Self-consistent calculation of the thermodynamic potential for superfluid states in ^{3}He near T_c

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A self-consistent diagram technique is developed in terms of a 4×4 Green's function and a symmetrized $4 \times 4 \times 4 \times 4$ interaction vertex to investigate the effect of spin fluctuations on the self-energy and thermodynamic potential in superfluid ^{3}He. In addition to reproducing the results of Brinkman, Serene, and Anderson (BSA), the resulting $\tilde{T}$-matrix approximation leads to the appearance of a new class of anomalous diagrams not considered by BSA. These new contributions involve a coupling of the particle-particle and particle-hole channels and give rise to an effective paramagnetic enhancement of the particle-particle T matrix below T_c. They are shown to further increase the stability of the Anderson-Morel state relative to that of the Balian-Werthamer state near T_c. Results are stated in terms of the five fourth-order invariants. The specific-heat discontinuity is calculated and discussed in the context of recent experimental results.

I. INTRODUCTION AND DISCUSSION OF RESULTS

In a recent paper Brinkman, Serene, and Anderson[1] (BSA) have shown in detail that, in superfluid ^{3}He below T_c, spin fluctuation effects,[2] if they are sufficiently strong, can stabilize the anisotropic Anderson-Morel (AM) state with respect to the pseudoisotropic Balian-Werthamer (BW) state. The anomalous diagrams included in their calculation of the free energy are the same as those considered previously by one of the present authors[3] in the self-consistent "T-matrix approximation." However, in view of the recent self-consistent theory of the random-phase-approximation (RPA) susceptibility in superfluid ^{3}He by one of the present authors,[4] it seems natural to include an additional class of anomalous diagrams in the free-energy calculation.

According to the self-consistent-approximation scheme[3] the 4×4 self-energy $\tilde{\Sigma}$ is given by the functional derivative

$$\tilde{\Sigma} = 2i\delta\Phi/\delta\tilde{G}\,,$$

where Φ is a certain approximate starting functional, related to the thermodynamic potential Ω, and $\tilde{G}$ is the full 4×4 single-particle Green's function. The irreducible vertex part $\tilde{\Gamma}$, occuring in the Bethe-Salpeter equation for the two-particle correlation function, is then given by[4]

$$\tilde{\Gamma} = -i\delta\tilde{\Sigma}/\delta\tilde{G}\,.$$

It was shown in Ref. 4 that in the simplest approximation one can think of, i.e., Hartree-Fock for Φ, or, equivalently, RPA for the susceptibility, $\tilde{\Gamma}$ becomes a completely antisymmetric tensor of the fourth rank with respect to the combined particle-hole and spin variables. The most interesting consequence is the appearance of new types of anomalous diagrams in both the particle-hole and the particle-particle scattering channels.

Since it is absolutely necessary to include these new types of anomalous diagrams in a self-consistent calculation of the RPA susceptibility below T_c, it appears natural to calculate the free energy in the corresponding approximation. At first glance one might assume that the contribution of the particle-particle channel to the free energy would be negligible in comparison to the strongly paramagnon-enhanced particle-hole channel. In a normal nearly-ferromagnetic system this is indeed true since, as is well known, the summing of particle-particle ladder diagrams merely leads to a replacement of a repulsive bare interaction by a *smaller* effective interaction, or "pseudopotential." Below T_c, however, new processes, which have no counterpart in the normal system, can occur: A particle-particle pair can, with the help of the anomalous propagators F and $\bar{F}$, effectively propagate in an intermediate state as a particle-hole pair as shown in Fig. 1. This intermediate particle-hole propagation can lead to an effective paramagnetic enhancement of the particle-particle T matrix below T_c. In a similar manner, the particle-hole channel can contain elements of the particle-particle channel. Our detailed analysis shows that proper inclusion of this new class of anomalous diagrams leads to substantial correction to the BSA free-energy difference between the AM and BW states.

Let us briefly describe more explicitly this approximation, which we call the generalized $\tilde{T}$-matrix approximation. It consists of setting up the Bethe-Salpeter equation for $\tilde{T}$ with an irreducible vertex part $\tilde{\Gamma}$, identical to that of the RPA correlation function, and then calculating the self-energy from the relation

$$\tilde{\Sigma} = 2i\,\delta\Phi/\delta\tilde{G} = -2i\tilde{T}\tilde{G}\,.$$

This approximation will be justified more directly by rewriting the S-matrix expression for the thermodynamic potential Ω (Ref. 5) in terms of Nambu four-component field operators and vertex functions $\tilde{\Gamma}$. In this way one sees that one obtains approximate functionals Φ from diagrams made up of $\tilde{G}$ and $\tilde{\Gamma}$. The particular functional $\Phi\{\tilde{G};\tilde{\Gamma}\}$ for the $\tilde{T}$-matrix approximation corresponds to the particle-hole T-matrix approximation for the normal system[6] and yields the self-energy given above. It includes correctly the infinite partial sums of diagrams which we believe are most important in a nearly-ferromagnetic superfluid system.

Finally, the thermodynamic potential can be obtained from the integral over the coupling constant λ,[3]

$$\Omega - \Omega_0 = -\tfrac{1}{4}i\int_0^1 \frac{d\lambda}{\lambda}\,\tilde{G}\tilde{\Sigma}\,,$$

by inserting the expression for $\tilde{\Sigma}$ given above (Ω_0 is the thermodynamic potential of the noninteracting system). This program is carried through with the aim of determining the difference in Ω between the AM and BW states.

We restrict our calculations in the present paper to temperatures near T_c since the Bethe-Salpeter equation for $\tilde{T}$ is rather complicated; there are four separate sets of four coupled equations for the 16 components of $\tilde{T}$ with respect to the particle-hole space and, in addition, each of these quantities depends on four spin variables. Accordingly, we solve these equations by systematic expansion in a series in terms of anomalous propagators F and $\bar{F}$, up to fourth order, corresponding to the fourth-order term in Δ of BSA. The spin variables in these equations are treated by expanding the components of $\tilde{\Gamma}$ and $\tilde{T}$ in terms of the complete set of tensor products of two Pauli matrices.[3]

Our generalized $\tilde{T}$-matrix approximation reproduces the result of BSA that the feedback effect due to the change in the spin fluctuations just below T_c can stabilize the AM state. The new anomalous diagrams considered here yield a correction to the fourth-order free-energy difference of BSA that also favors the AM state. The actual magnitude of this correction is, however, rather difficult to calculate accurately. An estimate yields a 20% correction to the BSA free-energy difference.

We have expressed the fourth-order spin-fluctuation energy for p-wave pairing in terms of the five invariants introduced by Mermin and Stare[7] and by Brinkman and Anderson.[8] We find that the various coefficients are affected quite differently by the new terms and thus the magnitude of the correction depends strongly on the type of state under consideration. For example, for the BW state the correction is large while for an AM state in a magnetic field in the A_1 phase[8,9] the new corrections have no effect at all.

We have also expressed the specific-heat discontinuity at T_c in terms of the Brinkman-Anderson spin-fluctuation parameter δ,[1,8] and find that the effect of the new corrections is that a smaller δ is now required to fit the specific-heat data. The specific-heat discontinuity at the polycritical point is reduced from the BSA value. We also suggest that the T_{AB}/T_c vs δ curve is shifted because the critical value of δ at which the free energies of the AM and BW states become equal is reduced from the BSA value of 0.47 to 0.41.

