PHYSICAL REVIEW B **109**, L220406 (2024)

Letter

Fidelity of the Kitaev honeycomb model under a quench

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(Received 29 August 2022; accepted 5 June 2024; published 14 June 2024)

We theoretically study the influence of quenched outside disturbances in an intermediately long-time limit. We consider localized imperfections, uniform fields, noise, and couplings to an environment within a unified framework using a prototypical but idealized interacting quantum device—the Kitaev honeycomb model. As a measure of stability we study the Uhlmann fidelity of quantum states after a quench. To treat the unperturbed dynamics as a free-fermion model without neglecting evolution of states between flux sectors, we push the flux degree of freedom into the perturbation. For noisy quenches, both gapped and gapless systems exhibit a universal form for the long-time fidelity, $Ce^{-\alpha t}t^{-\beta}$ where the values of C, α, and β depend on physical parameters such as system size and disturbance strength. Finally, we show that selective filling of the spinon Brillouin zone can be used to greatly increase the fidelity over the ground-state value. Our work provides estimates for the intermediate long-time stability of a quantum device, offering engineering guidelines for quantum devices in quench design and system size.

DOI: 10.1103/PhysRevB.109.L220406

Introduction. Processing quantum information requires stability of quantum states used to encode and transmit information [1–3]. A lack of information preservation can be disastrous for the reliability of computational results [4–9]. This has led to large efforts to develop fault tolerant devices and methods for error correction [8–22]. Thus, it is important to study the stability (with respect to environmental perturbations) of systems that might serve as the platform for a quantum computer/device.

Here, we study stability over long timescales using an asymptotic approach valid for a weak external influence [23,24]; this regime continues to become more relevant as technological advances are made that increasingly exclude outside disturbances [25,26]. Additionally, universal behavior is expected to emerge in this regime [27]. To observe emergent universal behavior, we study the impact of different classes of external disturbances on the stability of quantum states, treated using a quench that is turned on instantaneously and left on at all later times.

In particular, we study the effect of weak disturbances on the system. While current quantum computing platforms tend to be noisy, here we anticipate the need for a low enough physical error rate that fault-tolerant computing becomes possible [28]. Much of the work to be done in topological quantum computing is to ensure that a given platform will be capable of robust quantum computation, using logical rather than physical qubits, within the approaching weak-noise regime. This is the motivation for studying the regimes we have. Specifically, we study the stability of states within the flux-free sector, as these are states on which any anyonic excitations would be created, for use in topological quantum computing.

We consider disturbances captured via a perturbation to the Hamiltonian, as well as those requiring a Lindblad master equation approach [29,30] for which the most economical description involves nonpure density matrices. The latter captures effects appearing due to insufficient or leaky environmental shielding [24,30–32]. We employ the Uhlmann fidelity as a measure of the stability [33–35] of states in the Kitaev honeycomb model [36,37], which is exactly solvable, is a prototypical example of a 2D spin liquid [38,39], and is relevant for robust topological quantum computation [19] (through its anyonic excitations [19,40,41]). We find that the long-time stability in all cases we study has the same universal functional form, fidelity $\propto e^{-\alpha t}t^{-\beta}$, where the parameters α or β can be zero in certain situations. This result does not depend on the details of the physical system, such as the form of its excitation spectrum, but rather on general features such as system dimensionality and the nature of any band-touching points, which make dominant k-space contributions to the long-time fidelity, as seen in the asymptotic analysis we present here and in the Supplemental Material [42].

Insights into ground state stability of the Kitaev model are an important consideration that can complement the inherent stability of its anyonic excitations. Indeed, the ground state of Kitaev materials [43] must be robust enough to survive the introduction of anyonic excitations. We therefore study the long-time stability of the Kitaev ground state. We focus on important but relatively rarely studied noisy quenches or a sudden weak coupling to a heat bath (used to model localized holes in magnetic shielding of a device or coupling to the environment).

To supplement our ground state results we also consider excited spinon states. The study of stability of spinon states under quenches reinforces the relevance of the ground state stability for excited states.

We address this issue by considering the possibility of nontrivial evolution between flux sectors. We define an auxiliary Hamiltonian sharing a ground state with the Kitaev Hamiltonian, but without a flux degree of freedom. The dynamics of the flux degrees of freedom is pushed into the perturbed dynamics, a procedure which we check for consistency when the flux term is not necessarily small.

