Linewidth in exchange anisotropic paramagnets at the critical point

C. Bagnuls*† and C. Joukoff-Piette
Faculté des Sciences, Université Libre de Bruxelles, Brussels, Belgium
(Received 30 May 1974)

In this article the kinetic theory developed by Resibois and De Leener describing the temporal behavior of the spin autocorrelation functions near the critical point ($T \gtrsim T_c$) in the Weiss limit is extended to exchange anisotropic systems with arbitrary spin. From our model, the scaling laws proposed by Riedel and Wegner can be justified microscopically. Furthermore, we suggest an interpretation of experimental data of linewidths above T_c on the uniaxial antiferromagnet MnF_2 in terms of the variables $\kappa_{\parallel}/q$ and κ_Δ/q, where κ_Δ is directly related to anisotropy. Owing to the satisfactory agreement obtained in the above, we give scaling functions relative to various systems (uniaxial and planar ferromagnets and planar antiferromagnets).

I. INTRODUCTION

The angular and energetic analysis of magnetic neutron scattering[1] is related to the double Fourier transform, in time and space, of the spin autocorrelation functions (af) given by

$$\Gamma_{ab}^{\alpha\beta}(t)=\frac{\langle S_a^\alpha(t)S_b^\beta(0)\rangle}{\langle S_a^\alpha(0)S_b^\beta(0)\rangle}, \qquad (1.1)$$

where the denominator is included for reason of normalization. $S_a^\alpha(t)$ denotes the Heisenberg representation of spin component α ($\alpha = z, +, -$) at lattice point a, and the bracket $\langle\cdots\rangle$ indicates the average over the equilibrium canonical distribution. One also defines

$$\tilde{\Gamma}_q^{\alpha\beta}(\omega)=\sum_a\int_0^\infty dt\,\Gamma_{ab}^{\alpha\beta}(t)\exp i[q(a-b)-\omega t]. \quad (1.2)$$

The study of dynamics at the critical point of these af led to the dynamical scaling approximation (DSA) established by Halperin and Hohenberg[2] (HH):

$$\tilde{\Gamma}_q^{\alpha\beta}(\omega)=[\omega_\kappa^{\alpha\beta}(q)]^{-1}F^{\alpha\beta}\left(\frac{\omega}{\omega_\kappa^{\alpha\beta}(q)};\frac{\kappa}{q}\right), \qquad (1.3)$$

where $\omega_\kappa^{\alpha\beta}(q)$, the characteristic frequency or linewidth, is defined by

$$\int_{-1}^{+1}F(x;\kappa/q)\,dx=\tfrac{1}{2}. \qquad (1.4)$$

A further hypothesis expresses the homogeneity of the characteristic frequency

$$\omega_\kappa^{\alpha\beta}(q)=q^z f(\kappa/q). \qquad (1.5)$$

The DSA leads to $z=\frac{5}{2}$ and $\frac{3}{2}$ for both the ferromagnets and antiferromagnets, respectively.

These results received experimental confirmation in particular for the isotropic antiferromagnet $RbMnF_3$.[3] However, DSA gives no information on the universal scaling function $f(\kappa/q)$.

To justify DSA microscopically, essentially two methods have been used: kinetic equations[4] and mode-mode coupling.[5] Although these two approaches are different, their results are equivalent. However, if Kawasaki's theory is of more general application, the limitations of the dynamical molecular-field theory, developed by Resibois and De Leener[4] (R-DL) above the critical temperature are more directly apparent (Weiss limit or long-range forces). Resibois and Dewel[6] have extended the R-DL theory below T_c.

These works have provided a microscopic justification of DSA. Because of the increasing interest in the influence of dimensionality on critical phenomena[7] Hohenberg and R-DL[8] have shown using this theoretical scheme, that dynamical scaling applies up to $d=6$ above T_c, whereas below T_c, the theory fails at $d=4$.

Resibois and one of the authors[9] (C. J.-P.), going beyond DSA, have calculated the scaling functions for isotropic ferromagnets and antiferromagnets. Their theoretical results are in good agreement with the experimental data on[3] $RbMnF_3$ (antiferromagnet) and[10] Fe (ferromagnet).

The existence of a more pronounced minimum for ferromagnets has been confirmed.

According to the universality hypothesis formulated by Kadanoff,[11] in the vicinity of the critical point the properties of systems depend only on the symmetry, the range of the interactions and the dimensionality. Therefore, it appeared worthwhile to study anisotropic systems. To this end, Riedel and Wegner[12,13] (RW) generalized DSA and have introduced an anisotropy parameter Δ and a crossover index φ such that

$$\Delta \propto |\epsilon|^\varphi, \qquad (1.6)$$

where ϵ is the relative deviation from the critical temperature. Extending the DSA, the characteristic frequency is then written

$$\omega_\alpha(q,\kappa_{\parallel},\kappa_\Delta)=l^{-\psi/\nu}\omega_\alpha(ql,\kappa_{\parallel}l,\kappa_\Delta l), \qquad (1.7)$$

if $q, \kappa_{\parallel}, \kappa_\Delta \ll 1$ (in units of the lattice parameter a). Fixing κ_Δ, that is the symmetry and conservation laws, RW postulate

$$\omega_\alpha(q, \kappa_\parallel; \kappa_\Delta) = l^{-\psi'/\nu'}\omega_\alpha(ql, \kappa_\parallel l; \kappa_\Delta) , \qquad (1.8)$$

if $q, \kappa_\parallel \ll \kappa_\Delta$. They determine ψ'/ν' using Mori's[14] definition of the linewidth:

$$\omega_\alpha(q, \kappa_\parallel, \kappa_\Delta) = \frac{\mathrm{Re}\int_0^\infty dt \langle \dot{S}_q^{\alpha\dagger}(0)\dot{S}_{-q}^{\alpha}(t)\rangle}{\langle S_q^{\alpha\dagger} S_{-q}^{\alpha}\rangle} , \qquad (1.9)$$

where $\dot{S}_q^\alpha(t)$ is the time derivative of $S_q^\alpha(t)$. In the anisotropic regime where $q, \kappa_\parallel \ll \kappa_\Delta \ll 1$, they show that

$$\omega_\alpha(q, \kappa_\parallel, \kappa_\Delta) = \kappa_\Delta^{(\psi-\psi')/\nu}\kappa_\parallel^{\psi'/\nu'} W\left(\frac{q}{\kappa_\Delta^{(\nu-\nu')/\nu'}\kappa_\parallel}\right), \quad (1.10)$$

where W is a universal function in the sense used by HH. An important result, pointed out by HH,[2] is the existence of a wave number beyond which the system behaves as an isotropic system.

The interested reader is referred to Table I of Ref. 13 where the critical indices for uniaxial and planar ferromagnets and antiferromagnets are displayed. Their values were obtained taking $\eta = 0$ and $\varphi = \frac{4}{3}$. Adopting the mode-mode coupling theory of Kawasaki,[5,15] RW have especially studied the existence of the crossover effect and its temperature and wave-number dependence. They have also compared their results with experiments on the uniaxial antiferromagnets MnF_2[16,17] and FeF_2.[18]

Following the theory developed by R-DL, in Sec. II we recall briefly how to obtain the kinetic equations above T_c.

In Sec. III, we justify the scaling laws proposed by Riedel and Wegner and study the various scaling functions for some exchange anisotropic systems.