Recently Osheroff and Anderson[11] have noted that within the framework of BSA, a larger value of δ is required to fit the specific-heat discontinuity at the melting pressure than to fit NMR data or the measured T_{AB}/T_c ratio. We have also calculated the effect of the new anomalous diagrams on the determination of δ from the NMR data. This δ is shifted down by approximately the same amount as the specific heat δ. We also point out that the processes shown in Fig. 1 may have a significant effect on other quantities in the superfluid phase such as the Landau-Fermi liquid parameters and various collective modes.

In Sec. II we develop the general theory. This theory is applied in Sec. III to the calculation of the thermodynamic potential near T_c and a discussion of the results is given.

II. THERMODYNAMIC POTENTIAL IN THE $\tilde{T}$-MATRIX APPROXIMATION

The expression for the thermodynamic potential Ω as an integral over the coupling constant λ is

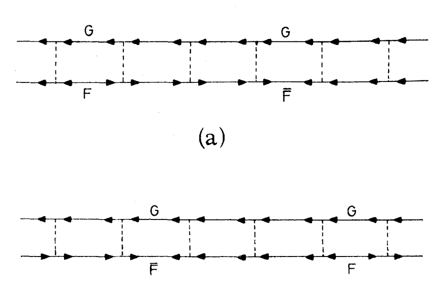

FIG. 1. Typical examples of anomalous diagrams dictated by self-consistency requirements. The particle-particle scattering (a) contains an intermediate particle-hole ladder, while the particle-hole scattering (b) contains a particle-particle ladder.

given by[3] (all expressions in this section are written for zero temperature):

$$\Omega - \Omega_0 = -\tfrac{1}{4} i \int_0^1 \frac{d\lambda}{\lambda} \tilde{G}(\bar{3}, \bar{1}^\dagger)\tilde{\Sigma}(\bar{1}^\dagger, \bar{3}) . \quad (1)$$

Here and in the following we use the notation: $1 = k_1\omega_1\alpha_1$, where $\alpha_1 = \uparrow, \downarrow, \uparrow^\dagger, \downarrow^\dagger$, etc.; the $1^\dagger$ means that $\alpha_1 \to \alpha_1^\dagger = \uparrow^\dagger, \downarrow^\dagger, \uparrow, \downarrow$, etc.; the bars on top of repeated numbers mean summation or integration over all the variables. Ω_0 is the thermodynamic potential of the noninteracting system.

In our "generalized $\tilde{T}$-matrix approximation" the self-energy is equal to

$$\tilde{\Sigma}(1^\dagger, 3) = 2i \frac{\delta\Phi}{\delta\tilde{G}(3, 1^\dagger)} = -2i\tilde{T}(1^\dagger, \bar{2}^\dagger, 3, \bar{4})\tilde{G}(\bar{4}, \bar{2}^\dagger) , \quad (2)$$

where the $\tilde{T}$ matrix is defined by the equation

$$\tilde{T}(1^\dagger, 2^\dagger, 3, 4) = \tilde{\Gamma}(1^\dagger, 2^\dagger, 3, 4) + i\tilde{\Gamma}(1^\dagger, \bar{5}^\dagger, \bar{6}, 4)\tilde{G}(\bar{6}, \bar{7}^\dagger) \times\tilde{G}(\bar{8}, \bar{5}^\dagger)\tilde{T}(\bar{7}^\dagger, 2^\dagger, 3, \bar{8}) . \quad (3)$$

The irreducible vertex part $\tilde{\Gamma}$ occuring in this equation is the same as that in the corresponding equation for the two-particle correlation function in the Hartree-Fock approximation.[4] The analytical expression of $\tilde{\Gamma}$ in terms of the bare (or model) interaction vertex function $\Gamma(1, 2, 3, 4)$ (where $1 = k_1\omega_1\sigma_1$, $\sigma_1 = \uparrow, \downarrow$, etc.) can be written as follows:

$$\tilde{\Gamma}(1^\dagger, 2^\dagger, 3, 4) = \tfrac{1}{2}[\Gamma(1, 2, 3, 4) + \Gamma(3, 4, 1, 2) - \Gamma(1, 3, 2, 4) - \Gamma(4, 2, 3, 1) - \Gamma(1, 4, 3, 2) - \Gamma(3, 2, 1, 4)] . \quad (4)$$

Here the prescription is to retain only the one Γ on the right-hand side which has the correct directions of arrowheads corresponding to $\alpha_1^\dagger$, $\alpha_2^\dagger$, α_3, α_4: outgoing (†) for the first two numbers and ingoing (no †) for the last two numbers. All the other components of $\tilde{\Gamma}$, for instance, the one for three ingoing and one outgoing arrows, vanish identically. Since the bare vertex function $\Gamma(1, 2, 3, 4)$ is taken to be antisymmetrized[5] with respect to 1 and 2, and also with respect to 3 and 4, one sees from Eq. (4) that $\tilde{\Gamma}$ is antisymmetric with respect to the interchange of any two variables.

The origin of this $\tilde{T}$-matrix approximation can be elucidated by constructing the functional Φ from the expression for $\Omega - \Omega_0$ in terms of the S matrix[5] and H_{int}, where

$$H_{\text{int}} = \tfrac{1}{4}c^\dagger(\bar{1})c^\dagger(\bar{2})\Gamma(\bar{1}, \bar{2}, \bar{3}, \bar{4})c(\bar{4})c(\bar{3}) . \quad (5)$$

In Ref. 3 it was shown that the starting functionals Φ of the self-consistent approximations can be obtained from the diagrammatic approximations to $\Omega - \Omega_0$ by replacing the zeroth-order Green's functions by dressed ones. Since we have to consider in the superfluid state not only the normal pairings $c^{\cdot}c^{\dagger\cdot}$ but also the anomalous pairings, $c^{\cdot}c^{\cdot}$ and $c^{\dagger\cdot}c^{\dagger\cdot}$, it is convenient to introduce a new interaction Hamiltonian defined by

$$\begin{aligned} H'_{\text{int}} &= \tilde{\phi}^\dagger(\bar{1})\tilde{\phi}^\dagger(\bar{2})\tilde{\Gamma}(\bar{1}^\dagger, \bar{2}^\dagger, \bar{3}, \bar{4})\tilde{\phi}(\bar{4})\tilde{\phi}(\bar{3}) \\ &= \tfrac{1}{2}[c^\dagger(\bar{1})c^\dagger(\bar{2})\Gamma(\bar{1}, \bar{2}, \bar{3}, \bar{4})c(\bar{4})c(\bar{3}) \\ &\quad + c(\bar{1})c(\bar{2})\Gamma(\bar{4}, \bar{3}, \bar{2}, \bar{1})c^\dagger(\bar{4})c^\dagger(\bar{3}) \\ &\quad - c^\dagger(\bar{1})c(\bar{2})\Gamma(\bar{1}, \bar{4}, \bar{3}, \bar{2})c^\dagger(\bar{4})c(\bar{3}) \\ &\quad - c(\bar{1})c^\dagger(\bar{2})\Gamma(\bar{3}, \bar{2}, \bar{1}, \bar{4})c(\bar{4})c^\dagger(\bar{3}) \\ &\quad - c^\dagger(\bar{1})c(\bar{2})\Gamma(\bar{1}, \bar{3}, \bar{2}, \bar{4})c(\bar{4})c^\dagger(\bar{3}) \\ &\quad - c(\bar{1})c^\dagger(\bar{2})\Gamma(\bar{4}, \bar{2}, \bar{3}, \bar{1})c^\dagger(\bar{4})c(\bar{3})] . \end{aligned} \quad (6)$$

