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Temperature-gradient instabilities in semiconductor junctions*

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A new type of instability is shown to occur if a temperature gradient is present across a nn^+, pp^+, or pn junction with no voltage applied. The critical temperature gradient depends on the size and steepness of the junction and is smaller for large and less abrupt junctions. The instability should occur in the best thermoelectric materials, characterized by a figure of merit $ZT > 1$. In other materials the required temperature difference between the homogeneous semiconductors at the end of the junction turns out to be too large.

I. INTRODUCTION

In a nn^+, pp^+, or pn junction the current carriers are comparable with a gas in a gravitational field. The Boltzmann equilibrium in the internal electric field of the junction corresponds to the equilibrium of a heavy gas which is described by the barometric formula. If the gas is heated from below, instabilities, convection and turbulence will appear successively. Suppose that the temperature gradient is directed downward and exceeds a certain critical value. Arbitrarily small temperature perturbations will grow then exponentially, leading to thermal convection. The critical gradient increases with the viscosity of the gas and decreases with the density, the gravitational acceleration and the thermal dilatation coefficient.[1]

The aim of this paper is to show that similar instabilities appear in semiconducting junctions, if the temperature gradient across the junction is big and has the corresponding direction. Similar to the case of a heavy gas heated from below, eddy currents and electrical noise will appear in semiconductor junctions without any applied voltage. In order to get a physical picture of these instabilities, it is convenient to express the underlying macroscopic perturbations of the carrier motions in terms of the well-known thermoelectric effects.

Let us consider a small spatially periodical temperature perturbation in the plane of an unbiased semiconductor junction. It will generate small Seebeck-current curls. The initial perturbation will be attenuated both through the Peltier heat of the Seebeck-current curls and through thermal conduction. However, if a large temperature gradient is present perpendicular to the junction plane, these losses can be overcompensated by the Thomson heat which also appears as a consequence of the mentioned Seebeck-current curls. Denoting by Q_A and Q_B, the thermoelectric powers of the materials forming the junction; by τ, the mean value of the Thomson coefficient in the junction; by l, the length of the junction; by j, the eddy current in a curl; and by ∇T, the temperature gradient, the condition of overcompensation of the Peltier heat by the Thomson heat in a volume unit is

$$(1/l)T\,(Q_B - Q_A)\,j < -\tau j \nabla T \ . \qquad (1)$$

Here the Peltier coefficient P has been expressed through the Thomson relation $P = TQ$ which is a consequence of the Onsager principle. Neglecting thermal conductivity we obtain the heuristic criterion

$$-\tau \nabla T > \epsilon T \qquad (2)$$

for instability, in which $\epsilon = (Q_B - Q_A)/l$ is the gradient of the Seebeck-coefficient in the junction. In Sec. II we shall take into account thermal conduction also.

II. SIMPLIFIED TREATMENT

Let σ be the electrical conductivity, κ the thermal conductivity, Q the Seebeck coefficient, μ the electrochemical potential of the electrons and $e > 0$ the elementary charge. Then the density $\vec{j}$ of the electric current and the density $\vec{w}$ of the thermal current are given by

$$\vec{j} = (1/e)\sigma\nabla\mu - \sigma Q\,\nabla T\ , \quad \nabla\vec{j} = 0\ , \qquad (3)$$

$$\vec{w} = (TQ - \mu/e)\vec{j} - \kappa\nabla T\ . \qquad (4)$$

Consider again a planar junction with free terminals, in which a uniform temperature gradient is present perpendicular to the junction plane. In the unperturbed state $\vec{j} = 0$. Denoting unperturbed values by the index 0, we obtain, from Eqs. (3) and (4),

$$(1/e)\nabla\mu_0 = Q_0 \nabla T_0\ , \qquad (5)$$

$$\vec{w}_0 = -\kappa\,\nabla T_0\ . \qquad (6)$$

The evolution of the perturbation generated by a small temperature variation δT is described by the following linearized equations:

$$\delta\vec{j} = \frac{1}{e}\sigma_0\nabla\delta\mu - \sigma_0 Q_0\nabla\delta T - \sigma_0\frac{\partial Q_0}{\partial T}\delta T\nabla T_0\ ,\ \nabla\delta\vec{j} = 0\ , \tag{7}$$

$$\delta\vec{w} = (T_0Q_0 - \mu_0/e)\,\delta\vec{j} - \kappa\nabla\delta T\ , \tag{8}$$

$$c\rho\frac{\partial\delta T}{\partial t} + \nabla\delta\vec{w} = 0\ . \tag{9}$$

Here c is the specific heat and ρ is the density. Let $x=0$ be the junction plane. Then T_0, Q_0, μ_0, and σ_0 do not depend on y and z. If we eliminate $\delta\vec{w}$, we obtain

$$c\rho\frac{\partial\delta T}{\partial t} + (\delta\vec{j})_x\frac{d}{dx}\left(T_0Q_0 - \frac{\mu_0}{e}\right) - \kappa\nabla^2\delta T = 0\ . \tag{10}$$

Equation (10) has to be considered together with Eq. (7).

In the simplified treatment of this section we shall neglect the spatial variation of σ_0 and $\partial Q_0/\partial T$. We shall also consider the dependence of Q_0 and T_0 on x to be linear. Actually, σ depends strongly on x both due to the spatial dependence of doping in the junction and because the temperature depends on x. Physically, the effect of this spatial variation is only a distortion of the $\vec{j}$ curls, which should not influence the stability criterion too much. The effect will be taken into account in Secs. III and IV.

With these simplifications, we try a solution of the form

$$\delta\vec{j} = \nabla\times\vec{A}\ ,\quad \nabla\vec{A} = 0\ ,$$
$$\vec{A} = \vec{a}e^{i\vec{k}\vec{r}+st}\ ,\quad \delta T = \theta e^{i\vec{k}\vec{r}+st}\ , \tag{11}$$

where $\vec{a}$, $\vec{k}$, θ and s are constants.

Taking the curl of Eq. (7), we obtain

$$k^2\vec{a} = i\theta\sigma_0\vec{k}\times\nabla Q_0 - i\sigma_0\theta\frac{\partial Q_0}{\partial T}\vec{k}\times\nabla T_0\ . \tag{12}$$

We calculate $i\vec{k}\times\vec{a}$ from Eq. (12) and substitute it for $\delta\vec{j}$ in Eq. (10). Using Eq. (5) we obtain the dispersion equation

$$c\rho s = -\sigma_0 T_0\frac{dQ_0}{dx}\left(\frac{dQ_0}{dx} - \frac{\partial Q_0}{\partial T}\frac{dT_0}{dx}\right)\times\left(1 - \frac{k_x^2}{k^2}\right) - \kappa k^2\ . \tag{13}$$

Let us denote by ϵ the big bracket which represents the isothermal gradient of the Seebeck coefficient, a characteristic of the junction. We obtain, finally,

$$c\rho s = -\sigma_0\epsilon(\tau\gamma + \epsilon T_0)\left(1 - \frac{k_x^2}{k^2}\right) - \kappa k^2\ ,$$
$$\epsilon \equiv \frac{\partial Q_0}{\partial x} > 0\ ,\quad \gamma \equiv \frac{dT_0}{dx}\ . \tag{14}$$

Here the Thomson coefficient τ was introduced with the Thomson relation

$$\tau = T_0\frac{\partial Q_0}{\partial T_0}\ . \tag{15}$$

Choosing by convention in each junction the direction of the x axis so that $\epsilon > 0$ we see that the heuristic criterion (2) is a necessary condition. In fact, instability means $s > 0$ and thus the criterion for instability is

$$-\sigma_0\epsilon(\tau\gamma + \epsilon\overline{T}) > \kappa k^2\ , \tag{16}$$

where we restricted ourselves to perturbations with $k_x = 0$ and where $\overline{T}$ is a mean temperature, corresponding for instance to the junction plane. Perturbations with $k_y = k_z = 0$ would never yield instabilities. Thus k_y and k_z should be considered equal to the inverse of a characteristic dimension of the junction [see Eqs. (49) and (51)].