At the outset of our discussion, we highlight a major feature of our methods. The Kitaev model is exactly solvable within a given flux sector, where flux on each plaquette is conserved. This complicates a straightforward use of an interaction picture in which to formulate the perturbation theory for weak disturbances, unless one simply neglects the flux degree of freedom to ensure the unperturbed Hamiltonian is a free-fermion model. However, we are interested in nontrivial evolution between flux sectors and therefore wish to keep this degree of freedom and carefully consider how it evolves. We therefore define an auxiliary Hamiltonian with the same ground state as the exact Kitaev Hamiltonian, but without a flux degree of freedom. The important nontrivial step then is to push the additional contributions from a dynamical flux degree of freedom into the perturbation, a treatment which is then checked for consistency when this term is not necessarily small. The physical way to understand this is that we consider not just a response to the change in Hamiltonian but also to possible fluctuations in flux degrees of freedom. In this way we leverage the exact-solvability of the free-fermion model without ignoring the fact that a disturbance will in principle lead to nontrivial evolution into other flux sectors, which would not be captured appropriately by a purely free-fermion treatment. This perturbative approach could also be applied to linear response theory (considering both the response to an external field and to the change in flux configurations), for example, and thus has implications reaching far beyond our study.

Model. We take the Kitaev honeycomb model [36,37] in Eq. (1) as a starting point:

$$H = -J_x \sum_{x-\text{bonds}} \sigma_i^x \sigma_j^x - J_y \sum_{y-\text{bonds}} \sigma_i^y \sigma_j^y - J_z \sum_{z-\text{bonds}} \sigma_i^z \sigma_j^z. \quad (1)$$

The equilibrium properties of this model are reviewed in the Supplemental Material (Sec. I) [42] to keep this work self-contained. Our first task is to study the stability of the Kitaev ground state. We use the Uhlmann fidelity $F(t)$ to quantify stability [24], $F(t) = \mathrm{Tr}[\rho_0(t)\rho(t)]$, where $\rho_0(t)$ is a density matrix evolved according to unperturbed dynamics, and $\rho(t)$ is subject to a quench at $t = 0$, where $\rho_0(0) = \rho(0) = |g\rangle\langle g|$, and $\langle g|$ is the ground state. For noisy disturbances we study the evolution to a mixed state through $\mathrm{Tr}[\rho(t)^2]$ [which contains information about decay of off-diagonal elements of $\rho(t)$]. We consider four idealized models for outside disturbances to this system.

First, we consider two varieties of noiseless disturbance: a uniform magnetic field and a local magnetic impurity. Because these disturbances can be captured as perturbations to the Hamiltonian, we refer to them as Hamiltonian types.

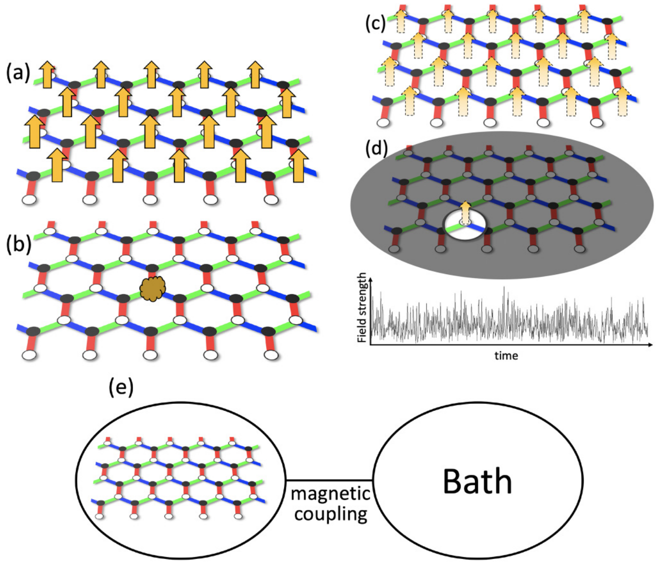

FIG. 1. Noiseless and noisy disturbances to the Kitaev honeycomb. (a) Kitaev honeycomb subject to a uniform magnetic field indicated by yellow arrows. (b) An impurity deposited onto the Kitaev model, shown as a brown speck of dirt, at some lattice point. We treat the case of a quenched magnetic impurity. (c) Kitaev honeycomb model coupled to a noisy but spatially uniform magnetic bath. We consider a uniform magnetic field subject to Gaussian white noise in the quantum master equation. (d) A hole in the system's magnetic shielding couples it locally to a noisy magnetic environment. We model this situation using a noisy magnetic impurity quench. (e) Coupling to a bath, similar to a noisy but spatially uniform magnetic field, is a physical scenario leading to a Lindblad operator in the quantum master equation.