Here we have defined, in analogy to the Nambu formalism four-component field operators by

$$\tilde{\phi}^\dagger(1) = (c_\uparrow^\dagger(k_1t_1), c_\downarrow^\dagger(k_1t_1), c_\uparrow(k_1t_1), c_\downarrow(k_1t_1)) , \quad (7)$$

such that the Green's function is equal to

$$\tilde{G}(1, 2) = -i\langle T\tilde{\phi}(1)\tilde{\phi}^\dagger(2)\rangle . \quad (8)$$

If we now replace in the S-matrix expression for $\Omega - \Omega_0$ the H_{int} by the H'_{int} of Eq. (6) and take, as in the normal phase, only the pairings

$$\tilde{\phi}(1)^{\cdot}\tilde{\phi}^\dagger(2)^{\cdot} = i\tilde{G}_0(1, 2) , \quad (9)$$

but not $\phi^{\cdot}\phi^{\cdot}$ and $\phi^{\dagger\cdot}\phi^{\dagger\cdot}$, we obtain the same types of diagrammatic contributions (with the correct signs and powers of i) as from the exact expression. This is because Eq. (6) contains all six different classes of permutations of the operators c and $c^\dagger$ in H_{int} [see Eq. (5)]. However, the numerical coefficient of each diagram must be determined by comparing it with the actual number of corresponding diagrams in the $c, c^\dagger, \Gamma$ technique. This is similar to the procedure in the normal phase where the factors in the diagrams in the symmetrized technique are determined by a comparison with those in the nonsymmetrized technique.[5] The diagrams of the $\tilde{G}, \tilde{\Gamma}$ technique merely serve as a guide to the different methods of pairing in the superfluid phase.

The functional Φ of the $\tilde{T}$-matrix approximation corresponds now to pairing the operators $\phi, \phi^\dagger$, of a given H'_{int} only with the operators of the two adjacent H'_{int}. In this way one obtains for the nth-order contribution in $\tilde{\Gamma}$, if we introduce an overall additional factor of $\frac{1}{2}$ (for the factor $1/n$ see Ref. 5):

$$\Phi^{(n)} = -(1/2n)\tilde{G}(\bar{3}, \bar{1}^\dagger)\tilde{T}^{(n)}(\bar{1}^\dagger, \bar{2}^\dagger, \bar{3}, \bar{4})\tilde{G}(\bar{4}, \bar{2}^\dagger) . \quad (10)$$

Inserting Eq. (10) into the middle term of Eq. (2) and making use of the antisymmetry properties of $\tilde{\Gamma}$ one finds that in fact to each order n the right-hand side of this equation is borne out. The numerical factors associated with the different diagrammatic contributions to $\Phi^{(n)}$ are obtained by inserting into Eq. (10) the different components of the $\tilde{T}$-matrix given by the equations in Figs. 2 and 3. A comparison of these numerical factors with the actual factors (obtained from the exact S-matrix expression by considering the corresponding pairings of the c and $c^\dagger$ operators) shows that our $\tilde{T}$-matrix approximation is correct in all orders $n \geq 3$. The first-order diagrams (Hartree-Fock, see Ref. 4) are overcounted two times, and the second-order diagrams three times. However, in calculating the spin-fluctuation contribution to Ω we shall subtract out the first-order contribution anyway (see below). The second-order diagrams could be subtracted also, but since only infinite-order collective effects are of interest this error can be neglected.

The spin-fluctuation contribution to the thermodynamic potential is obtained now from Eq. (1) by inserting Eq. (2) and subtracting from this expression the first-order contribution in $\tilde{\Gamma}$:

$$\Delta\Omega^s = -\frac{1}{2}\int_0^1 \frac{d\lambda}{\lambda}\, \tilde{G}(\bar{3},\bar{1}^\dagger)\tilde{G}(\bar{4},\bar{2}^\dagger) \times [\tilde{T}(\bar{1}^\dagger,\bar{2}^\dagger,\bar{3},\bar{4}) - \tilde{\Gamma}(\bar{1}^\dagger,\bar{2}^\dagger,\bar{3},\bar{4})]. \qquad (11)$$

We have to calculate now the $\tilde{T}$ matrix from Eq. (3), with the help of Eq. (4). This is a set of 2^4 equations for the 2^4 components with respect to the particle-hole space (having components with and without the dagger). This set of equations can be reduced in analogy to that for the correlation function[4] to four separate sets each consisting of three coupled equations for three independent components. Two sets of these equations are shown diagrammatically in Figs. 2 and 3. The independent particle-hole channel components are denoted by T, T', T'', and those of the particle-particle channel by S, S', and S''. The full-dotted vertices denote the outgoing arrowheads, the undotted vertices denote the ingoing arrowheads. Note that, in contrast to Γ and T in the normal phase, the components of $\tilde{T}$ are *not* restricted to only those having two ingoing and two outgoing arrows. The other two sets of equations for the components of the $\tilde{T}$ matrix can be obtained from the diagrammatic equations in Figs. 2 and 3 by replacing the dotted vertices by undotted ones, and vice versa.

These equations shown in Figs. 2 and 3 are handled by expanding all quantities in terms of the complete set of tensor products of two Pauli matrices, $\tau^\nu\tau^\mu$ (where $\nu, \mu = 0, 1, 2, 3$, and 0 denotes the unit matrix), as well as in a power series in terms of the anomalous propagators F and $\bar{F}$. For instance, one has for the component T the expansion:

$$T(1,2,3,4) = \sum_{\nu,\mu=0}^{3} [t^{(0)}_{\nu\mu}(1,2,3,4) + t^{(2)}_{\nu\mu}(1,2,3,4) + t^{(4)}_{\nu\mu}(1,2,3,4) + \cdots] \times \tau^{\nu}_{\sigma_1\sigma_4}\tau^{\mu}_{\sigma_2\sigma_3}. \qquad (12)$$

Here the superscripts (0), (2), (4), ... denote the order in F and $\bar{F}$ and, of course, the numbers on the right-hand side of Eq. (12) no longer include the spin variables. The spin-conserving form of the bare (or model) interaction function Γ can be written as follows:

$$\Gamma(1,2,3,4) = \sum_{\nu=0}^{3} V_\nu(1,2,3,4)\,\tau^{\nu}_{\sigma_1\sigma_4}\tau^{\nu}_{\sigma_2\sigma_3} = \sum_{\nu=0}^{3} V'_\nu\,\tau^{\nu}_{\sigma_1\sigma_2}\tau^{\nu}_{\sigma_4\sigma_3}, \qquad (13)$$

where the potentials are related by

$$V_1 = V_2 = V_3;\quad V'_0 = V'_1 = V'_3 = \tfrac{1}{2}(V_1 + V_0);\quad V'_2 = \tfrac{1}{2}(3V_1 - V_0). \qquad (14)$$