III. INTEGRAL CRITERION

Starting from Eqs. (7) and (10) we shall now avoid the drastic simplifications of Sec. II. Assuming in the junction

$$\sigma_0(x) = \sigma_0 e^{2bx}\ ,\quad b = \text{const.}\ ,\quad \sigma_0 = \text{const.} \tag{17}$$

as a realistic form, we consider a perturbation

$$\delta\vec{j} = \nabla\times\vec{A}\ ,\quad \nabla\vec{A} = 0\ ,\quad \text{with } \vec{A} = a(x)\vec{e}_2 e^{i\vec{k}\vec{r}+st}$$
and
$$\delta T = \theta(x)e^{i\vec{k}\vec{r}+st}\ . \tag{18}$$

Here $\vec{k}$ is a vector perpendicular to x and $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ are unit vectors of the three axes. From $\nabla\vec{A} = 0$, we obtain

$$k_y = 0\ ,\quad \vec{A} = a(x)\vec{e}_2 e^{ikz+st}\ . \tag{19}$$

Taking the curl of e^{-2bx} times Eq. (7) yields

$$k^2 a + 2b\frac{da}{dx} - \frac{d^2a}{dx^2} = i\sigma_0(x)\,\theta(x)k\frac{\partial Q_0}{\partial x}\ . \tag{20}$$

Applying the Fourier-transform method, we obtain

$$\delta i_x = ika = k^2\int_{-\infty}^{\infty}\frac{\chi(\xi)e^{ix\xi}d\xi}{\xi^2 + 2ib\xi + k^2}\ , \tag{21}$$

with

$$\chi(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\theta(x)\sigma_0(x)\frac{\partial Q_0}{\partial x}e^{-ix\xi}dx\ . \tag{22}$$

Thus, Eq. (10) yields

$$c\rho s\,\theta(x) + T_0(x)\frac{dQ_0}{dx}\frac{k^2}{2\pi}\int_{-\infty}^{\infty}\frac{d\xi e^{ix\xi}}{\xi^2 + i\beta\xi + k^2}\times\int_{-\infty}^{\infty}\theta(u)\sigma_0(u)\frac{\partial Q_0}{\partial u}e^{-iu\xi}du + \kappa\left(k^2 - \frac{d^2}{dx^2}\right)\theta(x) = 0\ . \tag{23}$$

All transformations are permitted since the perturbation is confined to the junction region.

After another Fourier transformation the Fredholm integral equation

$$\tilde{\theta}(r)=\int_{-\infty}^{\infty} K(r, v)\tilde{\theta}(v)\,dv \tag{24}$$

follows for the transform $\tilde{\theta}$ of θ, with

$$K(r, v)=\frac{-k^2\sigma_0/(b^2+k^2)^{1/2}}{4\pi(c\rho s+\kappa k^2+\kappa r^2)} \times\int_{-l/2}^{l/2}\int_{-l/2}^{l/2} T_0(x)\,\frac{dQ_0(x)}{dx}\,\frac{\partial Q_0(u)}{\partial u} \times\exp[-(b^2+k^2)^{1/2}\,|x-u|+b(x+u)-ixr+iuv]\,dx\,du\,. \tag{25}$$

Here the two integrations were limited to the width of the junction l, because $\partial Q_0(u)/\partial u$ and dQ_0/dx are both zero elsewhere. The temperature gradient is considered to be different from zero only within the width of the junction.