A comparison with the experimental data for the uniaxial antiferromagnet MnF_2[19] is made and because of the good agreement, we give the functions relative to uniaxial and planar ferromagnets and planar antiferromagnets. Because of some features of these cases perhaps new experiments could be undertaken.

II. KINETIC EQUATIONS

We consider a system of N identical spins ($|S|$ arbitrary) fixed on the sites of a three dimensional lattice. It is described by an exchange anisotropic Heisenberg Hamiltonian:

$$\mathrm{H} = -\sum_{i,j} J_{ij}S_i^z S_j^z + K_{ij}S_i^+ S_j^- . \qquad (2.1)$$

The magnitude of the anisotropy of the system is given by $\Delta = J/K$.

$\Delta > 1$ corresponds to the uniaxial case (easy axis of magnetization along the z direction) whereas $\Delta < 1$ gives the planar case (isotropy in the xy plane).

Naturally we recover the isotropic Heisenberg model for $\Delta = 1$.

Because of the anisotropy, the longitudinal (zz) and transverse ($+-$) af must be treated separately. Indeed, only the z component of the magnetization is conserved, i.e.,

$$\left[H, \sum_i S_i^z\right] = 0 . \qquad (2.2)$$

This property will play an important role in what follows as it implies that this component alone will satisfy a diffusion equation.

To study the time evolution of the af, we extended the theory developed by R-DL for isotropic systems to include anisotropic effects. We will recall that this method is based on an infinite resummation valid only in the Weiss limit (number of neighbors, $z \to \infty$, J and $K \to 0$, ZJ and ZK finite), of a formal perturbation expansion of $S_q^\alpha(t)$ in powers of the exchange integrals J and K. Consistently one also uses the Ornstein-Zernike approximation for the static correlation functions, the Fourier transforms of which are written

$$\Phi_q^L = J_q/(1 - \tfrac{1}{2}\beta J_q) , \qquad (2.3)$$

$$\Phi_q^T = K_q/(1 - \tfrac{1}{2}\beta K_q) . \qquad (2.4)$$

One knows that this procedure amounts to taking the value $\eta = 0$ for the exponent corresponding to the asymptotic behavior of the static correlation function.[20]

One of the authors[21] (C. B.) has shown that the consideration of arbitrary spin magnitude does not lead to new difficulties. Indeed, it is sufficient to introduce a following reduced variables

$$\tilde{t} = t[\tfrac{1}{3}4s(s+1)]^{1/2} , \qquad (2.5)$$

$$\tilde{\beta} = \beta[\tfrac{1}{3}4s(s+1)] . \qquad (2.6)$$

The extension of R-DL's theory to the anisotropic case is thus immediate and leads to a system of two coupled nonlinear non-Markovian integro-differential equations:

$$\partial_t \Gamma_{ab}^{zz}(t) = \sum_i \int_0^t d\tau\, G_{ai}^{zz}(t-\tau\,|\,\Gamma_{jk}^{+-}, \Gamma_{jk}^{zz})\Gamma_{ib}^{zz}(\tau), \quad (2.7)$$

$$\partial_t \Gamma_{ab}^{+-}(t) = \sum_i \int_0^t d\tau\, G_{ai}^{+-}(t-\tau\,|\,\Gamma_{jk}^{+-}, \Gamma_{jk}^{zz})\Gamma_{ib}^{+-}(\tau) . \quad (2.8)$$

In the Appendix we give the rules to obtain the kernels $G_{ai}^{\alpha\beta}$.

If one defines the space Fourier transform as

$$\tilde{G}_q = \sum_{\text{all } b} G_{ab}\, e^{iq(b-a)} , \qquad (2.9)$$

one obtains

$$\partial_t \tilde{\Gamma}_q^{zz}(t) = \int_0^t \tilde{G}_q^{zz}(t-\tau\,|\,\tilde{\Gamma}_{q'}^{zz}, \tilde{\Gamma}_{q'}^{+-})\tilde{\Gamma}_q^{zz}(\tau)\, d\tau , \quad (2.10)$$

$$\partial_t \tilde{\Gamma}_q^{+-}(t) = \int_0^t \tilde{G}_q^{+-}(t-\tau\,|\,\tilde{\Gamma}_{q'}^{+-}, \tilde{\Gamma}_{q'}^{zz})\tilde{\Gamma}_q^{+-}(\tau)\, d\tau . \quad (2.11)$$

The non-Markovian kernels $\tilde{G}$, which are highly nonlinear functionals of the af themselves, are given by a series of successive approximations

$$\tilde{G}_q(\tau \mid \{\tilde{\Gamma}^{zz}_{q'}\}, \{\tilde{\Gamma}^{+-}_{q'}\}) = \sum_{n=1}^{\infty} \tilde{G}^{(n)}_q(\tau \mid \{\tilde{\Gamma}_{q'}\}) . \tag{2.12}$$

However we will limit ourselves to the first approximation. This will be justified *a posteriori*.

III. LINEWIDTHS

We now analyze the linewidth for uniaxial and planar ferromagnets and antiferromagnets. In order to avoid reproducing similar calculations for ferromagnets and antiferromagnets, we shall develop the calculations for a ferromagnet in which case we only have two coupled kinetic equations.

We will then give and discuss the results for antiferromagnets.

A. Ferromagnets

The lowest-order kernels are given by

$$\tilde{G}^{zz(2)}_q(t-\tau) = \frac{2}{N} \sum_{q'} \tilde{\Gamma}^{+-}_{q+q'}(t-\tau) \tilde{\Gamma}^{+-}_{-q'}(t-\tau) \Phi^T_{q'} (K_{q+q'} - K_{q'})(1 - \tfrac{1}{2}\beta J_q) , \tag{3.1}$$

$$\tilde{G}^{+-(2)}_q(t-\tau) = \frac{1}{N} \left[\sum_{q'} (J_{q+q'} - K_{q'}) \Phi^T_{q'} (1 - \tfrac{1}{2}\beta K_q) \tilde{\Gamma}^{zz}_{q+q'}(t-\tau) \tilde{\Gamma}^{+-}_{-q'}(t-\tau) + \sum_{q'} (K_{q+q'} - J_{q'}) \Phi^L_{q'} (1 - \tfrac{1}{2}\beta K_q) \tilde{\Gamma}^{+-}_{q+q'}(t-\tau) \tilde{\Gamma}^{zz}_{-q'}(t-\tau) \right] , \tag{3.2}$$

where the summation on q' is restricted to the first Brillouin zone. Using the inversion symmetry of the exchange integral, one recovers Kawasaki's result[5,15] (see also Ref. 22).

