Letter | Editors' Suggestion

Gravitational wave analogs in spin nematics and cold atoms

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(Received 14 December 2023; revised 8 May 2024; accepted 9 May 2024; published 14 June 2024)

Large-scale gravitational phenomena are famously difficult to observe, making parallels in condensed matter physics a valuable resource. Here we show how spin nematic phases, found in magnets and cold atoms, can provide an analog to linearized gravity. In particular, we show that the Goldstone modes of these systems are massless spin-2 bosons, in one-to-one correspondence with quantized gravitational waves in flat spacetime. We identify a spin-1 model supporting these excitations and, using simulation, outline a procedure for their observation in a ^{23}Na spinor condensate.

DOI: 10.1103/PhysRevB.109.L220407

Introduction. Light has been a natural companion of humanity since our earliest days, shaping civilization as we know it. However, our attention to astrophysical gravitational waves is, by comparison, still in its infancy. The experimental detection of gravitational waves by the LIGO collaboration [1] marked the beginning of a new age of observational astronomy. That said, production of measurable gravitational radiation is far from being feasible due to the energy scales involved, unlike its photonic counterpart. Alternatives that provide laboratory access to such massless spin-2 waves would therefore provide many new opportunities.

Thus far, several condensed matter systems have been suggested to mimic features of gravity, with much focus on reproducing the effects of curved spacetimes. Tensor analogs leading to rich gravitational phenomena exist, and have been measured, in the context of superfluid ^{3}He [2–4]. Acoustic analogs of gravitational phenomena were suggested [5] and later measured [6], with further promising experimental candidates in semimetals [7,8], in quantum Hall systems [9], in optics [10,11] and in cold atoms [12–16]. Connections between elasticity and emergent gravitational phenomena have been explored theoretically, [17–22], and can also be found in models supporting fractons [23–28]. Related aspects of geometry also arise in magnetic models [29], and graphene [30]. Nonetheless, it remains an open question to identify an experimentally viable platform which provides access to massless spin-2 bosons in one-to-one analog with gravitational waves, as they appear in flat (3+1)-dimensional spacetime.

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Massless spin-2 bosons also arise as Goldstone modes in quantum spin nematics, a form of quantum liquid crystal found in both insulating magnets [31–41], and arrays of cold atoms [42–46]. In this Letter, we identify a parallel between gravitational waves and the Goldstone modes of quantum spin nematics, and suggest two routes for their experimental realization. We first review the description of gravitational waves within linearized gravity. We then show that an identical set of equations arises in the low-energy continuum field theory describing spin nematics. Through numerical simulation, we explore the real-time dynamics of a microscopic model with spin nematic order, showing how quadrupolar waves—equivalent to gravitational waves—are generated through the annihilation of topological defects. We conclude by suggesting an experimental protocol for the creation and observation of analogue gravitational waves in spin nematic phases, realized in either magnetic insulators or cold atoms.

Linearized gravity and gravitational waves. We now briefly summarize the key features of linearized gravity, leading up to gravitational waves. This treatment follows the conventions of standard textbooks, e.g., Refs. [47–49]. General relativity (GR) is a geometrical theory, describing the curvature of a four-dimensional spacetime. Fundamental to this is the metric tensor $g_{\mu\nu}$, a symmetric rank-2 tensor, which allows the definition of distance. Here the Greek indices μ, ν run over all four spacetime dimensions. In linearized gravity, spacetime is assumed flat up to small fluctuations $h_{\mu\nu}$ such that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric for a flat spacetime. The linearized theory is invariant under transformations

$$x'^{\mu} = x^{\mu} + \xi^{\mu}(x), \tag{2a}$$

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu, \tag{2b}$$

where x^μ denotes spacetime coordinates, and ξ_μ corresponds to an infinitesimal coordinate transformation. The existence of these transformations implies that not all degrees of freedom are independent, and in deriving a theory for gravitational waves, it is conventional to make the choice

$$h^{\mu}_{\ \mu}(x^\sigma) = 0 \text{ [traceless]}, \tag{3a}$$

$$h_{0\mu}(x^\sigma) = 0 \text{ [no scalar or vector components]}, \tag{3b}$$

$$\partial^n h_{nm}(x^\sigma) = 0\ ,$$

$$\Rightarrow \quad k^n h_{nm}(k^\sigma) = 0 \text{ [no longitudinal dynamics]}. \tag{3c}$$