The homogeneous equation (24) admits a nonzero solution only if s is the root of the dispersion relation

$$0=1-\frac{1}{1!}\int K(r, r)dr+\frac{1}{2!}\int \begin{vmatrix} K(u, u) & K(u, v)\\ K(v, u) & K(v, v)\end{vmatrix}\,du\,dv -\frac{1}{3!}\int |\cdots|\,du\,dv\,dw+\cdots \tag{26}$$

given by Fredholm's series. If this root s is positive, the system is unstable; otherwise stability prevails. This is the integral criterion, which can be handled with computers up to any order. In Sec. IV we shall give an analytical treatment, restricting ourselves to the first order of Eq. (26).

IV. FIRST-ORDER APPROXIMATION OF THE INTEGRAL CRITERION

Let us consider a spatial dependence of the form

$$T_0(x)=\begin{cases} T_{00}+\gamma x & \text{for } -\frac{l}{2}<x<\frac{l}{2}\\ \text{const.} & \text{otherwise}\,,\end{cases}$$

$$\frac{\partial Q_0(x)}{\partial x}=\begin{cases} \epsilon>0 & \text{for } -\frac{l}{2}<x<\frac{l}{2}\\ 0 & \text{otherwise}\,,\end{cases} \tag{27}$$

$$\tau(x)=\begin{cases} \tau_0+\tau^1 x & \text{for } -\frac{l}{2}<x<\frac{l}{2}\\ \text{const.} & \text{otherwise}\,,\end{cases}$$

where T_{00}, γ, ϵ, τ_0 and τ^1 are constants. Using the definitions in Eqs. (14) and (15) we obtain

$$T_0(x)\,\frac{dQ_0}{dx}\,\frac{\partial Q_0}{\partial x}=\epsilon[\tau(x)\gamma+\epsilon T(x)] =\begin{cases} \epsilon[\tau_0\gamma+\epsilon T_{00}+(\tau^1+\epsilon)\gamma x] & \text{for } -\frac{l}{2}<x<\frac{l}{2}\\ 0 & \text{otherwise}\,.\end{cases} \tag{28}$$

Usually $\tau^1\approx-\epsilon$. Indeed, considering for example the n-type part of the junction, we express the Seebeck coefficient in the form[2]

$$Q_0=-(k/e)(\ln N_C/N_D+\tfrac{3}{2}\ln m_{nd}/m +\tfrac{3}{2}\ln T+\tfrac{5}{2}+r)\,, \tag{29}$$

where the effective mass m_{nd} of the electrons, the free-electron mass m, k, e, and r are all constants. $N_c\equiv 2(2\pi m_{nd}kT/h^2)^{3/2}$ is the degeneration concentration, and the concentration of electrons has been approximated by the concentration of ionized donors N_D. If we assume that the concentration of ionized donors is controlled by an activation energy E_D, i.e.,

$$N_D/N_C\approx e^{-E_D/kT}\,,$$

we obtain the Thomson coefficient and its spatial gradient

$$\tau(x)\equiv T\,\frac{dQ_0}{dT}\approx-\frac{k}{e}\left(\frac{3}{2}-\ln\frac{N_C}{N_D}\right)\,,$$

$$\tau^1=\frac{k}{e}\,\frac{d}{dx}\ln\frac{N_C}{N_D}=\frac{k}{e}\,\frac{\partial}{\partial x}\ln\frac{N_C}{N_D} +\gamma\,\frac{k}{e}\,\frac{\partial}{\partial T}\ln\frac{N_C}{N_D}\,. \tag{30}$$

On the other hand,

$$\epsilon\equiv\frac{\partial Q_0}{\partial x}=-\frac{k}{e}\,\frac{\partial}{\partial x}\ln\frac{N_C}{N_D}=-\tau^1+\gamma\,\frac{k}{e}\,\frac{\partial}{\partial T}\ln\frac{N_C}{N_D}\,. \tag{31}$$

The last term in Eq. (31) would contribute to Eq. (28) a term which is quadratic in the temperature gradient γ. Such terms, however, are considered negligible in the present treatment which is based on the linear transport theory anyway.