For short wavelengths, the Fourier transform of the exchange integral takes the form

$$J(q) = ZJ(1 - q^2\delta^2 + \dots) , \tag{3.3}$$

with $q\delta \ll 1$. A similar relation exists for the transverse part:

$$\delta^2 = \frac{1}{6} \sum_R J(R) R^2 \Big/ \sum_R J(R) \tag{3.4}$$

gives the mean range of the interaction. In the Weiss limit where $2\beta_c ZJ = 2\beta_c^{\perp} ZK = 1$, we then have for the asymptotic forms of the static correlation functions

$$\Phi^L_q = \frac{ZJ}{\delta^2 (q^2 + \kappa_{\parallel}^2)} , \tag{3.5}$$

$$\Phi^T_q = \frac{ZK}{\delta^2 (q^2 + \kappa_{\parallel}^2 + \kappa_{\Delta}^2)} , \tag{3.6}$$

where one has put

$$\kappa_{\parallel}^2 = \frac{1}{\delta^2} \frac{T - T_c}{T_c} \ll 1 , \tag{3.7}$$

$$\kappa_{\Delta}^2 = (1/\delta^2)(\Delta - 1) \ll 1 . \tag{3.8}$$

Had we defined two critical temperatures, T_c and $T_c^{\perp}$, as Moriya[23] did, we would have introduced a transverse length given by

$$\kappa_{\perp}^2 = \frac{1}{\delta^2} \frac{T - T_c^{\perp}}{T_c^{\perp}} , \tag{3.9}$$

T_c and $T_c^{\perp}$ are respectively the temperatures at which the longitudinal and transverse static correlation functions diverge. One will note that

$$\kappa_{\Delta} = \kappa_{\perp}(T_c) . \tag{3.10}$$

Let us remark that the roles of the longitudinal and transverse parts are inverted in the uniaxial and planar cases. This happens because the critical temperature is, respectively, T_c and $T_c^{\perp}$. Moreover, in the planar case

$$\kappa_{\Delta}^2 = (1/\delta^2)(1/\Delta - 1) . \tag{3.11}$$

If one then assumes, as will be justified *a posteriori*, that small values of q' (Ref. 24) give the dominant contributions to the kernel (one recovers the mode-mode coupling), Eqs. (3.1) and (3.2) for the uniaxial case become

$$\tilde{G}^{zz(2)}_q = \gamma^2 \int d\vec{q}\,' [(\vec{q} + \vec{q}\,')^2 - q'^2] \frac{q^2 + \kappa_{\parallel}^2}{q'^2 + \kappa_{\parallel}^2 + \kappa_{\Delta}^2} \tilde{\Gamma}^{+-}_{q+q'}(t-\tau) \tilde{\Gamma}^{+-}_{q'}(t-\tau) , \tag{3.12}$$

$$\tilde{G}^{+-(2)}_q = -\tfrac{1}{2}\gamma^2 \left[\int d\vec{q}\,' \left([(\vec{q} + \vec{q}\,')^2 - q'^2 - \kappa_{\Delta}^2] \frac{q^2 + \kappa_{\parallel}^2 + \kappa_{\Delta}^2}{q'^2 + \kappa_{\parallel}^2 + \kappa_{\Delta}^2} \tilde{\Gamma}^{zz}_{q+q'}(t-\tau) \tilde{\Gamma}^{+-}_{q'}(t-\tau) + [(\vec{q} + \vec{q}\,')^2 - q'^2 + \kappa_{\Delta}^2] \frac{q^2 + \kappa_{\parallel}^2 + \kappa_{\Delta}^2}{q'^2 + \kappa_{\parallel}^2} \tilde{\Gamma}^{+-}_{q+q'}(t-\tau) \tilde{\Gamma}^{zz}_{q'}(t-\tau) \right) \right] , \tag{3.13}$$

where the sum on q' has been replaced by an integral and where

$$\gamma^2 = \delta^2 Z^2 K^2 a^3 / 4\pi^3 \ . \tag{3.14}$$

One recovers the isotropic case when $\kappa_\Delta = 0$. The longitudinal kernel tends to zero when $q \to 0$. This is a consequence of the fact that the z component of the magnetization is invariant (kinematic slowing down[1]). On the other hand, the transverse kernel does not vanish in the same limit.

Using the Markovian approximation, we get

$$\partial_t \tilde{\Gamma}_q^{zz}(t) = \left[\int_0^\infty \tilde{G}_q^{zz}(\tau \,|\, \tilde{\Gamma}_{q'}^{+-}, \tilde{\Gamma}_{q'}^{zz})\, d\tau\right] \tilde{\Gamma}_q^{zz}(t) \ , \tag{3.15}$$

$$\partial_t \tilde{\Gamma}_q^{+-}(t) = \left[\int_0^\infty \tilde{G}_q^{+-}(\tau \,|\, \tilde{\Gamma}_{q'}^{+-}, \tilde{\Gamma}_{q'}^{zz})\, d\tau\right] \tilde{\Gamma}_q^{+-}(t) \ . \tag{3.16}$$

The linewidth may then be defined by

$$\begin{aligned}\omega_L(q, \kappa_\parallel, \kappa_\Delta) &= \tilde{\Gamma}_q^{zz}(\omega = 0) = \left[\int_0^\infty \tilde{\Gamma}_q^{zz}(t)\, dt\right]^{-1} \\ &= \int_0^\infty \tilde{G}_q^{zz}(\tau \,|\, \tilde{\Gamma}_{q'}^{zz}, \tilde{\Gamma}_{q'}^{+-})\, d\tau \\ &\simeq \int_0^\infty \tilde{G}_q^{zz(2)}(\tau \,|\, \tilde{\Gamma}_{q'}^{zz}, \tilde{\Gamma}_{q'}^{+-})\, d\tau \ ,\end{aligned} \tag{3.17}$$

and

$$\omega_T(q, \kappa_\parallel, \kappa_\Delta) = \int_0^\infty \tilde{G}_q^{+-(2)}(\tau \,|\, \tilde{\Gamma}_{q'}^{zz}, \tilde{\Gamma}_{q'}^{+-})\, d\tau \ . \tag{3.18}$$

So,

$$\omega_L(q, \kappa_\parallel, \kappa_\Delta) = \int d\vec{q}\,'[(\vec{q} + \vec{q}\,')^2 - q'^2] \frac{q^2 + \kappa_\parallel^2}{q'^2 + \kappa_\parallel^2 + \kappa_\Delta^2} \frac{1}{\omega_T(q + q', \kappa_\parallel, \kappa_\Delta) + \omega_T(q', \kappa_\parallel, \kappa_\Delta)} \ , \tag{3.19}$$

$$\begin{aligned}\omega_T(q, \kappa_\parallel, \kappa_\Delta) = \frac{1}{2}\Bigg[\int d\vec{q}\,'\Bigg(&[(\vec{q} + \vec{q}\,')^2 - q'^2 - \kappa_\Delta^2] \frac{q^2 + \kappa_\parallel^2 + \kappa_\Delta^2}{q'^2 + \kappa_\parallel^2 + \kappa_\Delta^2} \frac{1}{\omega_L(q + q', \kappa_\parallel, \kappa_\Delta) + \omega_T(q', \kappa_\parallel, \kappa_\Delta)} \\ &+ [(\vec{q} - \vec{q}\,')^2 - q'^2 + \kappa_\Delta^2] \frac{q^2 + \kappa_\parallel^2 + \kappa_\Delta^2}{q'^2 + \kappa_\parallel^2} \frac{1}{\omega_T(q + q', \kappa_\parallel, \kappa_\Delta) + \omega_L(q', \kappa_\parallel, \kappa_\Delta)}\Bigg)\Bigg] \ .\end{aligned} \tag{3.20}$$

One easily sees that the linewidth is an homogeneous function which takes the form

$$\omega_{L,T}(q, \kappa_\parallel, \kappa_\Delta) = q^{5/2} f_{L,T}(\kappa_\parallel / q, \kappa_\Delta / q) \ . \tag{3.21}$$