Here, Roman indices n, m denote the spatial components, and the Einstein summation convention for repeated indices is assumed. Implementing these constraints, we arrive at a theory expressed in terms of a symmetric, traceless, rank-2 tensor [49], with dynamics governed by the action

$$\mathcal{S}_{\mathsf{LG}} = -\frac{c^3}{16\pi G}\int d^4x\,[\partial^\alpha h^{\mu\nu}\partial_\alpha h_{\mu\nu}], \tag{4}$$

where c is the speed of light, and G the gravitational constant. This leads to the equation of motion for massless waves

$$\frac{1}{c^2}\partial_t\partial^t h_{\mu\nu} - \partial_n\partial^n h_{\mu\nu} = 0, \tag{5}$$

where implicitly, only two of the 16 components of $h_{\mu\nu}$ have nontrivial independent dynamics. Once quantized [50], the solutions of this wave equation are spin-2 bosons (gravitons), with dispersion

$$\omega_{\mathsf{LG}}(\boldsymbol{k}) = c|\boldsymbol{k}|, \tag{6}$$

and two independent polarizations, $\sigma = +, \times$, such that

$$h_{\mu\nu}(t,\boldsymbol{x}) = \sum_{\sigma=+,\times}\int d^3k\frac{1}{\sqrt{\omega_{\mathsf{LG}}(\boldsymbol{k})}}\big[\epsilon^\sigma_{\mu\nu}a^\dagger_\sigma(\boldsymbol{k})e^{ik_\rho x^\rho} + (\epsilon^\sigma_{\mu\nu})^* a_\sigma(\boldsymbol{k})e^{-ik_\rho x^\rho}\big], \tag{7}$$

where $\epsilon^\sigma_{\mu\nu}$ is a tensor encoding information about polarization, and $a_\sigma(\boldsymbol{k})$ satisfies bosonic commutation relations

$$[a_\sigma(\boldsymbol{k}), a^\dagger_{\sigma'}(\boldsymbol{k}')] = \delta_{\sigma\sigma'}\delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{8}$$

For a wave with linear polarization, propagating along the z direction, $\epsilon^\sigma_{\mu\nu}$ takes the specific form

$$\boldsymbol{\epsilon}^+ = \frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix},$$

$$\boldsymbol{\epsilon}^\times = \frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix}. \tag{9}$$

Physically, gravitational waves correspond to a quadrupolar distortion of space, in which compression and dilation alternate. In Fig. 1, we visualize this by plotting surfaces of equal strain