Putting $\tau^1+\epsilon=0$ in Eq. (28) and performing the integrations in Eqs. (25) and (26), we obtain

$$\int_{-\infty}^{\infty} K(r, r)\,dr=\frac{l^2\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)m^2\{[(m^2+S)^{1/2}+(m^2+\mu^2)^{1/2}]\sinh\mu/\mu-\cosh\mu+e^{-[(m^2+S)^{1/2}+(m^2+\mu^2)^{1/2}]}\}}{2\kappa(m^2+\mu^2)^{1/2}(m^2+S)^{1/2}\{\mu^2-[(m^2+S)^{1/2}+(m^2+\mu^2)^{1/2}]\}} \tag{32}$$

where

$$m\equiv kl\,,\quad \mu\equiv bl\,,\quad \text{and}\quad S\equiv c\rho s l^2/\kappa \tag{33}$$

are dimensionless parameters. The first-order approximation of Eq. (26) is

$$\int_{\infty}^{\infty} K(r, r)\, dr = 1 \; . \qquad (34)$$

If we substitute Eq. (32) into (34), with the notations

$$(m^2+S)^{1/2}+(m^2+\mu^2)^{1/2}\equiv x\;,\quad (m^2+S)^{1/2} - (m^2+\mu^2)^{1/2}\equiv y\;, \qquad (35)$$

$$xy = S-\mu^2\;, \qquad (36)$$

$$x^2-y^2=4(m^2+S)^{1/2}(m^2+\mu^2)^{1/2}\;,$$
$$x^2+y^2-4\mu^2-2xy=4m^2\;, \qquad (37)$$

$$-\frac{l^2\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)}{2\kappa(x^2-\mu^2)}\left(x\,\frac{\sinh\mu}{\mu}-\cosh\mu+e^{-x}\right)\equiv P\;, \qquad (38)$$

we obtain

$$(x^2-y^2)/(x^2+y^2-4\mu^2-2xy)=P \qquad (39)$$

Here we changed from the variables m and S to x and y. From Eqs. (37) and (39) we see that

$$P>0 \qquad (40)$$

is a necessary condition for instability. Supposing $P>0$ we see that the discriminant of the quadratic equation

$$(1+P)z^2-2Pz-1+P+4rP=0\;; \qquad (41)$$

$$z\equiv y/z\;,\quad r\equiv\mu^2/x^2\;, \qquad (42)$$

which follows from Eq. (39), is always positive. There are thus always two real roots. Only the root

$$y/x=\{P-[1+4rP(P+1)]^{1/2}\}/(P+1) \qquad (43)$$

has physical meaning, as we see by comparison with the case $\mu=0$. From Eq. (36), (43) and $S>0$ the following necessary and sufficient instability criterion follows:

$$P(1-r)>1+r\;. \qquad (44)$$

Thus, the explicit form which corresponds to the first order approximation of the integral criterion reads

$$-l^2\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)\times\left(x_0\,\frac{\sinh\mu}{\mu}-\cosh\mu+e^{-x_0}\right)>2\kappa(x_0^2+\mu^2)\;, \qquad (45)$$

where

$$x_0\equiv m+(m^2+\mu^2)^{1/2}\;. \qquad (46)$$

Here the notations (33) are still used. Inequality (45) represents the most general form. The second bracket in the left-hand side of inequality (45) is always positive since $x_0>|\mu|$. Consequently, the heuristic criterium (2) is still necessary for the fulfillment of (45).