Upon inserting the first or the second form of Γ given in Eq. (13) into the equations shown in Figs. 2 and 3 one can separate everywhere the spin variables σ_1, σ_4 from σ_2, σ_3 in the same way as in Eq. (12). Then the equations can be solved by succes-

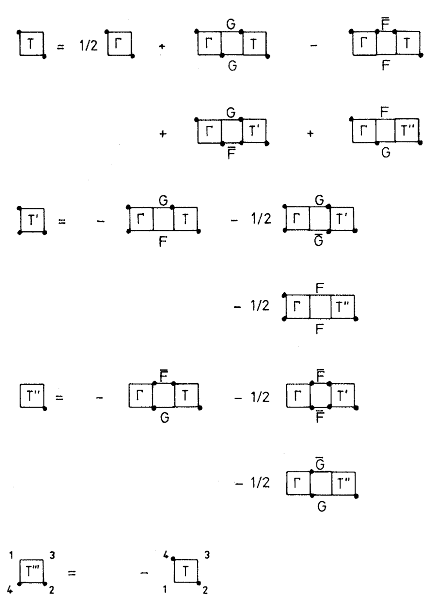

FIG. 2. Diagrammatic equations for the particle-hole components T, T', T'', of the $\tilde{T}$ matrix. The last equation shows the relation between the fourth component T''' and T. The dotted (undotted) vertices denote outgoing (ingoing) arrows. The full lines G and $\bar{G}$ are the normal components, and F and $\bar{F}$ are the anomalous ones of the 4×4 Green's function $\tilde{G}$.

sively equating equal powers of F and $\bar F$, and taking traces with respect to the spin variables. We make the usual approximation[6] of replacing the functions $V_0(\vec k_1, \vec k_2, \vec k_3, \vec k_4)$ and $V_1(\vec k_1, \vec k_2, \vec k_3, \vec k_4)$ by functions that are averaged in an appropriate way over the momentum variables such that they depend only on $\vec k_1 - \vec k_4 = \vec q$ for the particle-hole or $\vec k_1 + \vec k_4 = \vec q$ for the particle-particle channel. Further, we assume for simplicity that $V_0 = -V_1$, and thus $V_0' = V_1' = V_3' = 0$ (as is true for the repulsive contact interaction model where $V_0 = -\frac{1}{2}I$ and $V_1 = \frac{1}{2}I$).

For purposes of later reference we quote, in Appendix A, the complete results for the solutions of the equations in Figs. 2 and 3, up to and including the fourth order in F and $\bar F$. The enhancement factor due to the spin fluctuations occuring in these expressions is given by $[q = (\vec q, q_0)]$

$$t^{(0)}_{\nu\mu}(q) = \tfrac{1}{2} V_\nu(\vec q)[1 - V_\nu(\vec q)\chi(q)]^{-1}\delta_{\nu\mu}\,, \tag{15}$$

and the reduction factor due to the iteration in the particle-particle channel is

$$\bar t^{(0)}_{22}(q) = \tfrac{1}{2} V_2(\vec q)[1 + V_2(\vec q)\bar\chi(q)]^{-1}\,. \tag{16}$$

The generalized unenhanced "susceptibilities" occuring in the Eqs. (A1), (A2), (15), and (16), are defined as follows:

$$\begin{aligned}
&\chi(q) = 2iG(q+k)G(k)\,,\\
&\bar\chi(q) = -2iG(q-k)G(k)\,,\\
&\chi^{\nu\mu}_{\bar F F}(q) = -i\,\mathrm{Tr}[\bar\tau^{\nu}\bar F(q+k)\tau^{\mu} F(k)]\,,\\
&\chi^{\nu\mu}_{G\bar F}(q) = iG(q+k)\,\mathrm{Tr}[\tau^{\nu}\tau^{\mu}\bar F(k)]\,,\\
&\chi^{\nu\mu}_{FG}(q) = i\,\mathrm{Tr}[\tau^{\nu}F(q+k)\tau^{\mu}]G(k)\,,\\
&\chi^{\nu\mu}_{FF}(q) = i\,\mathrm{Tr}[\tau^{\nu}F(q+k)\tau^{\mu}F(k)]\,.
\end{aligned} \tag{17}$$

The solution of the $\tilde T$ matrix [see Eq. (12), etc., for the other components, and the Eqs. (A1) and (A2), etc., for the expansion coefficients] and the expression for $\tilde\Gamma$ [see Eqs. (4) and (13)] are now inserted into Eq. (11). After integration over the coupling constant λ we obtain for $\Delta\Omega^s$, up to and including the fourth-order terms in F and $\bar F$, the expression given in Appendix A [Eq. (A3)]. It should be pointed out that the particle-hole susceptibility $\chi(q)$ and the particle-particle susceptibility $\bar\chi(q)$ occuring in this expression depend on the "normal" Green's functions G (see Ref. 3) which still depends on the magnitude of the energy gap.

Since we are interested here mainly in the free-energy difference between the AM and BW states for T near T_c and, in analogy to the approximation of BSA, normalize the order parameter so that the second-order terms in F and $\bar F$ in Eq. (A3) are equal for the two states, we consider explicitly only the fourth-order terms in F and $\bar F$. Retaining only the terms with the highest number of enhancement factors [see Eq. (15)] and lowest number of reduction factors [see Eq. (16)], we obtain from Eq. (A3)

$$\Delta\Omega^{(4)} = i\int\frac{d^3q}{(2\pi)^3}\int\frac{dq_0}{2\pi}\sum_{\nu,\mu=1}^{3} t^{(0)}_{\nu\nu}(q)\,t^{(0)}_{\mu\mu}(q) \times[\chi^{\nu\mu}_{\bar F F}(q) - 4\bar t^{(0)}_{22}(q)\varphi^{\nu\mu}(q)][\chi^{\mu\nu}_{\bar F F}(q) - 4\bar t^{(0)}_{22}(q)\varphi^{\mu\nu}(q)]\,. \tag{18}$$

Here we have defined the new unenhanced "susceptibility" φ by

$$\varphi^{\nu\mu}(q) = \chi^{\nu 2}_{G\bar F}(q)\chi^{2\mu}_{G\bar F}(q) + \chi^{\nu 2}_{FG}(q)\chi^{2\mu}_{FG}(q)\,. \tag{19}$$

This expression for $\Delta\Omega^{(4)}$ agrees with the corresponding one of BSA, apart from the new correction terms $\varphi^{\nu\mu}$ due to the particle-particle scattering.