One then recovers Eq. (1.7), a result first given by RW.[13] Because of the form of the static correlation functions (3.5) and (3.6) we split the $(q, \kappa_\parallel)$ plane into an anisotropic region [I] (critical [I_a] and hydrodynamic [I_b]) such that $q^2 + \kappa_\parallel^2 \ll \kappa_\Delta^2$ and an isotropic region [II] (critical [II_a] and hydrodynamic [II_b]) with $q^2 + \kappa_\parallel^2 \gg \kappa_\Delta^2$. However, a more detailed study of the asymptotic behaviors shows that we must distinguish between two further regimes separated by the curve $q\kappa_\parallel \approx \kappa_\Delta^2$ in domain [II_b]. One will understand this result by noting that ω_L [Eq. (3.19)] in the region where $q, \kappa_\Delta \ll \kappa_\parallel$ contains only a diffusion type contribution ($q^2\kappa_\parallel^{1/2}$ isotropic behavior) whereas ω_T contains both a diffusive and nondiffusive contribution. The latter arises from a source term driven by the anisotropy κ_Δ.

Above the curve $q\kappa_\parallel \approx \kappa_\Delta^2$, region [$\mathrm{II}_{b_1}$], the diffusive term dominates whereas below it (region [II_{b_2}]) the nondiffusive contribution does, this leads to a $\kappa_\Delta^4 \kappa_\parallel^{-3/2}$ behavior. For more clarity, we give in Fig. 1 the various domains in the $(q, \kappa_\parallel)$ plane.

Let us insist on the fact that in the anisotropic critical region, ([I_a], where $\kappa_\parallel \ll q \ll \kappa_\Delta$) the longitudinal linewidth ω_L behaves as $q^4 \kappa_\Delta^{-3/2}$. We therefore have the characteristic q^4 behavior of Van Hove's conventional theory[25] which rests on the nondivergent character of Onsager's kinetic coefficient.[5] We give in Table I the asymptotic behavior

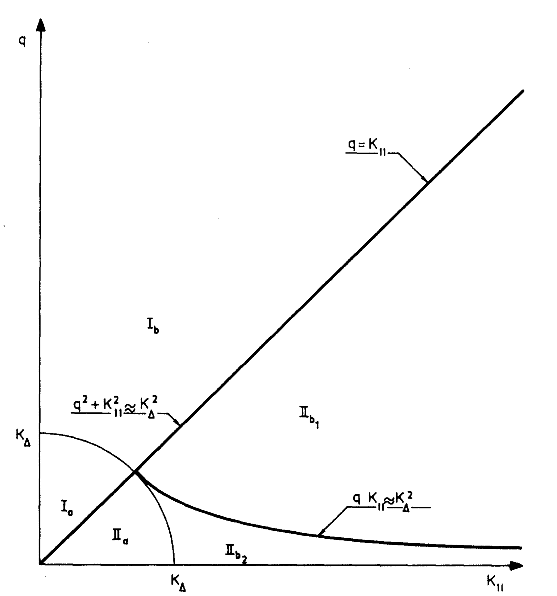

FIG. 1. Various regions in the $(q, \kappa_\parallel)$ plane. I and II correspond to the critical and hydrodynamic behaviors. The curve $q^2 + \kappa_\parallel^2 \approx \kappa_\Delta^2$ separates the anisotropic (I_a and II_a) and isotropic domain and $q\kappa_\parallel \approx \kappa_\Delta^2$ the diffusive (II_{b_1}) and nondiffusive (II_{b_2}) regimes.

in the various regions. We have studied numerically the scaling functions which satisfy the following equations for the uniaxial case

$$f_L(x,z)=2\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\, y^2(1+2y\Delta)\frac{1+x^2}{y^2+x^2+z^2}\times\left[(1+2y\Delta+y^2)^{5/4}f_T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1} \tag{3.22}$$

$$\begin{aligned}f_T(x,z)=\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\, y^2\Bigg\{&(1+2y\Delta-z^2)\frac{1+x^2+z^2}{y^2+x^2+z^2}\\&\times\left[(1+2y\Delta+y^2)^{5/4}f_L\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\\&+(1+2y\Delta+z^2)\frac{1+x^2+z^2}{y^2+x^2}\left[(1+2y\Delta+y^2)^{5/4}f_T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_L\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\Bigg\}\end{aligned} \tag{3.23}$$

where we have introduced the following notations

$$x=\kappa_\parallel/q;\quad y=q'/q;\quad z=\kappa_\Delta/q\ .$$

The numerical solutions are shown in Figs. 2 and 3. We have presented as reference the curve for the isotropic case which compared satisfactorily with the experimental results for Fe. This allows a better visualization of decoupling through the introduction of anisotropy, of the longitudinal and transverse part.

One clearly sees for ω_T that in region $[\mathrm{II}_{b_2}]$ characterized by $(\kappa_\parallel/q)(q/\kappa_\Delta)^2\approx 1$ (where $\kappa_\parallel/q>1$ and $\kappa_\Delta/q<1$) and $\kappa_\parallel/q\approx 1$, we effectively find the nondiffusive regime which expresses itself by a slower decay of the curve. The longitudinal part ω_L shows for small values of $\kappa_\parallel/q$ and κ_Δ/q, a characteristic bump.

It would be interesting to obtain an experimental confirmation of this feature. Krueger and Huber[26] made a similar calculation. However, as in their study of isotropic systems,[27] the use of a cutoff on the momentum severely restricts the prediction of their theory for small $\kappa_\parallel/q$ (cf. footnote 15 of Ref. 9).

Bearing in mind all the preceding remarks, we have also studied numerically the *planar* case for which the scaling functions satisfy

TABLE I. Asymptotic behaviors of linewidths for various exchange anisotropic systems in different regimes.