$$V(t, f_1, f_2, z) = \pm\ \text{const.}, \tag{10}$$

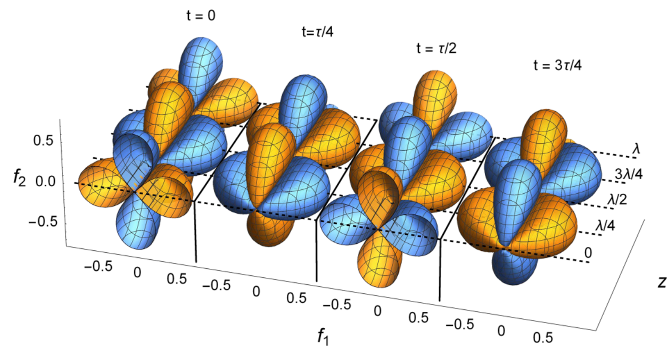

FIG. 1. Quadrupolar nature of gravitational waves, and Goldstone modes of spin-nematic order, visualized through the associated distortions of spacetime, or the spin-nematic ground state. Results are shown for a wave of wavelength λ and period τ, with polarization $\boldsymbol{\epsilon}^+$ [Eq. (9)], propagating along the z–axis. In the case of gravitational waves [Eq. (7)], f_1, f_2, represent x and y axes of spacetime, and the quantity plotted is a surface of constant strain [Eqs. (11) and (10)]. In the case of spin-nematic order [Eq. (19)], f_1, f_2, represent spin components S^x and S^y, and the quantity plotted is the change in the spin-nematic order parameter [Eqs. (20) and (21)]. Blue surfaces denote positive strain/deformation, while orange surfaces denote negative strain/deformation. An animated version of this figure is available in Ref. [51].

where the wave is taken to propagate in the z direction, and strain is visualized in the plane perpendicular to this, with $(f_1, f_2) \to (x, y)$. Here strain (squared) is defined by

$$V(t,\boldsymbol{x}) = \frac{x^m h_{mn}(t,\boldsymbol{x})x^n}{|\boldsymbol{x}|^2}, \tag{11}$$

with $m, n = 1, 2, 3$ running over the spatial components.

Linearized gravity analog in spin nematics. In the discussion above, we have seen how small fluctuations of the metric $g_{\mu\nu}$, [Eq. (1)], give rise to gravitational waves, which are linearly-dispersing massless spin-2 bosons, described by the action $\mathcal{S}_{\mathsf{LG}}$ [Eq. (4)]. In this sense, the search for analogs of gravitational waves can be cast as the search for a physical system which can be described in terms of a symmetric, traceless rank-2 tensor, with linearly-dispersing excitations governed by an action of the form $\mathcal{S}_{\mathsf{LG}}$.

The strategy we pursue in this Letter is to map fluctuations about a flat spacetime onto the Hilbert space of a quantum system with ground state order characterized by a symmetric, traceless rank-2 tensor. Analogues of gravitational waves can then be found in the Goldstone modes of this symmetry-broken state.

The order parameter for a nematic liquid crystal is a symmetric, traceless rank-2 tensor [52]. Here we consider a form of quantum liquid crystal known as a "quantum spin nematic," originally introduced as a magnetic state [31,32,36] which preserves time-reversal symmetry, but breaks spin-rotation symmetry through the quadrupole operators

$$\mathcal{Q}^{mn}_{ij} = \tfrac{1}{2}\big(S^m_i S^n_j + S^n_i S^m_j\big) - \tfrac{1}{3}\delta_{mn}S^n_i S^n_j. \tag{12}$$

Here S^m is a spin operator with components $m = x, y, z$, satisfying SU(2) commutation relations.

The simplest form of a quantum spin nematic is the "ferroquadrupolar" (FQ) state, a uniaxial nematic liquid crystal

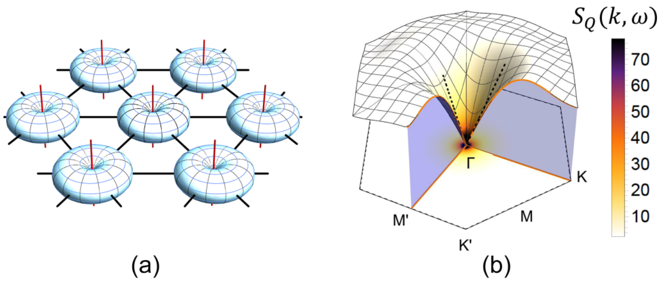

FIG. 2. Spin-nematic state on a triangular lattice, and its spin-2 excitations. (a) Ferroquadrupolar (FQ) ground state, in which quadrupole moments of spin align with a common axis. (b) Dispersion of excitations about the FQ state, as revealed by the quadrupolar structure factor $S_Q(\boldsymbol{k},\omega)$ [Eq. (23)]. The linear character of the dispersion at long wavelength, $\omega(\boldsymbol{k}) = v|\boldsymbol{k}|$ (black dashed line) is consistent with the predictions of the field theory [Eq. (16)]. The spin-2 nature of the long-wavelength excitations can be inferred from the high intensity of the structure factor at low energies. Results are shown for a spin-1 bilinear biquadratic (BBQ) model [Eq. (22)], with parameters $J_1 = 0$, $J_2 = -1$, as described in Ref. [55].