III. CALCULATION OF THE FREE-ENERGY DIFFERENCE BETWEEN AM AND BW

In this section we outline the calculation of the difference in $\Delta\Omega^s$ between the AM and BW states to order Δ^4 from Eq. (18). Generalizing to finite temperature T the frequency integration over $q_0/2\pi$ in Eq. (18) is replaced by iT times the sum over the frequencies $i\nu_m = i2m\pi T$, and the frequency integrations over $k_0/2\pi$ in Eqs. (17) by iT times the sum over $i\omega_n = i(2n+1)\pi T$. The integration over $|\vec q|$ is converted into an integration over $y = q/2k_F$. The potentials $V_\nu(\vec q)$ occuring in the expressions for the enhancement and reduction factors in Eqs. (15) and (16), respectively, are now specialized to

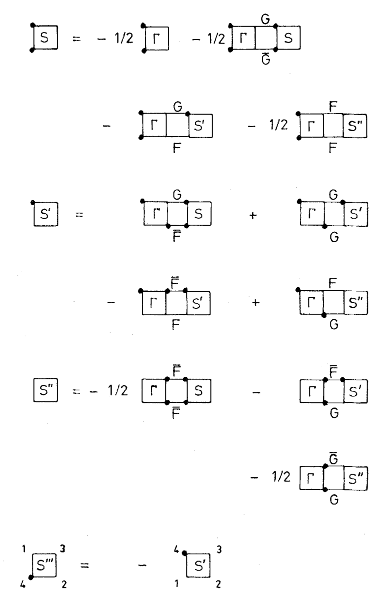

FIG. 3. Diagrammatic equations for the particle-particle components S, S', S'', of the $\tilde T$ matrix. The last equation shows the relation between the fourth component S''' and S'.

those of the repulsive contact interaction where $V_1 = V_2 = V_3 = \frac{1}{2}I$ [we define $\bar{I} = IN(0)$, where $N(0)$ is the density of states]. Introducing further dimensionless susceptibilities (denoted by a bar on top) by the definitions

$$\chi_{FF}^{\nu\mu}(\vec{q}, i\nu_m) = (\Delta/T)^2[N(0)T/2v_F q]\bar{\chi}_{FF}^{\nu\mu}, \tag{20}$$

$$\varphi^{\nu\mu}(\vec{q}, i\nu_m) = N(0)(\Delta/T)^2[N(0)T/2v_F q]\bar{\varphi}^{\nu\mu}, \tag{21}$$

we find from Eq. (18) the following expression to order Δ^4:

$$\Delta\Omega^{(4)} = -\frac{N(0)\Delta^4}{16E_F T}\int_0^1 dy\left(\frac{\bar{I}}{1-\bar{I}+\frac{1}{3}\bar{I}y^2}\right)^2 \times \sum_m \int \frac{d\Omega_q}{4\pi}\sum_{\nu,\mu=1}^{3}\frac{1}{4}\left[\bar{\chi}_{FF}^{\nu\mu} - \bar{\varphi}^{\nu\mu}\left(\bar{I}^{-1} + \frac{\bar{\chi}}{2N(0)}\right)^{-1}\right]\left[\bar{\chi}_{FF}^{\mu\nu} - \bar{\varphi}^{\mu\nu}\left(\bar{I}^{-1} + \frac{\bar{\chi}}{2N(0)}\right)^{-1}\right]. \tag{22}$$

In the expressions for the susceptibilities, Eq. (17), the integrations over the energy variable ϵ_k and the azimuthal angle $\varphi_\perp$ with respect to the polar axis along $\hat{q}$ can be carried out analytically. Then the results of the summations over ω_n can be expressed in terms of the digamma function ψ. For the AM state the gap matrix can be represented as[1]

$$\Delta(\hat{k}) = \sqrt{\tfrac{3}{2}}\,\Delta(\hat{k}_x + i\hat{k}_y)\tau^1 \quad \text{(AM)}, \tag{23}$$

and for the BW state it is equal to

$$\Delta(\hat{k}) = \Delta(-\hat{k}_x\tau^3 + i\hat{k}_y\tau^0 + \hat{k}_z\tau^1) \quad \text{(BW)}. \tag{24}$$

Going to the limit $\Delta/T \to 0$ we find for the *AM state:*

$$\bar{\chi}_{FF}^{\nu\mu} = \tfrac{3}{2}(1 + \hat{q}_z^2)\tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^1\tau^\mu\tau^1)\tilde{\chi}_{FF}(\alpha, m), \tag{25}$$

$$\bar{\varphi}^{\nu\mu} = \tfrac{3}{2}(\hat{q}_x^2 + \hat{q}_y^2)\delta_{\nu 3}\delta_{\mu 3}\tilde{\varphi}(\alpha, m), \tag{26}$$

where we have defined

$$\alpha = v_F q/2\pi T = (2E_F/\pi T)y. \tag{27}$$

The functions $\tilde{\chi}_{FF}$ and $\tilde{\varphi}$ are given by the following integrals over $x = \hat{q}\cdot\hat{k}$:

$$\tilde{\chi}_{FF}(\alpha, m) = \frac{\alpha}{\pi}\int_{-1}^{+1} dx\,\frac{1}{m^2 + \alpha^2 x^2} \times[\mathrm{Re}\,\psi(\tfrac{1}{2} + \tfrac{1}{2}m + \tfrac{1}{2}i\alpha x) - \psi(\tfrac{1}{2})], \tag{28}$$

$$\tilde{\varphi}(\alpha, m) = \frac{2}{\pi\alpha}\left(\int_{-1}^{+1} dx\,\frac{\alpha^2 x^2}{m^2 + \alpha^2 x^2} \times[\mathrm{Re}\,\psi(\tfrac{1}{2} + \tfrac{1}{2}m + \tfrac{1}{2}i\alpha x) - \psi(\tfrac{1}{2})]\right)^2. \tag{29}$$

Note that the integrands in Eqs. (28) and (29) are even in m. Analytical expressions for the integrals occuring in Eqs. (28) and (29) are given for $\alpha \gg m \geq 1$ in Appendix B.

For the *BW state* we obtain:

$$\begin{aligned}\bar{\chi}_{FF}^{\nu\mu} = {}&[\tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^3\tau^\mu\tau^3)(1 - \hat{q}_x^2) + \tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^0\tau^\mu\tau^0)(1 - \hat{q}_y^2) + \tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^1\tau^\mu\tau^1)(1 - \hat{q}_z^2)\\ &+ \tfrac{1}{2}i\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^3\tau^\mu\tau^0 - \tilde{\tau}^\nu\tau^0\tau^\mu\tau^3)\hat{q}_x\hat{q}_y\\ &+ \tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^3\tau^\mu\tau^1 + \tilde{\tau}^\nu\tau^1\tau^\mu\tau^3)\hat{q}_x\hat{q}_z\\ &+ i\tfrac{1}{2}\,\mathrm{Tr}(\tilde{\tau}^\nu\tau^0\tau^\mu\tau^1 - \tilde{\tau}^\nu\tau^1\tau^\mu\tau^0)\hat{q}_y\hat{q}_z]\tilde{\chi}_{FF}(\alpha, m),\end{aligned} \tag{30}$$

$$\bar{\varphi}^{\nu\mu} = (\delta_{\nu 1}\hat{q}_x + \delta_{\nu 2}\hat{q}_y + \delta_{\nu 3}\hat{q}_z)(\delta_{\mu 1}\hat{q}_x + \delta_{\mu 2}\hat{q}_y + \delta_{\mu 3}\hat{q}_z)\tilde{\varphi}(\alpha, m). \tag{31}$$