		Isotropic regime (II)			Anisotropic regime (I)	
			Hydrodynamic (II_b)		Hydrodynamic	critical
		Critical	Diffusive (II_{b_1})	Nondiffusive (II_{b_2})	(I_b)	(I_a)
Uniaxial	ω_L	$q^{5/2}$	$q^2\kappa_\parallel^{1/2}$	$q^2\kappa_\parallel^{1/2}$	$q^2\kappa_\parallel^2\kappa_\Delta^{-3/2}$	$q^4\kappa_\Delta^{-3/2}$
Ferro.	ω_T	$q^{5/2}$	$q^2\kappa_\parallel^{1/2}$	$\kappa_\parallel^{-3/2}\kappa_\Delta^4$	$\kappa_\Delta^{5/2}$	$\kappa_\Delta^{5/2}$
Planar	ω_L	$q^{5/2}$	$q^2\kappa_\parallel^{1/2}$	$\kappa_\parallel^{-3/2}\kappa_\Delta^4$	$\kappa_\Delta\kappa_\parallel^{3/2}$	$q^{3/2}\kappa_\Delta$
Ferro.	ω_T	$q^{5/2}$	$q^2\kappa_\parallel^{1/2}$	$q^2\kappa_\parallel^{1/2}$	$q^2\kappa_\parallel^{-1/2}\kappa_\Delta$	$q^{3/2}\kappa_\Delta$
	ω_L^T	$q^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^2\kappa_\Delta^{-1/2}$	$q^2\kappa_\Delta^{-1/2}$
Uniaxial Antiferro.	ω_T^T	$q^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\Delta^{3/2}$	$\kappa_\Delta^{3/2}$
	ω_L^0	$q^{3/2}$	$q^2\kappa_\parallel^{-1/2}$	$q^2\kappa_\parallel^{-1/2}$	$q^2\kappa_\Delta^{-1/2}$	$q^2\kappa_\Delta^{-1/2}$
	ω_T^0	$q^{3/2}$	$q^2\kappa_\parallel^{-1/2}$	$\kappa_\parallel^{-5/2}\kappa_\Delta^4$	$\kappa_\Delta^{3/2}$	$\kappa_\Delta^{3/2}$
	ω_L^T	$q^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^{3/2}$	$q^{3/2}$
Planar	ω_T^T	$q^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\parallel^{3/2}$	$\kappa_\Delta^{3/2}$	$\kappa_\Delta^{3/2}$
Antiferro.	ω_L^0	$q^{3/2}$	$q^2\kappa_\parallel^{-1/2}$	$\kappa_\parallel^{-5/2}\kappa_\Delta^4$	$\kappa_\Delta^{3/2}$	$\kappa_\Delta^{3/2}$
	ω_L^0	$q^{3/2}$	$q^2\kappa_\parallel^{-1/2}$	$q^2\kappa_\parallel^{-1/2}$	$q^2\kappa_\parallel^{-1/2}$	$q^{3/2}$

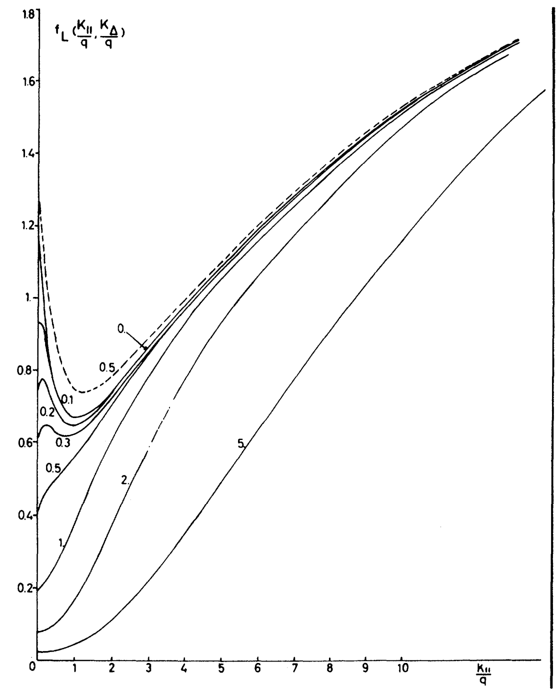

FIG. 2. Temperature dependence of the longitudinal (i.e., parallel to the easy axis of magnetization) scaling functions of uniaxial ferromagnets for various values of κ_Δ/q. The curve for $\kappa_\Delta = 0$ corresponds to the isotropic ferromagnet. The dashed curve refers to transverse part for $\kappa_\Delta/q = 0.5$.

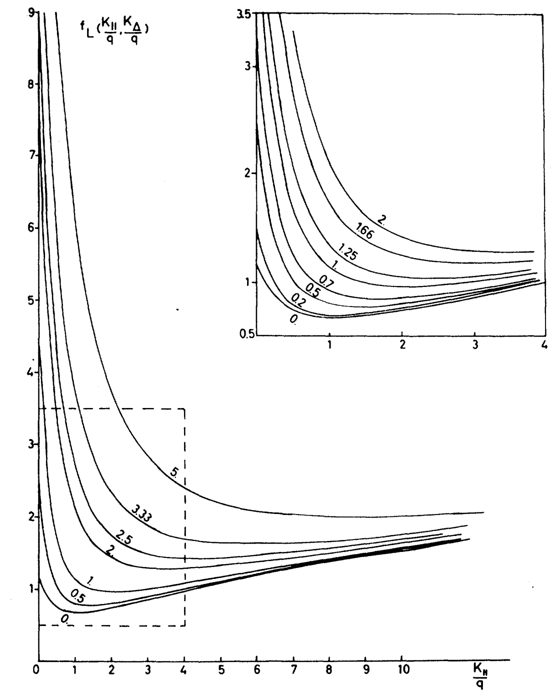

FIG. 4. Temperature dependence of the longitudinal (i.e., parallel to the easy plane of magnetization) scaling functions of planar ferromagnets for various values of κ_Δ/q. We recall that the roles of longitudinal and transverse parts are inverted in the uniaxial and planar cases. The insertion is given for clarity.

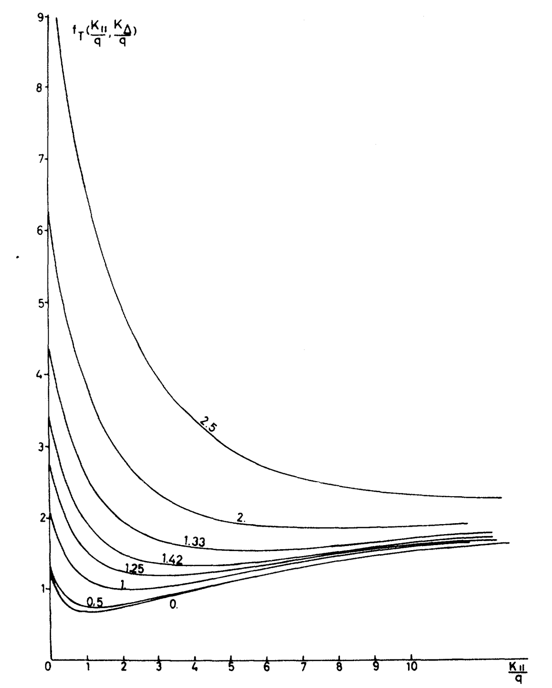

FIG. 3. Temperature dependence of the transverse scaling functions of uniaxial ferromagnets of various values of κ_Δ/q.

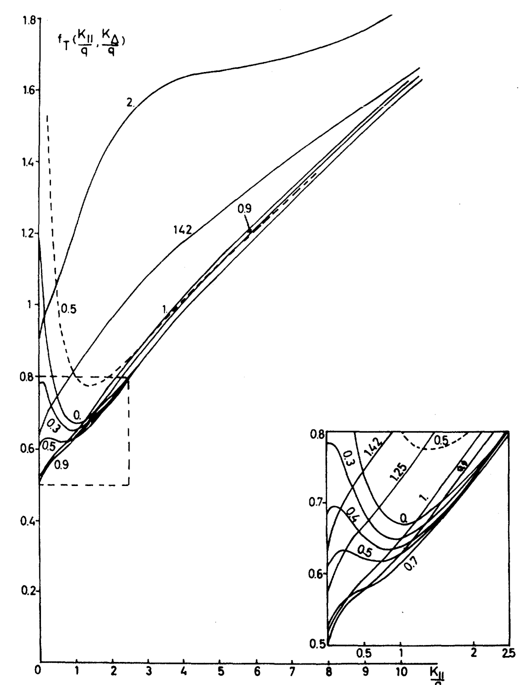

FIG. 5. Temperature dependence of the transverse scaling functions of planar ferromagnets for various values of κ_Δ/q. The insertion is given for clarity.