This distinguishes them from perturbations which must be considered using Lindblad jump operators and treating the system as "open."

We model Hamiltonian-type disturbances as a perturbation V in $H = H_0 + V$, with H the Hamiltonian for the full system, H_0 the unperturbed Kitaev Hamiltonian in Eq. (1), and V small relative to H_0. In the case of Hamiltonian-type quenches turned on at time $t = 0$, only pure states enter $F(t) = |\langle g|e^{iH_0 t}e^{-i(H_0+V)t}|g\rangle|^2 \equiv |G(t)|^2$, where $G(t)$ is the Loschmidt echo [27,44,45].

A simple disturbance to a magnetic system is a uniform magnetic field in the z direction $V = h\sum_i \sigma_i^z$ (Fig. 1(a) and Ref. [46]). We restrict ourselves to the case of small field strength relative to the Kitaev couplings. Such disturbances are experimentally relevant in situations where a weak, constant magnetic field might suddenly become coupled to the system, such as a background magnetic field from Earth. Another interesting case is that in which an impurity, perhaps a magnetic piece of dirt, is deposited in the system, Fig. 1(b). This is modeled by a quench with a local impurity operator, $V = \lambda\sigma_l^z$, where we consider an arbitrary site l.

Next, we consider quenches involving noise; we treat the case of Gaussian white noise. The same formalism described below is also conventionally used to describe coupling to a heat bath in open quantum systems. Both effects can be captured in the same formalism employing density matrices.

TABLE I. Asymptotic behavior ($\mathbf{t} \to \infty$) of the Uhlmann fidelity, $\mathbf{F(t)}$, measuring the stability of the Kitaev ground state when a quench is applied. λ, impurity strength; A, system size; h magnetic field strength; κ, noise strength. For noiseless cases we can identify $F(t) = |G(t)|^2$. Quenches studied are a magnetic impurity, magnetic field, noisy magnetic impurity, and environmental coupling, each for both gapped and gapless models. For the noisy cases, a universal form $Ce^{-\alpha t}t^{-\beta}$ is found, where the specific form of α and β depends on the type of quench. C is constant in the long-time limit. The magnetic impurity is modeled by a perturbation $V = \lambda\sigma_l^z$ and the magnetic field by $V = h\sum_i \sigma_i^z$, where the sum is over sites in the honeycomb lattice. Noise is treated using a generalization of these operators in a Lindblad formalism, where a small parameter κ determines the strength of the noise.

$F(t) \sim Ce^{-\alpha t}t^{-\beta}$	Impurity		Magnetic field		Noisy impurity		Environmental coupling	
	α	β	α	β	α	β	α	β
Gapped	0	0	0	0	$\kappa - \kappa^2 \times$ const.	$\propto \kappa^2$	$\propto Ah^2\kappa - A^2h^4\kappa^2 \times$ const.	$\propto A^2h^4\kappa^2$
Gapless	0	$\propto \lambda^2$	0	$\propto Ah^2$	$\kappa - \kappa^2 \times$ const.	$\propto \kappa^2$	$\propto Ah^2\kappa - A^2h^4\kappa^2 \times$ const.	$\propto A^2h^4\kappa^2$

Within a Lindblad master equation approach, density matrices evolve into mixed states [24],

$$\frac{d\rho}{dt} = -i[H_0, \rho(t)] + \kappa\mathscr{L}[\rho(t)], \tag{2}$$

where $\mathscr{L}[\rho(t)] = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$, and L is a Lindblad operator [47] that, similar to V in the Hamiltonian case, is used to describe a quench, except that L describes both noisy couplings and couplings to a bath with strength parameter κ. In this case, $\rho(t)$ in $F(t)$ is a state initialized as $|g\rangle\langle g|$ and evolved under Eq. (2). For weak disturbances F(t) can be treated via a modified second-order cumulant expansion, detailed in Sec. II of the Supplemental Material [42].

The first example of a Lindblad operator we study is a noisy uniform magnetic field, Fig. 1(c), modeled by the Lindblad operator [31], $L = h\sum_i \sigma_i^z$, which describes coupling to a magnetic bath, Fig. 1(e). Thus, one can interpret the results of our analysis as pertaining to both the noisy uniform field and bath situations. Here we consider the weak-coupling regime, for which $\kappa h^2 \ll J_i$. As magnetic shielding becomes more reliable in quantum applications, this weak-coupling regime becomes increasingly relevant.

