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Minority carrier injection in relaxation semiconductors

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On the basis of normalized continuity equations solved by numerical procedures, it is shown that the predicted Van Roosbroeck depletion of majority carriers does indeed occur, not only in materials of near-zero lifetime τ_0 but whenever $\tau_0/\tau_D < 1$, where τ_D is the dielectric relaxation time. A recombination front appears for $\tau_0/\tau_D \ll 1$ and/or high currents. Carrier concentrations, field, and recombination rate are evaluated as a function of distance from the injecting boundary, and the implications for the voltage-current characteristic are analyzed. In contrast to the lifetime case, the computed characteristics show an extended linear region, resulting from the opposed tendencies of of majority-carrier depletion (sublinear) and minority-carrier injection (superlinear). Total resistances higher than those calculated on the basis of the unperturbed bulk resistivity have not been found for the conditions investigated.

I. INTRODUCTION

The concept of the relaxation semiconductor and some of the corresponding transport relationships have been introduced by Van Roosbroeck and co-workers.[1-3] These papers are concerned with materials in which $\tau_D > \tau_0$, where τ_0 is the minority-carrier lifetime and τ_D is the dielectric relaxation time, the contention being that this inequality implies (a) a region of majority-carrier depletion, (b) a recombination front, (c) a critically important region of minimum conductivity which implies a specimen resistance greater than that expected from the bulk resistivity value, and (d) a sublinear voltage-current relationship. In a recent paper by Kiess and Rose,[4] this was described as an "attractive and provocative idea," but the final conclusion of the authors was that the model was internally inconsistent, inasmuch as it failed to ensure current continuity in the presence of Van Roosbroeck space charge and conductivity contours.

In his analysis, Van Roosbroeck concerned himself with explicit solutions, obtained by neglecting the spatial derivatives of the carrier concentrations. As Kiess and Rose point out, the insertion of diffusion terms by itself does not correct the internal inconsistencies of the carrier concentration and field contours which resulted from the Van Roosbroeck analysis. Their conclusions are, however, excessively far reaching. It will be shown below that the majority-carrier depletion predicted by Van Roosbroeck can in fact occur in a manner consistent with the demands of current continuity. Accordingly, a computer analysis of the *complete* transport equations yields self-consistent concentration contours. They also demonstrate that there is a continuous transition from the lifetime case ($\tau_0 > \tau_D$) to the relaxation case ($\tau_0 < \tau_D$). $\tau_0 = \tau_D$ represents the *approximate* boundary condition for which majority-carrier depletion first appears.

Beyond this point, the carrier depletion can be increased to the limit given by $pn = n_i^2$ in two ways: (a) by increasing τ_D/τ_0, or (b) by increasing the current. In the depletion region, the current is carried by diffusion as well as drift of minority carriers, in the undisturbed material by equilibrium carriers drift. A recombination front appears between these two regions, if the depletion is sufficient. The calculated concentration contours show that a minimum value of $n\mu_n + p\mu_p$ does not always exist. When it does, its value is not necessarily equal to $n_i\mu_n + p_i\mu_p$, nor does it have a controlling influence over the resistance of the specimen. The minimum occurs within the recombination front, and because of the strong diffusion, the minimum-conductivity concept has no functional meaning. A sublinear voltage-current relationship is not intrinsically associated with the relaxation case, though it can appear in special circumstances. For $\mu_n = \mu_p$ a significant sublinearity has not been found. The most significant difference between the present results and Van Roosbroeck's predictions is that the total resistance of the sample is *lower*, not higher, than that evaluated from the unperturbed bulk resistivity.

Previous numerical calculations concerning relaxation semiconductors are limited by the assumption $np = n_i^2$, which implies zero recombination rate throughout. This assumption is not made in the present calculations, which show that departures from this condition appear and have an essential function in defining solutions. For this reason, definitions[5] which link the character of a relaxation semiconductor to $np = n_i^2$ are best avoided.