$$f_L(x,z)=\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\, y^2\left\{\frac{(x^2+2xy\Delta+z^2)(x^2+1)}{y^2+1}\right.$$
$$\times\left[(1+2y\Delta+y^2)^{5/4}f_T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_L\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}$$
$$\left.+\frac{(1+2y\Delta+z^2)(1+x^2+z^2)}{y^2+x^2}\left[(1+2y\Delta+y^2)^{5/4}f_L\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\right\}, \tag{3.24}$$

$$f_T(x,z)=2\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\, y^2(1+2y\Delta)\frac{1+x^2}{y^2+x^2+z^2}$$
$$\times\left[(1+2y\Delta+y^2)^{5/4}f_L\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{5/2}f_L\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}. \tag{3.25}$$

The longitudinal part is that relative to the plane of magnetization. In these curves (Figs. 4 and 5), the nondiffusive part appears in the longitudinal function, whereas the transverse function exhibits the bumps for $\kappa_\Delta/q<1$ and $\kappa_\parallel/q<1$. However, when κ_Δ/q grows, we note a behavior for f_T quite different from that of a uniaxial ferromagnet.

Let us also recall that the planar ferromagnet is formally equivalent to liquid helium.[2]

B. Antiferromagnets

The study of the antiferromagnets differs from that of the ferromagnets in that we must now consider both the critical behavior around $q=0$ and $q=\tau$ where the exchange integrals show their maximum value (τ is a vector of the reciprocal lattice). We are thus led to study coupled kinetic equations relative to the total ($q=0$) and staggered ($q=\tau$) magnetizations. Only the z component of the total magnetization is an invariant. Taking these differences into account and limiting ourselves again to the lowest-order kernel and the Markovian approximation we obtain for the uniaxial antiferromagnets, where we have put

$$\gamma^2=\frac{Z^2JKa^3}{4\pi^3}, \tag{3.26}$$

$$\omega_L^\tau=\int d\vec{q}\,'\frac{q^2+\kappa_\parallel^2}{q'^2+\kappa_\parallel^2+\kappa_\Delta^2}\frac{1}{\omega_T^0(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_T^\tau(q',\kappa_\parallel,\kappa_\Delta)}, \tag{3.27}$$

$$\omega_T^\tau=\frac{1}{2}\int d\vec{q}\,'\left(\frac{q^2+\kappa_\parallel^2+\kappa_\Delta^2}{q'^2+\kappa_\parallel^2+\kappa_\Delta^2}\frac{1}{\omega_L^0(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_T^\tau(q',\kappa_\parallel,\kappa_\Delta)}+\frac{q^2+\kappa_\parallel^2+\kappa_\Delta^2}{q'^2+\kappa_\parallel^2}\frac{1}{\omega_T^0(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_L^\tau(q',\kappa_\parallel,\kappa_\Delta)}\right), \tag{3.28}$$

$$\omega_L^0=\int d\vec{q}\,'\frac{(\vec{q}+\vec{q}\,')^2-q'^2}{q'^2+\kappa_\parallel^2+\kappa_\Delta^2}\frac{1}{\omega_T^\tau(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_T^\tau(q',\kappa_\parallel,\kappa_\Delta)}, \tag{3.29}$$

$$\omega_T^0=\frac{1}{2}\int d\vec{q}\,'\left(\frac{(\vec{q}+\vec{q}\,')^2-q'^2-\kappa_\Delta^2}{q'^2+\kappa_\parallel^2+\kappa_\Delta^2}\frac{1}{\omega_L^\tau(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_T^\tau(q',\kappa_\parallel,\kappa_\Delta)}\right.$$
$$\left.+\frac{(\vec{q}+\vec{q}\,')^2-q'^2+\kappa_\Delta^2}{q'^2+\kappa_\parallel^2}\frac{1}{\omega_T^\tau(q+q',\kappa_\parallel,\kappa_\Delta)+\omega_L^\tau(q',\kappa_\parallel,\kappa_\Delta)}\right). \tag{3.30}$$

Let us stress the fact that it is the only situation where we have the possibility of comparison with experiment. The linewidths are again homogeneous functions but the exponent is now $\frac{3}{2}$ and for example, for the staggered longitudinal part we have

$$\omega_L^\tau(q,\kappa_\parallel,\kappa_\Delta)=q^{3/2}f_L^\tau(\kappa_\parallel/q,\kappa_\Delta/q). \tag{3.31}$$

In the regime $q,\kappa_\Delta\ll\kappa_\parallel$, one then recovers the diffusive and nondiffusive region for ω_T^0.

The scaling functions satisfying

$$f_L^\tau(x,z)=2\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\frac{y^2(1+x^2)}{y^2+x^2+z^2}\left[(1+2y\Delta+y^2)^{3/4}f_T^0\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2}f_T^\tau\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}, \tag{3.32}$$

$$f_T^\tau(x,z)=\pi\int_0^\infty dy\int_{-1}^{+1}d\Delta\, y^2\left\{\frac{1+x^2+z^2}{y^2+x^2+z^2}\left[(1+2y\Delta+y^2)^{3/4}f_L^0\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2}f_T^\tau\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\right.$$

$$+\frac{1+x^2+z^2}{y^2+x^2}\left[(1+2y\Delta+y^2)^{3/4} f_T^0\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2} f_L^T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\Bigg\}, \tag{3.33}$$

$$f_L^0(x,z)=2\pi\int_0^\infty dy\int_{-1}^{+1} d\Delta\, y^2\frac{1+2y\Delta}{y^2+x^2+z^2}\left[(1+2y\Delta+y^2)^{3/4} f_T^T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2} f_T^T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}, \tag{3.34}$$

$$f_T^0(x,z)=\pi\int_0^\infty dy\int_{-1}^{+1} d\Delta\, y^2\Bigg\{\frac{1+2y\Delta-z^2}{y^2+x^2+z^2}\left[(1+2y\Delta+y^2)^{3/4} f_L^T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2} f_T^T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}$$

$$+\frac{1+2y\Delta+z^2}{y^2+x^2}\left[(1+2y\Delta+y^2)^{3/4} f_T^T\left(\frac{x}{(1+2y\Delta+y^2)^{1/2}};\frac{z}{(1+2y\Delta+y^2)^{1/2}}\right)+y^{3/2} f_L^T\left(\frac{x}{y};\frac{z}{y}\right)\right]^{-1}\Bigg\}, \tag{3.35}$$

have been calculated numerically and are reproduced in Figs. 6 and 7.

The nondiffusive behavior of ω_T^0 is characterized here by a faster decay of f_T^0 in the regime considered. The bumps characteristic of the conserved part are again present in f_L^0.

Because of the possibility of separating experimentally the longitudinal and transverse parts of the spin af in MnF_2, Schulhof, Nathans, Heller and Linz[16,17] were able to study the temperature and wave-number behavior of the linewidths.

We therefore propose a microscopically based interpretation of their experimental results as a function of the parameters $\kappa_\parallel/q$ and κ_Δ/q.

