has been derived previously by Chiu and Quinn[5] for the free-electron gas.

To summarize, we have derived expressions for the electrical susceptibility $\overleftrightarrow{\chi}$ and magnetic spin susceptibility $\overleftrightarrow{\alpha}$ of a two-dimensional electron liquid in the presence of a dc magnetic field. The fundamental cyclotron resonance absorption occurs at $\omega = \alpha_1 \omega_c = \alpha_1 cB/m^* c$, where m^* is the quasiparticle effective mass. It follows from Galilean invariance that $m^* = \alpha_1 m$, where m is the band mass and is independent of electron-electron interactions. Thus in a cyclotron resonance experiment[7] we would not expect m to depend on the electron concentration N except for nonparabolicity of the bands or electron-phonon effects. In contrast, the amplitude of quantum oscillations depends on $\hbar\omega_c/kT$. This quantity is inversely proportional to m^*, the quasiparticle effective mass, which depends on α_1 and therefore is sensitive to the electron concentration.[8] We have also investigated the magnetoplasma modes of the two-dimensional electron liquid. We find a sequence of "cyclotron harmonic" waves, which begin for very small values of qv_F/ω_c at $\omega = \alpha_l l\omega_c$, for $l = 2, 3, 4, \ldots$. In addition to the cyclotron harmonic modes, we find the usual two-dimensional plasmon. As discussed by Stern,[6] the plasmon begins at $q = 0$ with the frequency $\omega = \epsilon_0^{-1/2} cq$; for larger values of q, $\omega = (\alpha_1 \epsilon_0^{-1} aq)^{1/2}$. The plasmon mode and cyclotron modes are coupled, and one obtains a splitting of the coupled modes in the vicinity of their crossing point. For very large values of q, the cyclotron mode which begins at $\omega = l\alpha_l \omega_c$ approaches asymptotically the value $\omega = (l-1)\omega_c$. It is interesting to note that there exists a long-lived excitation of frequency ω_c (for the $l = 2$ case at large values of q) whose frequency is close to but not equal to the fundamental cyclotron resonance frequency $\alpha_1\omega_c$. It is tempting to think of ascribing the subharmonic structure of cyclotron resonance seen by Kotthaus *et al.*[9] to those propagating modes, but the lack of sensitivity to the electron concentration of the position and amplitude of the subharmonic structure makes this surmise unlikely.

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