II. TRANSPORT EQUATIONS

The continuity relationships for electrons and holes have a common recombination term which can be written[2]

$$R = (np - n_e p_e)/\tau_0(n_e + p_e), \tag{1}$$

where n_e, p_e are the equilibrium concentrations and τ_0 is a carrier lifetime, here assumed to be independent of the injection level. With this term, the two standard continuity equations, together with Poisson's equation, can be written, for a trap-free n-type material as

$$\frac{\delta N}{\delta t} = \frac{\delta(NE)}{\delta X} + \frac{\delta^2 N}{\delta X^2} - A\left(NP - \frac{p_e}{n_e}\right), \tag{2}$$

$$\frac{\delta P}{\delta t} = -\frac{\mu_p}{\mu_n}\left(\frac{\delta(PE)}{\delta X} - \frac{\delta^2 P}{\delta X^2}\right) - A\left(NP - \frac{p_e}{n_e}\right), \tag{3}$$

$$\frac{\delta E}{\delta X} = P - N + 1 - \frac{p_e}{n_e}, \tag{4}$$

where N is the electron concentration normalized to n_e, P is the hole concentration normalized to n_e, E is the electric field normalized to kT/eL_{Dn}, (voltage normalized to kT/e), X is the spatial coordinate normalized to L_{Dn}, t is the time normalized to τ_{Dn}, with

$$L_{Dn} = (kT\epsilon/e^2 n_e)^{1/2}, \tag{5}$$

the Debye length defined for equilibrium electrons, and

$$\tau_{Dn} = \epsilon/e\mu_n n_e, \tag{6}$$

the dielectric relaxation time, defined for equilibrium electrons, and practically equal to the dielectric relaxation time of the n-type material in equilibrium, which is

$$\tau_D = \epsilon/e(\mu_n n_e + \mu_p p_e),$$

$$A = \tau_{Dn}/\tau_0(1 + p_e/n_e). \tag{7}$$

It is immediately clear that the parameter A, which for usual materials is approximately equal to τ_{Dn}/τ_0, is a very important quantity in these equations. It governs the balance between recombination on the one hand and neutrality restoration on the other. In this respect, the case $A = 1$ represents a significant boundary: when $A = 1$, Eq. (2) is compatible with $N = 1$ for all t and X, irrespective of P. Next we will see how A affects the manner in which a system recovers after a temporary departure from equilibrium.

III. RECOVERY IN THE HOMOGENEOUS CASE

Consider that $\Delta P = \Delta P_0$, and $\Delta N = \Delta N_0 = 0$ at $t = 0$, all over the sample. In Eqs. (2) and (3), the first term depends, via Eq. (4), on departures from neutrality, while the third term depends on recombination. For present purposes we assume that carrier concentrations remain uniform during the recovery process, i.e., the second terms in Eqs. (2) and (3) are neglected. The magnitude of A controls which process is the more important in determining the rate of change. If $A \ll 1$ or $A \gg 1$, the recovery process always has two well defined stages as shown below.

For $A \ll 1$, the recovery of neutrality takes place first, followed by recombination of electrons and holes in equal numbers. This in the normal ("lifetime") case and is denoted by the contour IBO in Fig. 1(a). In the first stage, the equations show that the electron concentration increases ($dN/dt > 0$), while the hole concentration decreases ($dP/dt < 0$). This decrease is insignificant as long as $P \ll 1$. As ΔN approaches ΔP (neutrality) the field terms tend to vanish. After that, the last terms in Eqs. (2) and (3) prevail. Since they are the same, the process is then governed by recombination, implying $dN/dt = dP/dt < 0$. If A were zero, the contour would be IB′C.

For $A \gg 1$, the case of a relaxation semiconductor, recombination prevails first (contour IC). Its rate diminishes with time and becomes nearly zero around C (actually zero at C'). This is a zero recombination point which does not, however, represent equilibrium, because we have not yet returned to neutrality. Neutrality is restored approximately along the hyperbola (contour CO). As $A \to \infty$, $C \to C'$, a limiting case which corresponds to the Van Roosbroeck approximation. Majority carrier depletion is associated with all stages of this process. The time dependence of the two processes is shown schematically in Figs 1(b) and 1(c).