As they suggested in Fig. 18 of their paper,[17] we have drawn in Fig. 8 the following normalized longitudinal and transverse scaling functions

$$f_{L,T}^T\left(\frac{\kappa_\parallel}{q},\frac{\kappa_\Delta}{q}\right)\Big/ f_{L,T}^T\left(0,\frac{\kappa_\Delta}{q}\right). \tag{3.36}$$

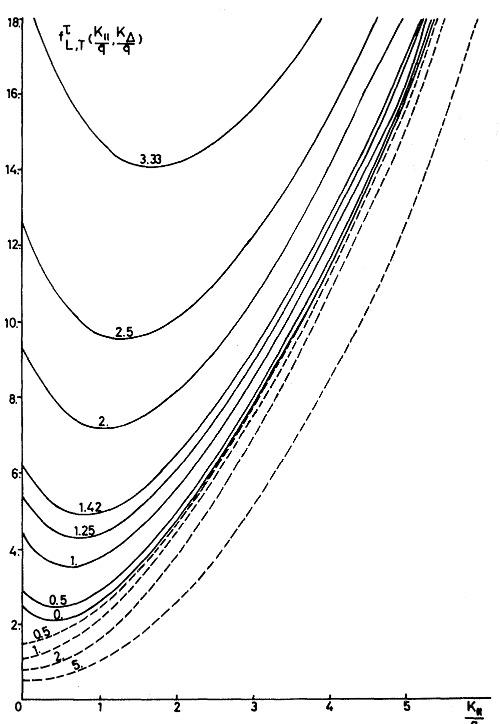

FIG. 6. Temperature dependences of the staggered longitudinal (dashed curves) and transverse (plain curves) scaling functions of uniaxial antiferromagnets for various κ_Δ/q.

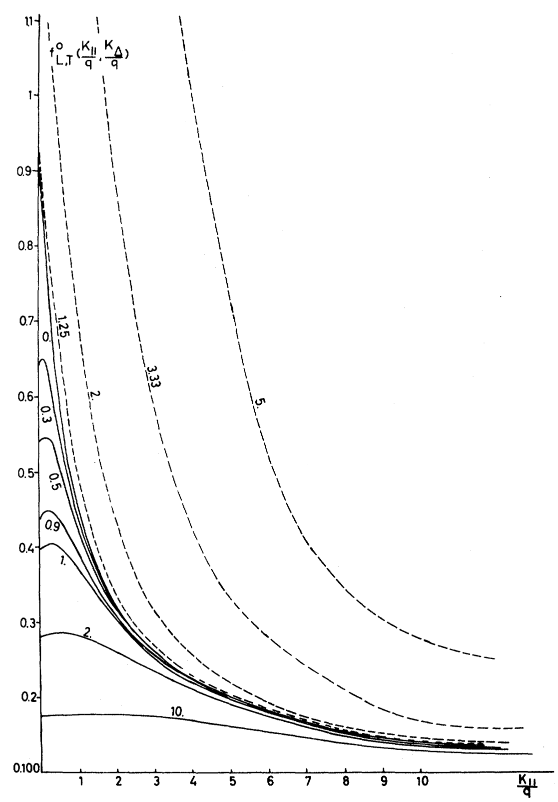

FIG. 7. Temperature dependence of the total longitudinal (plain curves) and transverse (dashed curves) scaling functions of uniaxial antiferromagnets for various κ_Δ/q.

It clearly appears that the agreement with the experimental results is better than when the latter were compared with the isotropic theoretical function. The influence of anisotropy seems thus not negligible. We have taken the value 0.054 Å^{-1} proposed by RW,[12] for the anisotropy parameter κ_Δ.[28] Although, when $\kappa_\parallel/q<1$ the comparison with experimental results of the transverse part f_T^τ is gratifying, in the region $\kappa_\parallel/q<1$, where most of the experimental results lie, the influence of anisotropy is not so evident. Indeed, it appears that the transverse part f_T^τ uncouples more slowly from the isotropic function than the longitudinal part.

This reasonable fit with experiment comes from the fact that we have taken $\kappa_\parallel$ as independent variable. So, we were able to bypass the difficulty related to the molecular field approximation in the static behavior of the correlation function ($\nu_{\mathrm{mol.field}}=\frac{1}{2}$ instead of $\nu_{\mathrm{DSA}}=\frac{2}{3}$ or $\nu_{\mathrm{exp}}=0.634\pm0.2$). This could be understood as an indication that the Weiss limit does not play in the dynamics of spin systems a dominant role at the critical point.

The staggered part of the *planar* antiferromagnet which has been obtained along the same line is given in Fig. 9.

The study of anisotropy-exchange spin systems, using the method developed by Resibois and De Leener thus permits a microscopic verification, in the Weiss limit, of the phenomenological scaling laws proposed by Riedel and Wegner.

Owing to a satisfactory interpretation of the ex-

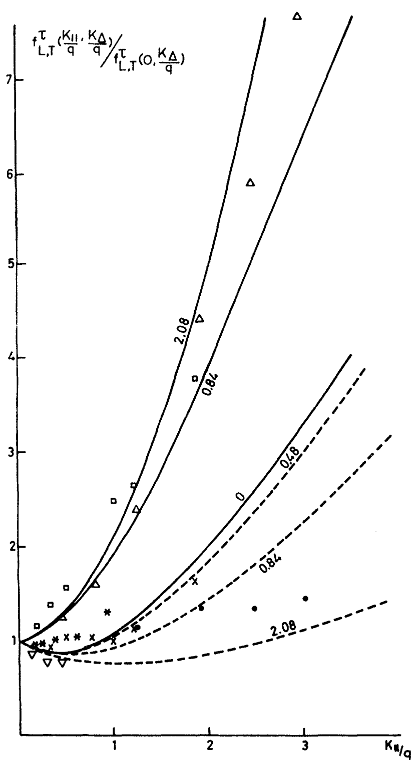

FIG. 8. Temperature dependence of the staggered longitudinal (plain curves) and transverse (dashed curves). scaling functions for uniaxial antiferromagnets. The curves have been normalized to 1. We compare the theoretical results to experimental data on MnF_2 where: Δ and $\square$ correspond to the longitudinal part for $\kappa_\Delta/q=2.08$ and 0.84, respectively; $\bullet$, $\times$, $*$, and ∇ correspond to the transverse part for κ_Δ/q 2.08, 0.84; 0.42, and 0.21, respectively.

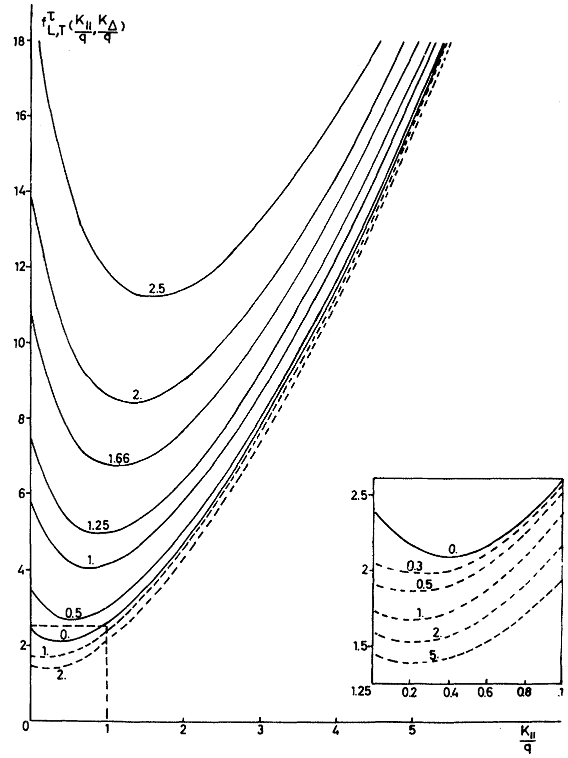

FIG. 9. Temperature dependence of the staggered longitudinal (dashed curves) and transverse (plain curves) scaling functions for planar antiferromagents.