in which all quadrupole moments are aligned [Fig. 2(a)]. As in conventional liquid crystals [53], such a state can be characterized by a director $\boldsymbol{d}$, and its symmetry dictates that it supports two, degenerate Goldstone modes [54], which have the character of massless, spin-2 bosons [34,37,55]. We will now show how these correspond to the massless spin-2 bosons found in linearized gravity.

We start by promoting $\mathcal{Q}^{mn}$ to a tensor field $Q^{\mu\nu}$ providing a low-energy effective description, identifying $Q^{mn} = \mathcal{Q}^{mn}$, where $Q^{mn} = Q_{mn}$, and by setting components $Q_{0\mu} = Q_{\mu 0} = 0$. In analogy with Eq. (1), we consider fluctuations $Q^{\mathsf{E}}_{\mu\nu}$ about a state with uniform spin nematic order $Q^{\mathsf{GS}}_{\mu\nu}$, viz

$$Q_{\mu\nu} = Q^{\mathsf{GS}}_{\mu\nu} + Q^{\mathsf{E}}_{\mu\nu}, \tag{13}$$

requiring that these fluctuations occur in the transverse channel, i.e., that the change affects the direction but not the magnitude of the quadrupolar order. This assumption is appropriate for the low-energy physics of spin nematics [34,56,57].

What remains is to match the fluctuations of quadrupolar order to the fluctuations associated with a gravitational wave. The former occur in spin space and are transverse to the order of the ground state, while the latter occur in spacetime, and are transverse to the direction of propagation. The fields which describe the fluctuations in these two different coordinate systems can be related through the unitary transformation

$$\tilde{Q}_{\mu\nu}(\boldsymbol{k}) = C_{\mu\nu}{}^{\rho\sigma}(\boldsymbol{k},\boldsymbol{d}) Q^{\mathsf{E}}_{\rho\sigma}(\boldsymbol{d}), \tag{14}$$

where $C_{\mu\nu}{}^{\rho\sigma}(\boldsymbol{k},\boldsymbol{d})$ acts on quadrupole excitations with wave vector $\boldsymbol{k}$ about a FQ state characterized by director $\boldsymbol{d}$. Further details of this transformation are given in Ref. [51].

For an appropriate choice of $C_{\mu\nu}{}^{\rho\sigma}(\boldsymbol{k},\boldsymbol{d})$, $\tilde{Q}_{\mu\nu}$ satisfies the conditions

$$\tilde{Q}^{\mu}{}_{\mu} = Q^{\mu}{}_{\mu} = 0 \text{ [traceless]}, \tag{15a}$$

$$\tilde{Q}_{0\mu} = Q_{0\mu} = 0 \text{ [no scalar or vector components]}, \tag{15b}$$

$$k_m \tilde{Q}_{mn}(k^{\sigma}) = 0 \text{ [no longitudinal dynamics]}. \tag{15c}$$

The low-energy fluctuations of the spin nematic can be described in terms of a quantum nonlinear sigma model [34,56,57]. Given these physical constraints, and the decomposition described by Eq. (13), we arrive at an action which exactly parallels linearized gravity [Eq. (4)], via

$$\mathcal{S}_{\mathsf{FQ}} = -\frac{1}{2}\int dt d^d x[-\chi_\perp(\partial^t \tilde{Q}^{\mu\nu}\partial_t \tilde{Q}_{\mu\nu}) + \rho_s(\partial^n \tilde{Q}^{\mu\nu}\partial_n \tilde{Q}_{\mu\nu})], \tag{16}$$

where $\chi_\perp$ is the transverse susceptibility and ρ_s the stiffness, associated with spin-nematic order [56,57].