For the limiting case $A = 1$ the recovery takes place along the line IO. This means that electrons brought from the outside in an attempt to recover neutrality recombine immediately; the electron concentration thus remains unchanged. This happens because the rate at which electrons are brought from the outside in an attempt to recover neutrality is equal to the rate at which they recombine with the excess holes; the electron concentration thus remains unchanged in time and as we will see below, also in space.

IV. STEADY STATE; EQUATIONS AND BOUNDARY CONDITIONS

In the transient relaxation case, some useful results can be derived by neglecting the current divergence terms as shown above. However, in the steady state, no terms can be neglected without sacrificing the possibility of evaluating car-

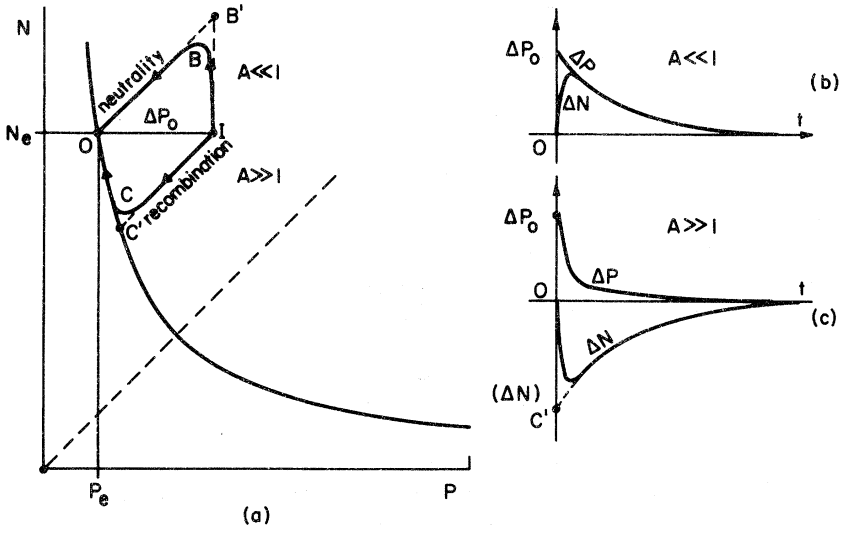

FIG. 1. Recovery of equilibrium in the homogeneous case. (a) Recovery process in the N-P plane. (b) and (c) Time evolution of the recovery process for lifetime case and relaxation case, respectively.

rier concentration contours. Thus, when the time derivatives are zero, Eqs. (2)–(4) become

$$\frac{d^2N}{dX^2} = A\left(NP - \frac{p_e}{n_e}\right) - \frac{d(NE)}{dX}, \tag{8}$$

$$\frac{d^2P}{dX^2} = A\left(NP - \frac{p_e}{n_e}\right)\frac{\mu_n}{\mu_p} + \frac{d(PE)}{dX}, \tag{9}$$

$$\frac{dE}{dX} = P - N + 1 - \frac{p_e}{n_e}. \tag{10}$$

Correspondingly, in the same terms, the equations for current densities are

$$J_n = \frac{\mu_n}{\mu_p}\frac{L_p}{L_{Dn}}\frac{n_e}{p_e}\left(NE + \frac{dN}{dX}\right), \tag{11}$$

$$J_p = \frac{L_p}{L_{Dn}}\frac{n_e}{p_e}\left(PE - \frac{dP}{dX}\right), \tag{12}$$

J_n and J_p being normalized to $eD_p p_e/L_p$.