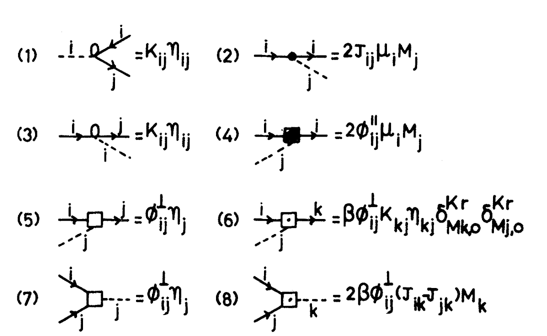

FIG. 10. Elementary vertices and their contribution.

perimental results on MnF_2 (uniaxial antiferromagnet) as a function of the parameters $\kappa_{\parallel}/q$ and κ_{Δ}/q, we have proposed the scaling functions for uniaxial and planar ferromagnets, hoping they might suggest new experiments.

ACKNOWLEDGMENTS

We would like to thank Professor P. Résibois for suggesting this work and for a critical proof reading. We acknowledge fruitful discussions with Dr. P. Borckmans, G. Dewel, and D. Walgraef. We also thank Professor P. Heller for kindly making available to us the precise experimental data on MnF_2. One of us (C. B.) wishes to express his gratitude to Professor I. Prigogine for his kind hospitality.

APPENDIX

The kernels which appeared in the kinetic equations, Eqs. (2.7) and (2.8), were calculated according to the rules presented in R-DL[4] taking into account the following modifications introduced because of the anisotropy.

The contributions of the elementary vertices (Figs. 10 and 11 of R-DL) are now given in Fig. 10.

One must also pay attention to the fact that, because of the anisotropy, plain and dotted lines are renormalized by different af, respectively Γ^{+-} and Γ^{zz}. Furthermore, the traces are taken in the same way as we have taken the magnitude of the spin into account by working with reduced variables [(Eqs. (2.5) and (2.6)].

---

*PA Service de Physique du Solide et de Résonance Magnétique Centre d'Etudes Nucléaires de Saclay, BP No. 2 91190 GiP/Yvette, France.

†Part of a thesis to be presented by C.B. at the Université Paris Sud-Orsay, for the degree of Docteur ès Sciences. This thesis is registered with the CNRS under the number AO 10531.

[1] P. G. de Gennes, in *Magnetism*, edited by G. T. Pado and H. Suhl (Academic, New York, 1963), Vol. 3, p. 115; W. Marshall and R. D. Lowde, Rep. Prog. Phys. B 31, 705 (1968).

[2] (a) B. I. Halperin and P. C. Hohenberg, Phys. Rev. 177, 952 (1969); (b) P. C. Hohenberg, in *Proceedings of the Enrico Fermi Summer School of Physics, Varenna* (Academic, New York, 1972).

[3] A. Tucciarone, H. Y. Lau, L. M. Corliss, A. Delapalme, and J. M. Hastings, Phys. Rev. B 4, 3206 (1971); A. Tucciarone, J. M. Hastings, and L. M. Corliss, *ibid.* 8, 1003 (1973).

[4] P. Résibois and M. De Leener, Phys. Rev. 178, 806 (1969); 178, 819 (1969).

[5] K. Kawasaki, J. Phys. Chem. Solids 28, 1277 (1967); Prog. Theor. Phys. 39, 285 (1968).

[6] P. Résibois and G. Dewel, Ann. Phys. (N.Y.) 69, 299 (1972).

[7] An extensive review is contained in K. G. Wilson and J. B. Kogut, Phys. Rep. 12C, 76 (1974).

[8] P. C. Hohenberg, M. De Leener, and P. Résibois, Physica 65, 506 (1973).

[9] P. Résibois and C. Piette, Phys. Rev. Lett. 24, 514 (1970); Phys. Lett. A 42, 531 (1973).

[10] J. Als-Nielsen, Phys. Rev. Lett. 25, 730 (1970); R. Kahn and G. Parette, J. Phys. 32, C1-523 (1970). The agreement with the data for Fe, which is an itinerant ferromagnet, may appear surprising at first sight. It would be an argument in favor of the universality of scaling laws which express that the system is not very sensitive to the exact microscopic structure of the Hamiltonian but depends on the over-all characteristics.

[11] L. P. Kadanoff, in Ref. 2(b); see also, R. B. Griffiths, Phys. Rev. Lett. 24, 1479 (1970), and G. Paul and H. E. Stanley, Phys. Rev. B 5, 2578 (1972).

[12] E. Riedel and F. Wegner, Z. Phys. 225, 195 (1969); Phys. Lett. A 32, 273 (1970); J. Appl. Phys. 42, 1383 (1971); J. Phys. 32, C1-519 (1970); see also, T. Kawasaki, Prog. Theor. Phys. 41, 1381 (1969).

[13] E. Riedel and F. Wegner, Phys. Rev. Lett. 24, 730 (1970).

[14] H. Mori, Prog. Theor. Phys. 33, 423 (1965).

[15] K. Kawasaki, Ann. Phys. (N.Y.) 61, 1 (1970); Prog. Theor. Phys. 39, 1183 (1968); 40, 11 (1968); 40, 706 (1968); 40, 930 (1968).

[16] M. P. Schulhof, P. Heller, R. Nathans, and A. Linz, Phys. Rev. B 1, 2304 (1970).

[17] M. P. Schulhof, R. Nathans, P. Heller, and A. Linz, Phys. Rev. B 4, 2254 (1971).

[18] A. M. Gottlieb and P. Heller, Phys. Rev. B 3, 3615 (1971); M. T. Hutchings, M. P. Schulhof, and H. J. Guggenheim, Phys. Rev. B 5, 154 (1974).

[19] Preliminary results concerning the uniaxial antiferromagnetic case were published in Phys. Lett. A 47, 95 (1974).

[20] M. E. Fisher, J. Math. Phys. 5, 944 (1964); M. E. Fisher and R. J. Burford, Phys. Rev. 156, 583 (1967); M. E. Fisher and D. S. Ritchie, Phys. Rev. B 5, 2668 (1972).

[21] C. Bagnuls, Bull. Cl. Sci. Acad. R. Belgique 60, 191 (1974).

[22] J. Winter, J. Phys. 33, 1099 (1972).

[23] T. Moriya, Prog. Theor. Phys. 28, 371 (1962).

[24] M. De Leener showed [Physica (Utr.) 56, 62 (1971)] that this statement was exact to all orders in the asymptotic limit.

[25] L. Van Hove, Phys. Rev. 95, 1374 (1954).

[26] D. A. Krueger and D. L. Huber, Phys. Rev. B 1, 3152 (1970).

[27] D. L. Huber and D. A. Krueger, Phys. Rev. Lett. 24, 111 (1970).

[28] We have not made the comparison for FeF_2 as we have no detailed experimental data for this substance.