If we neglect the variation of σ, we obtain from Eq. (45) with $\mu\to 0$,

$$-l^2\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)(2m-1+e^{-2m})>8\kappa m^2\;. \qquad (47)$$

Denoting by D the transversal dimension of the junction, we observe that (a) in the case $l\ll D$ of abrupt junctions with large diameter, k which is given approximately by π/D, will be small. Thus, m is small and expanding the exponential we obtain from inequality (47),

$$-\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)>\kappa(2/l)^2\;. \qquad (48)$$

Consequently, in inequality (16), k should also be replaced in this case by a

$$k_{\mathrm{eff}}=2/l\;. \qquad (49)$$

This is not unexpected because the overall k of inequality (16) has always to reflect the smallest dimension of the junction. The integral criterion incorporates this result.

(b) In the case $l\gg D$, m should be large and neglecting the exponential we obtain from inequality (47):

$$-\sigma_0\epsilon(\tau_0\gamma+\epsilon T_0)>\kappa\,4k/l\approx\kappa(4/l)\,\pi/D\;. \qquad (50)$$

Thus, the geometrical mean

$$k_{\mathrm{eff}}=2\pi/lD \qquad (51)$$

has to be substituted into inequality (16) in this case.

In the most general case, k_{eff} is defined by dividing inequality (45) or (47) by the second bracket onthe left-hand side and by l^2.

We also mention that the criteria (45) and (47) can be illustrated graphically, drawing in the xy plane, the hyperbolas $S=$ const., the straight lines $m=$ const. which are inclined to 45° and the dispersion relation (43). In all cases, the dispersion curve starts from the origin, passes through a minimum and approaches finally the asymptote $y=-x$.

V. DISCUSSION

The criteria (16), (45), (47), (48), and (50) are all of the form

$$\xi(\eta+\xi)+\alpha=0\;, \qquad (52)$$

if the critical condition is just satisfied. Here

$$\xi\equiv\epsilon>0\;,\quad \eta\equiv(\tau_0/T_0)\gamma<0\;, \qquad (53)$$

and α differs in the mentioned criteria but is always positive. The minimum of the absolute value of η in Eq. (52) is obtained for

$$\xi_m=\sqrt{\alpha}\ \text{ with }\ \eta_m=-2\sqrt{\alpha}\;. \qquad (54)$$

Thus, for a junction with $\xi=\xi_m$ the threshold gradient of temperature required for instability is smallest. With $\alpha=\kappa\sigma^{-1}T_0^{-1}k_{\mathrm{eff}}^2$ we obtain

$$\gamma_{\min}=-2k_{\mathrm{eff}}\tau_0^{-1}(\kappa T_0/\sigma)^{1/2} \qquad (55)$$

for

$$\epsilon_m = k_{eff}(\kappa\sigma^{-1}T_0^{-1})^{1/2} . \quad (56)$$

If we consider $l \ll D$, we set $k_{eff} = 2/l$ as in case a, Sec. IV. By multiplying Eqs. (55) and (56) with l we obtain the total temperature variation and the total isothermal variation of the Seebeck temperature coefficient across the sample

$$\Delta T = -2\tau_0^{-1}(\kappa T_0/\sigma)^{1/2} , \quad (57)$$

$$\Delta Q = (\kappa\sigma^{-1}T_0^{-1})^{1/2} . \quad (58)$$

Let us discuss two special cases: (i) In the case of a thermoelectric material with a given figure of merit $Z \equiv Q^2\sigma/\kappa$ we take the square of Eq. (58) and approximate ΔQ by the maximal value of the Seebeck coefficient in the isothermal sample, Q. This yields

$$ZT_0 \approx 1 . \quad (59)$$

Thus, only the best thermoelectric materials, with $ZT \geq 1$ should exhibit thermal gradient instabilities. Furthermore, the presence of τ_0^{-1} in the denominator of Eq. (55) favors the occurrence of instabilities in nn^+ or pp^+ junctions and pn^+ or np^+ junctions with respect to symmetric pn junctions. (ii) In the case of a metallic solid which obeys the Wiedemann-Franz law