The model here used is that of a semi-infinite trap-free n-type material. The space origin of the distributions has been taken at the injection plane, where the field is equal to zero. This notion comes from junction and contact theory and represents conventional practice. As a simplifying assumption, the injection efficiency is taken as unity ($J_{n0} = 0$ at $X = 0$). The above equations could be solved, in principle, if N_0, P_0, dN/dX, dP/dX, and E_0 were given. The solutions would automatically conserve a certain current $J = J_0$. However, what is needed here are solutions for a semi-infinite material. This implies zero space charge in the bulk and a finite limiting field E_∞ which secures a drift current (equal to the injected current) in the unperturbed bulk. Since these are two additional conditions, N_0 and P_0 can no longer be arbitrarily chosen. Self-consistent values have to be found by a search procedure involving successive approximations. It can be shown that the equations automatically ensure $N_\infty = N_e = 1$, $P_\infty = P_e = p_e/n_e$, $(dN/dX)_\infty = (dP/dX)_\infty = 0$.

According to these considerations, the expressions for the boundary conditions are (a) at $X = 0$,

$$\left(\frac{dP}{dX}\right)_0 = -\frac{L_{Dn}}{L_p}\frac{p_e}{n_e}J, \tag{13}$$

and

$$\left(\frac{dN}{dX}\right)_0 = 0, \tag{14}$$

$$E_0 = 0, \tag{15}$$

(b) at $X = \infty$,

$$\left(\frac{dE}{dX}\right)_\infty = 0, \tag{16}$$

and

$$E_\infty = \frac{L_{Dn}}{L_p}\frac{p_e}{n_e}\frac{J}{\mu_n/\mu_p + p_e/n_e}. \tag{17}$$

It can be shown that the search for proper values of N_0 and P_0 is aided by the fact that $|N_0 - N_e| < |N_{00} - N_e|$, where N_{00} is the value of N_0 which corresponds to zero curvature of the electron concentration contour, i.e., to $(dN^2/dX^2)_0 = 0$. It should be noted that $N_\infty - N_e$ is positive, zero or negative, for $A < 1$, $A = 1$, and $A > 1$, respectively. From the pattern of computed contours, an assessment can be made as to the sign of $\delta P_0 = P_0 - P_{0(\text{exact})}$ and $\delta N_0 = N_0 - N_{0(\text{exact})}$, and this assessment guides the process of successive approximation.

V. STEADY-STATE CONCENTRATION AND FIELD CONTOURS

The results which follow show the effect of changing the parameter A, when the injected current of (minority) holes is kept constant. Equal mobilities of electrons and holes have been assumed in an n-type semiconductor with $p_e/n_e = 10^{-2}$.

For purposes of comparison, Fig. 2 shows computer solutions for a typical lifetime case. Neutrality is not complete at $X = 0$, but becomes an increasingly useful approximation. It is this initial departure from neutrality which produces the field which, in turn, ensures the electron flux necessary for recombination with the injected holes. In addition the field serves to compensate for an opposed diffusion gradient of majority electrons. The various components of the current are also shown; they add (as, indeed, they must) to the constant total current. The recombination rate is given by the top curve.

Figure 3 refers to a case close to the boundary $A = 1$. It may be seen that $\Delta N \approx 0$ throughout, as expected. The space charge is due to holes and all of it now serves to generate the electron field current which ensures recombination. The electron diffusion current is, of course, zero throughout. (Computations of the actual boundary case $A = 1$ would be very time consuming and would not yield additional insights.)

We go now to the relaxation case. When the lifetime becomes shorter than the relaxation time it

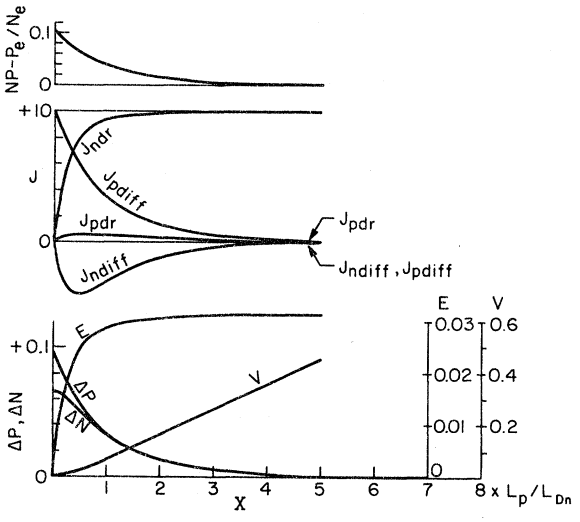

FIG. 2. Spatial contours of concentrations, field, voltage drop, current components, and recombination for a moderate lifetime case; $\tau_{Dn}/\tau_0 = 0.1 (A \simeq 0.099)$; $J = 10$, $\mu_n = \mu_p$, $p_e/n_e = 10^{-2}$. Magnitudes are normalized as shown in Sec. II. $NP - N_e P_e$ is proportional to the recombination rate.