The low-lying excitations of this theory are massless spin-2 bosons, satisfying the wave equation [cf. Eq. (5)]

$$\frac{1}{v^2}\partial_t \partial^t \tilde{Q}_{\mu\nu} - \partial_n \partial^n \tilde{Q}_{\mu\nu} = 0, \tag{17}$$

with $v = \sqrt{\rho_s/\chi_\perp}$, and with dispersion

$$\omega_{\mathsf{FQ}}(\boldsymbol{k}) = v|\boldsymbol{k}|. \tag{18}$$

The solutions to Eq. (17) have exactly the same structure as those for gravitons [cf. Eq. (7)]

$$\tilde{Q}_{\mu\nu}(t,\boldsymbol{x}) = \sum_{\sigma=+,\times}\int d^3k \frac{1}{\sqrt{\omega_{\mathsf{FQ}}(\boldsymbol{k})}}\left[\epsilon^{\sigma}_{\mu\nu} b^{\dagger}_{\sigma}(\boldsymbol{k}) e^{ik_\rho x^\rho} + \left(\epsilon^{\sigma}_{\mu\nu}\right)^* b_\sigma(\boldsymbol{k}) e^{-ik_\rho x^\rho}\right], \tag{19}$$

where $b_\sigma(\boldsymbol{k})$ satisfies bosonic commutation relations [Eq. (8)] and the tensors $\epsilon^{\sigma}_{\mu\nu}$ are given by Eq. (9).

Just as we can visualize gravitational waves through surfaces of constant strain, so we can visualize the Goldstone modes of a spin-nematic state through surfaces of equal distortion of the spin-nematic order parameter. In analogy with Eq. (11), we consider the wave function amplitude

$$V(\boldsymbol{S},(t,\boldsymbol{x})) = \frac{S^m \tilde{Q}_{mn}(t,\boldsymbol{x}) S^n}{|\boldsymbol{S}|^2}. \tag{20}$$

In Fig. 1, we show surfaces of equal amplitude in the (S^x, S^y) plane, for a quadrupole wave propagating in the z direction,

$$V((f_1, f_2, 0), (t, 0, 0, z)) = \pm \text{ const.}, \tag{21}$$

with $(f_1, f_2) \rightarrow (S^x, S^y)$, in analogy with Eq. (10). Plotted in this way, the identity between the two types of excitation is clear.

From the analysis above, we learn that quadrupolar waves in a quantum spin nematic are in one-to-one correspondence with quantized gravitational waves (gravitons) in a flat, four-dimensional spacetime. However, there is a critical distinction regarding the spaces these waves arise in, which has important implications for realizing them in experiment. Gravitational waves involve quadrupolar distortions of space, transverse to the direction of propagation. This implies that a minimum of three spatial dimensions is required to support a gravitational wave. In contrast, the quadrupolar waves found in spin nematics arise in a spin space which is automatically three-dimensional, regardless of the number of spatial dimensions. For this reason, it is possible to explore analogs of gravitational waves in two-dimensional spin systems. It is this subject which we turn to next.