The same formalism may be used to model a scenario in which information is lost from the system locally, such as a case in which a hole appears in the shielding of a device, coupling it locally to the environment or allowing magnetic noise to enter, Fig. 1(d). We treat this situation using a local Lindblad operator [31], $L = \lambda\sigma_l^z$.

We noted earlier that care must be taken to ensure we capture possible fluctuations between flux sectors, as the Kitaev model is only reduced to a free-fermion model within a given choice of flux eigenvalues. The unperturbed Hamiltonian can be written in the form $H_0 = H_1 + H_2$, where H_1 contains no flux degree of freedom, and H_2 captures interactions between the spinless fermions and the flux excitations. Crucially, H_0 and H_1 share a ground state and act identically on states within the flux-free sector. This invites a repartitioning of the dynamics, Eq. (2), such that

$$\begin{aligned}\frac{d\rho}{dt} &= -i[H_1, \rho(t)] - i[H_2, \rho(t)] + \kappa\mathscr{L}[\rho(t)] \\ &\equiv -i[H_1, \rho(t)] + \tilde{\mathscr{L}}[\rho(t)].\end{aligned} \tag{3}$$

The interaction picture is then defined with respect to the free Hamiltonian H_1, and the flux degree of freedom is pushed into the perturbing dynamics. Of course, this must be checked for consistency when H_2 is not small. This concern is treated carefully in the Supplemental Material (Sec. III) [42].

Results. We now turn our discussion to explicit expressions of the fidelity for the quenches we consider. Results for the long-time behavior of the stability are summarized in Table I. In particular, we highlight the similarities of the fidelity across cases, as well as the dependence on parameters such as field strength and system size. We expect these results to be universal, because they do not depend on the precise dispersion, but rather on the dimensionality of k space and the structure of band crossings, as is detailed in the Supplemental Material (Sec. IV) [42]. We first present an analysis of noiseless quenches in a gapped system.

For long times, we find a general expression for the fidelity valid for any couplings $\mathbf{J}$ that give rise to a gapped phase, i.e., $J_x > J_y + J_z$. Referring to the fidelity under uniform field and impurity quenches as $|G_u|^2$ and $|G_l|^2$, respectively, we find

$$\begin{aligned}|G_u(t)| &\sim \exp\left[4Ah^2\left(-c_u + \frac{1}{4\pi\sqrt{J_xJ_yJ_z}}\frac{\gamma(t)}{t}\right)\right],\\ |G_l(t)| &\sim \exp\left[\lambda^2\left(-c_l + \frac{1}{4\pi\sqrt{J_xJ_yJ_z}}\frac{\gamma(t)}{t}\right)\right],\\ \gamma(t) &= -\frac{\sin(\Delta Et)}{\sqrt{2}\Delta E^{3/2}} + \frac{2\cos\left(\frac{1}{2}(\Delta E + \omega + \delta)t\right)}{(\Delta E + \omega + \delta)^{3/2}}\\ &\quad + \frac{2\cos\left(\frac{1}{2}(\Delta E + \omega - \delta)t\right)}{(\Delta E + \omega - \delta)^{3/2}} + \frac{\sin(\omega t)}{\sqrt{2}\omega^{3/2}}\end{aligned} \tag{4}$$

, where $c_{u/l}$ are constants dependent on the particular choice of parameters $\mathbf{J}$.

Therefore, the fidelity performs damped oscillation around an asymptotic value, indicating finite stability even at long times. Furthermore, there are three natural energy scales of the system which determine the oscillation frequencies: $\Delta E = 2(J_x - J_y - J_z)$ is the band minimum (half the band gap), $\omega = 2(J_x + J_y + J_z)$ the band maximum (half the bandwidth), and $\delta = 4(J_y - J_z)$ is the separation between saddle points, as shown in Fig. 2(a). Interestingly, the oscillatory behavior means that the fidelity will have periodic revivals. The energy scales given above determine the timescale on which any revivals in the stability occur. We note that because of the $1/t$ factor, revivals are most pronounced early in the evolution.