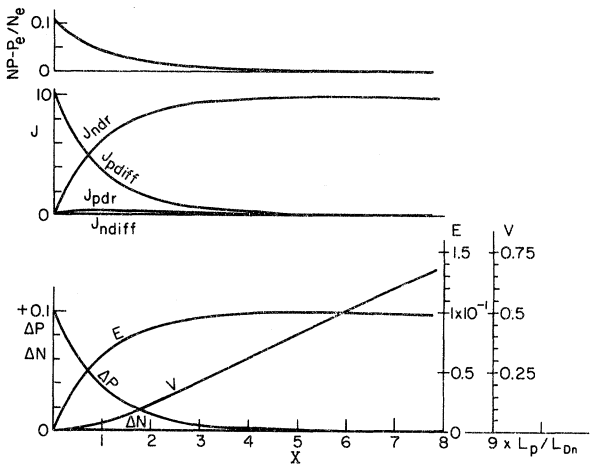

FIG. 3. Spatial contours of concentrations, field, voltage drop, current components, and recombination for the boundary case. Parameters are as for Fig. 1, except that $A \simeq 1$ $(\tau_{Dn}/\tau_0 = 0.99099)$.

is more and more difficult to secure electrons for recombination, and electron depletion appears. Figure 4 corresponds the case of $A = 99$, in which the electron depletion can be clearly seen. The depletion has three helpful consequences: (a) The electron-concentration gradient is now reversed and the electron diffusion now adds to the electron drift. (b) The electron depletion increases the positive space charge which builds up the field. (c) The recombination rate is reduced through electron depletion. There is a minimum of $N+P$, smaller than the equilibrium value. However, the effect of this general depletion is not critical, because it appears in a region with strong diffusion currents. $N+P$ as such therefore does not

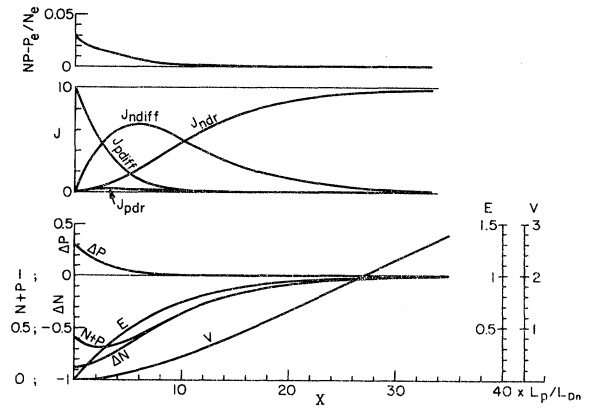

FIG. 4. Spatial contours of concentrations, field, voltage drop, current components, and recombination for a relaxation case. Parameters as for Fig. 1, except that $\tau_{Dn}/\tau_0 = 10^2$ $(A \simeq 99)$.

control the conduction process in any simple way. In particular, this minimum does not lead to a field higher than the field in the unperturbed bulk.

In all the cases presented above the recombination played an essential role in current continuity, beginning from the injection plane, where it had its maximum value. The situations which follow are different.

Figure 5 shows a pronounced relaxation case ($A = 990$) for the same current as above. The space charge and concentration of the injected holes now becomes so high that from the injected plane, to a certain depth in the material, the holes are able to carry the entire current by themselves. As is usual under space-charge-limited current conditions, the current gradually changes from diffusion to drift as the field increases. On the other hand, the field near the injecting contact, also to a certain depth inside the material, is insufficient to secure the influx of electrons for recombination. As a consequence, there are practically no electrons in this region; the recombination rate is practically zero and plays next to no role in current continuity. When the field resulting from the hole space charge becomes great enough to secure the necessary electrons, a recombination front appears, as predicted by Van Roosbroeck. It is within this front that the change over from hole current to electron current takes place. $N+P$ has a minimum smaller than the equilibrium value, situated in the recombination front where diffusion currents are prominent. The highest field is again in the unperturbed bulk.