Inserting Eqs. (25)–(31) into Eq. (22) and carrying out the summations over ν and μ from 1 to 3, and the average over the solid angle Ω_q, we obtain the following final result for the *AM state:*

$$\Delta\Omega_{\mathrm{AM}}^{(4)} = -\frac{N(0)\Delta^4}{16E_F T}\int_0^1 dy\left(\frac{\bar{I}}{1-\bar{I}+\frac{1}{3}\bar{I}y^2}\right)^2[3.15S_1(\alpha) + 0.9S_2(\alpha) + 1.2S_3(\alpha)], \tag{32}$$

and the *BW state,*

$$\Delta\Omega_{\mathrm{BW}}^{(4)} = -\frac{N(0)\Delta^4}{16E_F T}\int_0^1 dy\left(\frac{\bar{I}}{1-\bar{I}+\frac{1}{3}\bar{I}y^2}\right)^2[S_1(\alpha) - S_2(\alpha) + S_3(\alpha)]. \tag{33}$$

Here we have defined the following sums over m:

$$S_1(\alpha)=\sum_{m=-\infty}^{+\infty}[\tilde{\chi}_{\bar{F}F}(\alpha,m)]^2\,, \quad (34)$$

$$S_2(\alpha)=\sum_{m=-\infty}^{+\infty}\tilde{\chi}_{\bar{F}F}(\alpha,m)\tilde{\varphi}(\alpha,m)\left(\frac{1}{\bar{I}}+\frac{\bar{\chi}(\alpha,m)}{2N(0)}\right)^{-1}, \quad (35)$$

$$S_3(\alpha)=\sum_{m=-\infty}^{+\infty}\tfrac{1}{4}[\tilde{\varphi}(\alpha,m)]^2\left(\frac{1}{\bar{I}}+\frac{\bar{\chi}(\alpha,m)}{2N(0)}\right)^{-2}. \quad (36)$$

Comparing the $S_1(\alpha)$ terms in Eqs. (32) and (33) we see that these terms agree with those of BSA, yielding the same value, 3.15, for the ratio of AM to BW. The new terms involving $S_2(\alpha)$ lower the free energy of the AM state still further while raising the BW free energy. The terms involving $S_3(\alpha)$ yield, as we shall see below, a negligibly small contribution.

The new correction terms in Eqs. (32) and (33) depend critically on the "particle-particle" susceptibility $\bar{\chi}$ defined in Eq. (17). In calculating this quantity we have taken as an upper estimate for the cutoff in the integration over ϵ_k the (bare) Fermi energy E_F and find to a good approximation:

$$\frac{\bar{\chi}(\vec{q},i\nu_m)}{2N(0)}=\frac{1}{2}\ln\left(\frac{m^2+4(E_F/2\pi T)^2}{m^2+\alpha^2}\right)+1-(m/\alpha)\arctan(\alpha/m)\,. \quad (37)$$

We have calculated numerically the three sums defined in Eqs. (34)–(36) with the help of Eqs. (28), (29), and (37). Since $1-\bar{I}$ is small we have approximated $1/\bar{I}$ in Eqs. (35) and (36) by 1. In Table I we present the numerical values for a number of values of α. One sees that all three sums depend rather weakly on the parameter α for the range of values of α, or y [see Eq. (27)], which are most important in the integrals over y in Eqs. (32) and (33). Note that α is very large, more precisely, $\alpha\sim 1.2\times 10^3 y$ as estimated from Eq. (27) with $E_F\sim 6$ K and $T_c\sim 2.5$ mK. Moreover, the ratio of $S_2(\alpha)$ to $S_1(\alpha)$ is seen to be almost constant, i.e., $S_2(\alpha)/S_1(\alpha)\simeq 0.15$.

We have not actually calculated the integrals over y in Eqs. (32) and (33), but from the numerical values presented in Table I it is clear that the correction terms due to $S_2(\alpha)$ lower the energy in the AM state still further by about 5% while raising the BW free energy by about 15%.

Finally, we have calculated the corrections to the coefficients of the five fourth-order invariants which have been introduced by Mermin and Stare[7] and by Brinkman and Anderson.[8] The general form of the gap matrix can be written for p-wave pairing as follows[1]:

$$\Delta(\hat{k})=d_{\lambda i}\hat{k}_\lambda(\tau^i i\tau^2)\,. \quad (38)$$

In terms of this 3×3 order parameter $d_{\lambda i}$ the dimensionless susceptibilities [defined by the Eqs. (20) and (21)] take on the following form:

$$\bar{\chi}^{\nu\mu}_{\bar{F}F}=\Delta^{-2}(\delta_{\alpha\beta}-\hat{q}_\alpha\hat{q}_\beta)[\delta_{\nu\mu}d^*_{\alpha i}d_{\beta i}-(d^*_{\alpha\nu}d_{\beta\mu}+d^*_{\alpha\mu}d_{\beta\nu})]\tilde{\chi}_{\bar{F}F}(\alpha,m)\,, \quad (39)$$

$$\bar{\varphi}^{\nu\mu}=\Delta^{-2}\tfrac{1}{2}[(d^*_{\alpha\nu}\hat{q}_\alpha)(d_{\beta\mu}\hat{q}_\beta)+\text{c.c.}]\tilde{\varphi}(\alpha,m)\,. \quad (40)$$

Inserting these expressions into Eq. (22), carrying out the average over $d\Omega_q$, and expressing the sums over m in terms of the functions S_1 and S_2 [see Eqs. (34) and (35)], we find the following result:

$$\begin{aligned}\Delta\Omega^{(4)}=-\frac{N(0)}{16E_FT}\int_0^1 dy\left(\frac{\bar{I}}{1-\bar{I}+\frac{1}{3}\bar{I}y^2}\right)^2 &[(1-S_2/S_1)\,|d^2_{\alpha i}|^2+0.5d^*_{\alpha i}d^*_{\alpha j}d_{\beta i}d_{\beta j}\\ &+(7+3S_2/S_1)d^*_{\alpha i}d^*_{\beta i}d_{\alpha j}d_{\beta j}-(2+5S_2/S_1)(|d_{\alpha i}|^2)^2\\ &+(5.5+5S_2/S_1)d^*_{\alpha i}d^*_{\beta j}d_{\alpha j}d_{\beta i}]\,.\end{aligned} \quad (41)$$

The first numbers occuring in the brackets have been calculated from S_1 by setting $S_1=30$ (see Table I); these numbers agree with those which have been given by BSA as the coefficients of the five fourth-order invariants. The second terms involving the ratio $S_2/S_1\simeq 0.15$ (see Table I) yield the corrections of these coefficients due to the new susceptibility $\varphi^{\nu\mu}$. One notes that the various coefficients in Eq. (41) are affected quite differently by the new terms, the coefficient of the fourth invariant, for instance, being enhanced by about 37%.

It should be pointed out that we have also calculated the effect of $\delta\chi_{GG}=\chi_s-\chi_n$ on the fourth-order free energy by expanding the zeroth- and second-order terms in F and $\bar{F}$ occuring in Eq. (A3) up to order Δ^4. It follows that one should add $\delta\chi_{GG}$ to the expressions in square brackets in Eq. (18) and replace the $t^{(0)}_{\nu\nu}$ by their normal-state values. We have done this and find that the only appreciable change is a replacement $2\rightarrow 1.98$ in the coefficient of the fourth term in Eq. (41). We shall not include this effect in the following discussion.