$$\frac{\kappa}{\sigma T_0} = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 = 2.45\times 10^{-8}\,\frac{\text{W}\Omega}{\text{K}^2} , \quad (60)$$

we obtain, from Eqs. (55) and (56),

$$\gamma_{min} = -3.13\times 10^{-4}\,(\text{V/K})k_{eff}\,T_0/\tau_0 , \quad (61)$$

$$\epsilon_m = 1.565\times 10^{-4}\,k_{eff}\ \text{V/K} . \quad (62)$$

It is important to note that the linear theory of transport phenomena on which the present work is based will always be applicable for large enough samples, no matter how large the applied temperature difference ΔT required by Eq. (57) is. This important scaling property can also be recognized directly on any form of the instability criterion. Whenever nonlinear effects become likely, we only increase all linear dimensions of the sample by a suitable factor and observe the instability with the same total temperature and Seebeck variation across the larger sample.

As an example, let us consider inequality (48) for a n-type PbTe sample. Simultaneous determinations of thermoelectric properties for this substance[3] have led to the values $\kappa = 0.039$ W/cm, $\sigma_0 = 185\ (\Omega\ \text{cm})^{-1}$, $Q = 2.37\ 10^{-4}$ V/K, $T_0 = 811$ K, $\tau_0 = 1.2\times 10^{-3}$ V/K. The melting point of PbTe is 1197 K. With $l = 20$ cm, $k_{eff} = 0.1\ \text{cm}^{-1}$, $\epsilon = 1.185\times 10^{-5}$ V/cm K, we obtain

$$1.2\times 10^{-3}|\gamma| > \frac{0.039\times 10^{-2}\ \text{cm}^{-1}}{185\times 1.185\times 10^{-5}} + 1.185\times 10^{-5}\times 811\ \text{K cm}^{-1} , \quad (63)$$

$$|\gamma| > 156\ \text{K/cm} .$$

This gradient is about three times larger than the gradient which can be sustained over the length l of the junction without reaching the melting point of the substance. This is not unexpected, since $ZT_0 \approx 1$ is required by Eq. (59), whereas the substance considered has $ZT_0 = 0.216$ only. Since values of the ZT_0 product which are more than four times larger have been achieved for some thermoelectric materials, there is no doubt that the instability criterion can be satisfied with them. As an example, for the compound Cu_8Te_3S a value $Z = 1.5\times 10^{-3}$ and a melting point of 930 °C have been reported.[4] Choosing $T_0 = 602$ K a product $ZT_0 = 0.9$ can be expected. In the same way, for Bi_2Te_3 with[4] $Z = 2\times 10^{-3}$ and a melting point of 575 °C, we obtain $ZT_0 = 0.848$ at $T_0 = 424$ K. Unfortunately, we do not have for these substances the systematic measurements of Q as a function of temperature which allowed for the calculation of τ_0 in the case of PbTe. In some cases, values of $Z = 3.4\times 10^{-3}$ have been reported[5] but we do not even know the melting point of the corresponding compound. For a Bi_2Te_3 + (BiSb)Te alloy $Z = 2.65\times 10^{-3}$ at $T = 300$ K has been reported,[6] which corresponds to $ZT \approx 0.8$.

Experimentally, the instability should be characterized by an abrupt increase in the electrical noise of the junction at the critical temperature gradient in the absence of any applied voltage.

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[1]S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Instability* (Oxford U.P., Oxford, 1960).

[2]J. Tauc, *Photo and Thermoelectric Effects in Semiconductors* (Pergamon, Oxford, 1962), Eqs. 42.13a and 43.3 and pp. 152 and 153.

[3]J. D. Richards, Energy Conversion 9, 73 (1969).

[4]D. L. Wright, Brit. J. Appl. Phys. 15, 217 (1964).

[5]W. M. Yim, E. Fitzke, F. D. Rosi, J. Mater. Sci. 52, (1966).

[6]A. W. Penn, Energy Conversion 8, 65 (1968).