The complete majority carrier depletion we have seen in a pronounced relaxation case may also appear in a moderate relaxation case ($A = 9.9$) when high recombination rates are imposed by very high currents (Fig. 6). The current is now a hundred times greater than in the previous situations. The diffusion currents are negligible everywhere, except in close proximity of the injection plane. In contrast to previous cases, $N+P$ now has no minimum and the field is again greatest in the unperturbed bulk.

VI. VOLTAGE-CURRENT RELATIONSHIPS

Two important Van Roosbroeck predictions remain to be discussed: (a) the prediction of higher-than-bulk resistance, and (b) the prediction of sublinear behavior. In Van Roosbroeck's system of approximation, (a) and (b) are closely related, both being a consequence of an $N+P$ depletion region in which the field is higher than in the bulk.

Figure 7 represents voltage-current characteristics, the voltage being measured at three arbitrary distances from the injection plane. I_0 is an arbitrary reference current, taken as the lowest value for which the above computations have been made. The voltage has been suitably normalized to permit easy comparison with the unperturbed bulk, which corresponds to the broken line. Up to moderate currents the curves are nearly linear (in distinct contrast with the lifetime case), with a small but very real tendency towards sublinearity, though not pronounced enough to be visible on Fig. 7, computed for $\mu_n = \mu_p$. (Preliminary calculations show that this sublinear behavior becomes more pronounced when $\mu_p < \mu_n$.) At high currents (high injection levels), a pronounced superlinear tendency prevails. The physics of this behavior depends on the fact that there are always two tendencies, one due to the injected minority carriers which *lowers* the local effective resistivity, and one due to the depleted minority carriers which *increases* it. Which of these effects "wins" de-

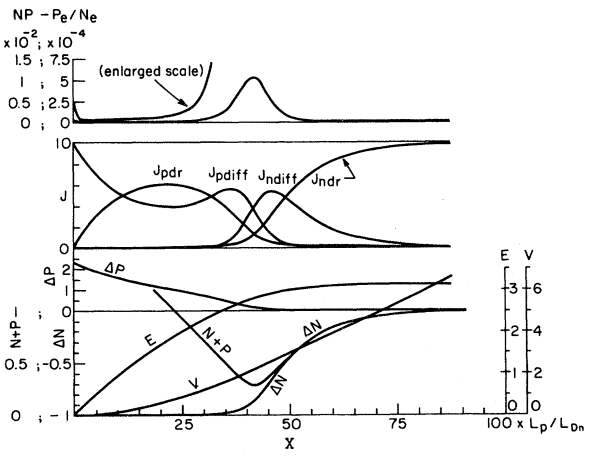

FIG. 5. Spatial contours of concentrations, field, voltage drop, current components, and recombination for a pronounced relaxation case. Parameters as for Fig. 1 except that $\tau_{Dn}/\tau_0 = 10^3$ ($A \simeq 990$).

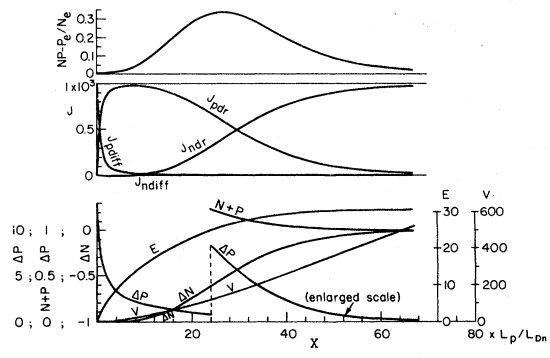

FIG. 6. Spatial contours of concentrations, field, voltage drop, current components, and recombination for a moderate relaxation case and high current. Parameters as for Fig. 1, except that $\tau_{Dn}/\tau_0 = 10$, $J = 10^3$.