Consider, for example, the AM state in the presence of a magnetic field[8]:

TABLE I. Numerical values for the sums in Eqs. (34)–(36) as a function of the parameter α, or y [see Eq. (27)].

α	2	5	10	20	50	100	200	500	1000
$S_1(\alpha)$	5.7	11.4	15.9	19.8	23.8	26.0	27.5	28.8	29.4
$S_2(\alpha)$	0.55	1.62	2.53	3.27	3.85	3.99	4.04	4.05	4.24
$S_3(\alpha)$	0.01	0.06	0.11	0.16	0.22	0.27	0.33	0.49	0.74
S_2/S_1	0.10	0.14	0.16	0.17	0.16	0.16	0.15	0.14	0.14

$$\Delta(\hat{k}) = (\hat{k}_x + i\hat{k}_y)\sqrt{\tfrac{3}{2}}\begin{pmatrix}\Delta_1 & 0\\ 0 & \Delta_2\end{pmatrix}. \tag{42}$$

The bracketed expression in Eq. (41) now becomes equal to

$$[(7+3S_2/S_1)9|\Delta_1|^2|\Delta_2|^2 - (2+5S_2/S_1)\tfrac{9}{4}(|\Delta_1|^2+|\Delta_2|^2)^2 + (5.5+5S_2/S_1)\tfrac{9}{4}(|\Delta_1|^2+|\Delta_2|^2)^2]. \tag{43}$$

One notes that the relatively large corrections of the last two terms in Eq. (43) cancel. Thus for the A_1 phase of Ambegaokar and Mermin[9] the new corrections have no effect at all.

We turn now to a discussion of the specific-heat discontinuity which is inversely proportional to the coefficient of the fourth-order term in the free-energy expansion at T_c. Including the weak-coupling contribution as well as the BSA spin-fluctuation terms and our corrections we obtain for the transition from normal Fermi liquid to either AM or BW:

$$\frac{\Delta C^{\text{AM}}}{C_n} = \frac{5}{6}\,\frac{1.43}{1-\frac{21}{40}(1+\eta)\delta}, \tag{44}$$

$$\frac{\Delta C^{\text{BW}}}{C_n} = \frac{1.43}{1-\frac{1}{5}(1-\eta')\delta}. \tag{45}$$

The Brinkman-Anderson parameter δ is defined by

$$\delta = \Delta F^{\text{BSA}}_{\text{BW}}/(F^0_{\text{BW}} - F^0_{\text{AM}}). \tag{46}$$

The superscript zero denotes weak coupling. It should be kept in mind that δ is defined in terms of $\Delta F^{\text{BSA}}_{\text{BW}}$, the fourth-order spin-fluctuation free energy *in the BSA approximation,* i.e., Eq. (33) with $S_2(\alpha) = S_3(\alpha) = 0$. η accounts for the new corrections and has values $\eta \cong 0.04$ for AM and $\eta' \cong 0.15$ for BW.

A truly satisfactory theory should, of course, provide an accurate theoretical value for δ. The present theory, however, as discussed by BSA, yields a δ much too large to fit experiment, if no cutoff is employed in Eq. (33). Since an improved theory would involve a refined treatment of the spin-fluctuation mechanism,[10] we follow BSA and consider δ a parameter to be determined from experiment.

One observes from Eqs. (44) and (45) that, for fixed δ, the corrections to BSA increase (decrease) $\Delta C/C_n$ in the AM (BW) state. The slope of the $\Delta C/C_n$ vs δ curve, and hence the pressure dependence of $\Delta C/C_n$, is increased (decreased) in the AM (BW) state. The "critical" crossover δ_c at which the BW state becomes more stable than the AM state as δ decreases is reduced from the BSA value of 0.47 to 0.41. Assuming the B phase to be a BW state, $\Delta C/C_n$ at the "polycritical point" can be obtained by inserting δ_c into Eqs. (44) and (45). We find $\Delta C/C_n \cong 1.54$ for both BW and AM. The corresponding BSA value is 1.58. Note that the specific-heat discontinuity, including correction terms in the BW state is quite close to the BCS value of 1.43.

We have also calculated the slope ratio $4\beta/\alpha$ of the temperature dependences of the NMR frequency shift $\Delta(\nu_0)^2$ in the A and A_1 phases.[11] The constants α and β are defined in Ref. 11 by $(\Delta_1)^2 = \alpha(t'+1)$, $(\Delta_2)^2 = 0$ for $-1 \leq t' \leq 0$ and $(\Delta_1)^2 = \alpha + \beta t'$, $(\Delta_2)^2 = \beta t'$ for $0 \leq t'$, where t' is the new linear temperature scale. We find with the help of Eq. (43)

$$4\frac{\beta}{\alpha} = 4\,\frac{1-\frac{7}{80}\delta}{1-\frac{21}{40}(1+\eta)\delta}. \tag{47}$$

Here η is the correction to the expression given in Ref. 11 and is the same as that occuring in Eq. (44). Inserting the experimental value $4\beta/\alpha \cong 5.33$ at the melting pressure[11] one obtains from Eq. (47) $\delta \cong 0.52$ instead of 0.54 in the BSA approximation.

An independent determination of δ can be obtained at the melting pressure where, experimentally, $\Delta C/C_n = 1.85$ (Ref. 12) for the transition into the A phase. From Eq. (44) this yields $\delta \cong 0.68$ in the BSA approximation and $\delta \cong 0.65$ in the present theory. Since the new corrections have the effect of shifting the δ's calculated from specific-heat data and NMR data by approximately the same amount, the discrepancy between these values stated in Ref. 11 remains. Concerning the calculation of δ from T_{AB}/T_c data[1,11] we observe that, since δ_c is smaller in the present theory, the BSA curve of T_{AB}/T_c vs δ should be shifted so as to yield a smaller δ at the experimental T_{AB}/T_c. This would be in line with the shift in the NMR and specific-heat discontinuity δ's. However, in order to make a meaningful comparison with the δ determined from the T_{AB}/T_c data, the new corrections should also be applied in this case.

Finally, it should be pointed out that our estimate of the magnitude of the contribution of the new anomalous diagrams is uncertain due to the cutoff required in $\bar{\chi}$. In Eq. (37) we employed as a cutoff the bare Fermi energy $E_F \sim 6$ K. A smaller cutoff would lead to a larger contribution. In conclusion we have thus confirmed the importance of the feedback effect of Anderson and Brinkman within the framework of a self-consistent and conserving theory and given additional theoretical evidence that the A phase is indeed an anisotropic AM state.