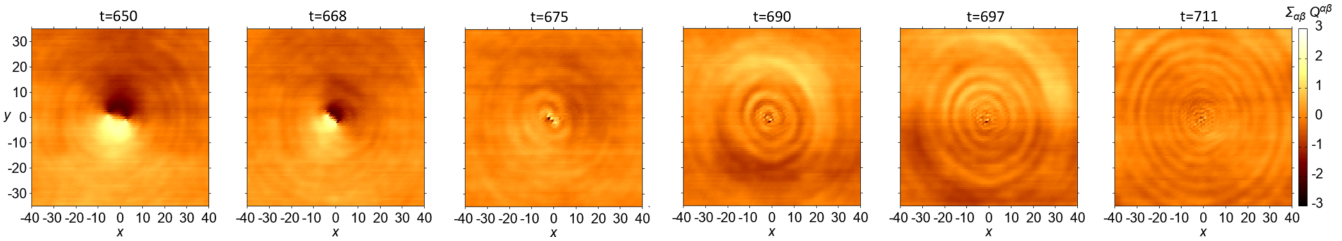

FIG. 3. Numerical simulation of vortices within a spin-nematic state, showing how quadrupole waves, analogous to gravitational waves, are created when a pair of vortices in-spiral and annihilate. Individual frames are taken from dynamical simulation of a ferroquadrupolar state (FQ) in the spin-1 bilinear biquadratic (BBQ) model on a triangular lattice [Eq. (22)], with further detail given in Ref. [51] (see also Ref. [74] therein), where an animated version of this result is also available.

Simulation using cold atoms. The idea of using cold atoms to simulate a quantum spin nematic has a long history [42–46]. The majority of proposals build on "spinor condensates" of atoms, such as ^{23}Na, ^{39}K, or ^{87}Rb, whose internal hyperfine states mimic the magnetic basis of a spin-1 moment [42,58,59]. The interactions between these effective spin-1 moments depend on the details of their scattering and, where attractive, can lead to spin-nematic order [42]. Condensates described by the order parameter $\mathcal{Q}^{\alpha\beta}$ [Eq. (12)] have already been observed in experiment [60]. On symmetry grounds, the Goldstone modes of these systems must be described by $\mathcal{S}_{\mathsf{FQ}}$ [Eq. (16)], making this an analog of linearized gravity.

Optical lattices can be arranged in a wide array of geometries, including triangular lattices [61], and cold atom experiments with ^{23}Na atoms are carried out in many laboratories, e.g., Refs. [46,62,63]. Realizing an analog of gravitational waves on a lattice therefore also seems a realistic possibility.

Realization of gravitational waves in a microscopic lattice model. In addition to realization of analog gravitational waves using quantum fluids as suggested above, spin-nematic phases can also be found in solid state magnetic systems. The simplest microscopic model supporting a quantum spin-nematic state is the spin-1 bilinear biquadratic (BBQ) model

$$\mathcal{H}_{\mathsf{BBQ}} = J_1 \sum_{\langle ij\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle ij\rangle} (\boldsymbol{S}_i \cdot \boldsymbol{S}_j)^2, \tag{22}$$

known to support FQ order for a wide range of $J_2 < 0$, irrespective of lattice geometry [31,33,64]. Particular attention has been paid to the BBQ model on a triangular lattice [37,55,56,65–68], where studies have been motivated by $NiGa_2S_4$, a candidate spin-nematic material. It has also been argued that ^{23}Na atoms in an optical lattice could be used to realize the BBQ model [Eq. (22)], with parameters falling into the range relevant to FQ order [44,69].

Explicit calculations of FQ dynamics within the BBQ model reveal two, degenerate bands of excitations, with linear dispersion at long wavelength [37,55,68]. The quadrupolar (spin-2) nature of these excitations at low energy is manifest in the dynamical structure factor for quadrupole moments

$$S_{\mathcal{Q}}(\boldsymbol{k},\omega) = \sum_{\alpha,\beta} \int \frac{dt}{2\pi} e^{i\omega t} \langle \mathcal{Q}^{\alpha\beta}(\boldsymbol{k},t)\mathcal{Q}^{\alpha\beta}(-\boldsymbol{k},0)\rangle, \tag{23}$$

shown in Fig. 2(b), for calculations carried out at a semiclassical level [55]. Starting from Eq. (22), it is also possible to parametrize the continuum field theory Eq. (16), obtaining results in quantitative agreement with the microscopic model, as shown in Fig. 2(b).