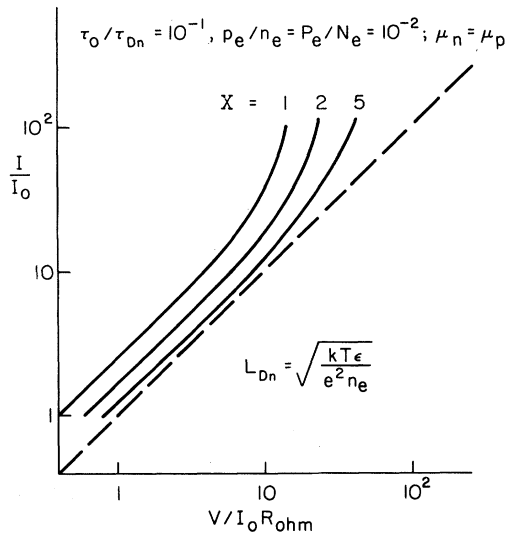

FIG. 7. *I-V* curves normalized for comparison with the unperturbed bulk behavior (broken line). $\mu_n = \mu_p$, $p_e/n_e = 10^{-2}$, I_0 corresponds to $J = 10$. Voltage is measured at the distance X (normalized to L_{Dn}) from the injection plane. R_{ohm} is the resistance of the 0-X region calculated on the basis of the unperturbed bulk resistivity.

pends on detailed circumstances, for which there is no immediate intuitive assessment. Even when the sublinearity component is too small to predominate, it increases the region of approximate linearity. In the cases here discussed, the total resistance is not higher but lower than the resistance calculated on the basis of the unperturbed bulk resistivity (curves on Fig. 7 above the broken line). This is a principal difference between the present results and Van Roosbroeck predictions.

The question arises, could a greater-than-bulk resistance be obtained if unequal carrier mobilities and traps were introduced into the model? To answer this, one could argue as follows: For a higher-than-bulk resistance the field in the region perturbed by injection must be *greater* than the limit field in the bulk, and not lower as it is in the situations here discussed. Since the field is zero at $X = 0$, it must have a maximum somewhere, and this must be a location where the space charge changes from positive to negative. This calls for a significant negative space charge, which means that excess electrons must be present, either free or trapped. If all charges were trapped, the systen would be self-consistent; however, the normal assumption is that even though most of the excess electrons might be trapped, some must be free. Thus, for a higher than bulk resistance, the majority carrier concentration must not only increase from total depletion to normal in the bulk, but most overshoot. Kiess and Rose have already shown that such an overshoot cannot occur in a region in which the field is higher than in the bulk. Unequal mobilities cannot alter this situation; nor can traps. One possibility is that the overshoot might, in certain circumstances (e.g., very high currents) occur in a region in which the field is still lower than bulk, i.e., closer to the injecting contact. Whether this corresponds to a valid solution of the equations remains to be seen. Under such conditions the computation is particularly laborious, which explains why a quick and reliable answer is still unavailable.

VII. SUMMARY OF CONCLUSIONS

Majority carrier depletion is the logical outcome of minority carrier injection into a relaxation semiconductor, defined by $A > 1$ ($\tau_{Dn}/\tau_0 > 1 + p_e/n_e$). The depletion can become complete by enhancing the total recombination rate, either by increasing A or by increasing the injected current. Under such conditions, there is a distinct region in which the current changes from minority carriers to majority carriers, i.e., a recombination front. Between the injection plane and the recombination front the current is minority carrier space charge limited; beyond it it is majority carrier dominated, and eventually becomes the drift current in the unperturbed bulk. The computed voltage-current characteristics show an extended linear region, resulting from the opposed tendencies of majority carrier depletion (sublinear) and minority carrier injection (superlinear). Total resistances higher than those calculated on the basis of the unperturbed bulk resistivity have not been found for the conditions investigated.

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