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APPENDIX A

Solution of equations for T, T', T'', in Fig. 2 in terms of expansion coefficients t, t', t'' [see Eq. (12), etc.]:

$$t'^{(1)}_{\nu\mu}(q) = -\delta_{\nu 2} 4\bar{t}^{(0)}_{22}(q)\chi^{2\mu}_{GF}(q)t^{(0)}_{\mu\mu}(q)\,;$$

$$t''^{(1)}_{\nu\mu}(q) = -\delta_{\nu 2} 4\bar{t}^{(0)}_{22}(-q)\chi^{2\mu}_{\bar{F}G}(q)t^{(0)}_{\mu\mu}(q)\,;$$

$$t^{(2)}_{\nu\mu}(q) = 2t^{(0)}_{\nu\nu}(q)[\chi^{\nu\mu}_{\bar{F}F}(q)t^{(0)}_{\mu\mu}(q) + \chi^{\nu 2}_{G\bar{F}}(q)t'^{(1)}_{2\mu}(q) + \chi^{\nu 2}_{FG}(q)t''^{(1)}_{2\mu}(q)]\,;$$

$$t'^{(3)}_{\nu\mu}(q) = -\delta_{\nu 2} 2\bar{t}^{(0)}_{22}(q)\sum_\delta [2\chi^{2\delta}_{GF}(q)t^{(2)}_{\delta\mu}(q) + \chi^{2\delta}_{FF}(q)t''^{(1)}_{\delta\mu}(q)]\,;$$

$$t''^{(3)}_{\nu\mu}(q) = -\delta_{\nu 2} 2\bar{t}^{(0)}_{22}(-q)\sum_\delta [2\chi^{2\delta}_{\bar{F}G}(q)t^{(2)}_{\delta\mu}(q) + \chi^{2\delta}_{\bar{F}\bar{F}}(q)t'^{(1)}_{\delta\mu}(q)]\,;$$

$$t^{(4)}_{\nu\mu}(q) = 2t^{(0)}_{\nu\nu}(q)\sum_\delta [\chi^{\nu\delta}_{\bar{F}F}(q)t^{(2)}_{\delta\mu}(q) + \chi^{\nu\delta}_{G\bar{F}}(q)t'^{(3)}_{\delta\mu}(q) + \chi^{\nu\delta}_{FG}(q)t''^{(3)}_{\delta\mu}(q)]\,. \tag{A1}$$

Solution of equations for S, S', S'', in Fig. 3 in terms of expansion coefficients s, s', s'':

$$s^{(0)}_{\nu\mu}(q) = -2\bar{t}^{(0)}_{22}(q)\delta_{\nu 2}\delta_{\mu 2};$$

$$s'^{(1)}_{\nu\mu}(q) = -4t^{(0)}_{\nu\nu}(q)\chi^{\nu 2}_{G\bar{F}}(q)\bar{t}^{(0)}_{22}(q)\delta_{\mu 2};$$

$$s^{(2)}_{\nu\mu}(q) = -4\bar{t}^{(0)}_{22}(q)\sum_\delta \chi^{2\delta}_{GF}(q)s'^{(1)}_{\delta 2}(q)\delta_{\nu 2}\delta_{\mu 2}\,;$$

$$s''^{(2)}_{\nu\mu}(q) = 4\bar{t}^{(0)}_{22}(-q)\left[\chi^{22}_{\bar{F}\bar{F}}(q)\bar{t}^{(0)}_{22}(q) - \sum_\delta \chi^{2\delta}_{\bar{F}G}(q)s'^{(1)}_{\delta 2}(q)\right]\delta_{\nu 2}\delta_{\mu 2}\,;$$

$$s'^{(3)}_{\nu\mu}(q) = 2t^{(0)}_{\nu\nu}(q)\left[\chi^{\nu 2}_{G\bar{F}}(q)s^{(2)}_{22}(q) + \sum_\delta \chi^{\nu\delta}_{\bar{F}F}(q)s'^{(1)}_{\delta 2}(q) + \chi^{\nu 2}_{FG}(q)s''^{(2)}_{22}(q)\right]\delta_{\mu 2}\,;$$

$$s^{(4)}_{\nu\mu}(q) = -2\bar{t}^{(0)}_{22}(q)\left[\sum_\delta 2\chi^{2\delta}_{GF}(q)s'^{(3)}_{\delta 2}(q) + \chi^{22}_{FF}(q)s''^{(2)}_{22}(q)\right]\delta_{\nu 2}\delta_{\mu 2}\,. \tag{A2}$$

The spin-fluctuation contribution to the thermodynamic potential, up to and including fourth-order terms in F and $\bar{F}$:

$$\Delta\Omega^s = i\int \frac{d^3q}{(2\pi)^3}\int \frac{dq_0}{2\pi}\Big(-\frac{1}{2}\sum_\nu \{\ln[1 - V_\nu(\vec{q})\chi(q)] + V_\nu(\vec{q})\chi(q)\} - \{\ln[1 + V_2(\vec{q})\bar{\chi}(q)] - V_2(\vec{q})\bar{\chi}(q)\}$$
$$+ \sum_\nu t^{(0)}_{\nu\nu}(q)[\chi^{\nu\nu}_{\bar{F}F}(q) - 4\bar{t}^{(0)}_{22}(q)\varphi^{\nu\nu}(q)] - \sum_\nu \tfrac{1}{2}V_\nu(\vec{q})\chi^{\nu\nu}_{\bar{F}F}(q)$$
$$+ \sum_{\nu,\mu} t^{(0)}_{\nu\nu}(q)t^{(0)}_{\mu\mu}(q)[\chi^{\nu\mu}_{\bar{F}F}(q) - 4\bar{t}^{(0)}_{22}(q)\varphi^{\nu\mu}(q)][\chi^{\mu\nu}_{\bar{F}F}(q) - 4\bar{t}^{(0)}_{22}(q)\varphi^{\mu\nu}(q)]$$
$$+ \sum_\nu 8[\chi^{\nu 2}_{G\bar{F}}(q)\chi^{2\nu}_{\bar{F}G}(q)\chi^{22}_{FF}(q) + \chi^{2\nu}_{GF}(q)\chi^{\nu 2}_{FG}(q)\chi^{22}_{\bar{F}\bar{F}}(q)][\bar{t}^{(0)}_{22}(q)]^2 t^{(0)}_{\nu\nu}(q) + 4\chi^{22}_{\bar{F}\bar{F}}(q)\chi^{22}_{FF}(q)[\bar{t}^{(0)}_{22}(q)]^2 + \cdots\Big)\,. \tag{A3}$$

APPENDIX B

$$\frac{\alpha}{\pi}\int_{-1}^{+1}\frac{dx}{m^2+\alpha^2x^2}\left[\mathrm{Re}\,\psi(\tfrac{1}{2}+\tfrac{1}{2}m+\tfrac{1}{2}i\alpha x) - \psi(\tfrac{1}{2})\right] \sim (1/m)[\psi(m+\tfrac{1}{2}) - \psi(\tfrac{1}{2})] \quad \text{for } 1 \leq m \ll \alpha\,. \tag{B1}$$

$$\int_{-1}^{+1} dx\frac{\alpha^2x^2}{m^2+\alpha^2x^2}\left[\mathrm{Re}\,\psi(\tfrac{1}{2}+\tfrac{1}{2}m+\tfrac{1}{2}i\alpha x) - \psi(\tfrac{1}{2})\right] \sim \ln[\tfrac{1}{4}(1+m)^2+\tfrac{1}{4}\alpha^2] - 2\left[1 - \frac{m}{\alpha}\arctan\left(\frac{\alpha}{1+m}\right)\right]$$
$$- 2\psi(\tfrac{1}{2})\left[1 - \frac{m}{\alpha}\arctan\left(\frac{\alpha}{m}\right)\right] - \pi\frac{m}{\alpha}\psi(m+\tfrac{1}{2}) \quad \text{for } m \ll \alpha\,. \tag{B2}$$

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