Quench dynamics, simulation, and measurement. We now turn to the question of how gravitational-wave analogs could be created and observed in experiment. For concreteness, we consider a FQ state in an explicitly two-dimensional system, which we model as set of spin-1 moments on a lattice [cf. Eq. (22)]. Consistent with the Mermin-Wagner theorem, for low-dimensional systems to exhibit anything besides exponentially-decaying correlations at low temperature, they must undergo topological phase transitions e.g., of the BKT type [70,71]. The FQ state in a 2D magnet has been argued to be connected to the high-temperature magnetic phase via a vortex-induced topological phase transition [72].

The excitations which mediate this phase transition are no longer the integer vortices of the Berezinskii-Kosterlitz-Thouless (BKT) transition [73], but rather $\mathcal{Z}_2$ vortices of homotopy group π_1, specific to the nematic order parameter [53]. Cooling rapidly through the transition (quenching) leads to a state rich in pairs of $\mathcal{Z}_2$ vortices, which are subject to attractive interactions, and spiral towards one another in much the same way as gravitating masses. In the process, vortices radiate energy in the form of quadrupolar waves, Eq. (19), and eventually annihilate. This process is clearly visible in simulations of the BBQ model [Eq. (22)], as illustrated in Fig. 3 and the accompanying animation [51] (see also Ref. [74] therein).

As can be seen from these simulations, the dynamics of vortices is very slow compared to that of quadrupolar waves, and the timescale associated with the annihilation of $\mathcal{Z}_2$ vortices is of order $10^2 J_2^{-1}$. Observing vortices in experiment will therefore typically demand long-lived condensates. Nevertheless, successful imaging of nematic vortices within a spinor-condensate of ^{23}Na ions has already been realized, over timescales of $\sim$1 s [75]. Direct measurements of the quadrupole operators $\mathcal{Q}_{mn}$ are possible for spinor condensates, and have been carried out in ^{87}Rb [76], and condensates populating the nematic phase can be distinguished from magnetic phases [46]. Additionally, real-space imaging of the quadrupolar channel of a spinor condensate is accessible through the imprint of the tensor polarizability of the atoms on light passing through the system [69,77]. It is also possible to indirectly access quadrupolar correlations in magnetic insulators through Raman scattering [78], or RIXS measurements [79,80].

Taken together, this all jointly suggests that it is a realistic possibility to realize and observe such spin nematic gravitational waves analogs.

Conclusions. Gravitational waves are of fundamental interest, but hard to study in experiment, because the energy scales of excitations are so large, and their amplitudes so small. In this Letter we have shown how a theory in direct correspondence to linearized gravity arises in systems with spin-nematic order. The Goldstone modes of this spin-nematic state are massless spin-2 bosons, which behave as exact analogs of quantized gravitational waves (gravitons). These results imply that it is possible to simulate various aspects of linearized gravity, including gravitons and topological excitations, in magnets or assemblies of cold atoms which realize a spin-nematic state [81–85]. A more general analog of aspects of gravitational physics could be realizable via an appropriate emergent gauge theory in spin liquids.

Acknowledgments. The authors are pleased to acknowledge helpful discussions with Yuki Amari, Andrew Smerald and Hiroaki T. Ueda. This work was supported by Japan Society for the Promotion of Science (JSPS) KAKENHI Grants No. JP19H05822, No. JP19H05825, No. JP20K14411, No. JP20H05154, No. JP24K00546, and No. JP22H04469, MEXT as "Program for Promoting Researches on the Supercomputer Fugaku" (Grant No. JPMXP1020230411), JST PRESTO Grant No. JPMJPR2251, and by the Theory of Quantum Matter Unit, OIST. Numerical calculations were carried out using HPC facilities provided by OIST and the Supercomputer Center of the Institute of Solid State Physics, the University of Tokyo.

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