The leading $t \to \infty$ asymptotic behavior for the two cases is $|G_u(t)| \approx e^{-c_uAh^2+\mathcal{O}(1/t)}$ and $|G_l(t)| \approx e^{-c_l\lambda^2+\mathcal{O}(1/t)}$. Indeed,

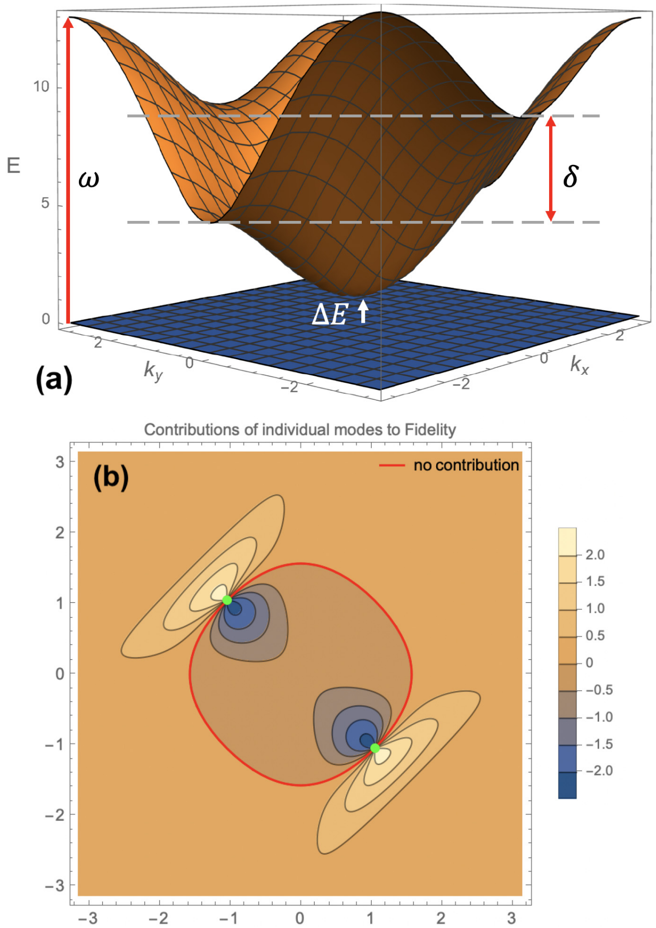

FIG. 2. (a) Oscillation-determining energy scales. The natural energy scales (ΔE, ω, and δ) of the problem determine the frequencies of oscillations for the Loschmidt echo. ΔE and ω are the band minima and maxima, respectively, with ΔE also being half the band gap. δ is the energy difference between saddle points in the spectrum, and is inherently 2D. (b) Spinon contributions to stability. Map of regions giving positive (yellow) and negative (blue) contributions to the fidelity relative to the ground state coherence for magnetic field quench. This is shown for the specific case of $\mathbf{J} = (1, 1, 1)$. Dirac points (green) at $\mathbf{k} = (\pm\pi/3, \mp\pi/3)$ give singular contributions to the integral in Eq. (5), and regions around them contribute most heavily to modifying the fidelity relative to the ground state result. In particular, the plot is of the integrand in Eq. (5) at a particular time; the contribution only becomes more heavily localized around the Dirac points at later times. By virtue of being near the Dirac points, the heaviest contributions also occur in the regions of lowest energy.

the limit $t \to \infty$ is time independent in both cases, corresponding to a finite overlap with the Kitaev ground state as time $t \to \infty$. Notably, an orthogonality catastrophe does not manifest in either case (which would lead to a vanishing fidelity). This long-time result is one of our key findings, in addition to the results summarized in Table I.

The long-time form for $|G_u(t)|$ may be exploited for quantum device design: for a uniform field the finite fidelity at long times may be tuned via sample area size A and coupling strength h, where larger Ah^2 leads to reduced fidelity. For fixed h, larger system size A corresponds to reduced fidelity, while smaller system sizes enhance fidelity. In the impurity case, the impurity coupling strength λ solely determines the asymptotic limit of $|G_l(t)|$, which is system size independent. To a first approximation we also find that decay should be weaker when the gap is larger, because for a larger gap c_l should be smaller—this can be seen in the integral form of c_l given in the Supplemental Material (Sec. IV) [42].

The behavior of the gapless phase strongly contrasts that in the gapped phase. In the gapless case, band crossings in the dispersion make important contributions to the long-time fidelity. Details about the calculation of the long-time behavior of a system with gapless dispersion can be found in Sec. IV of the Supplemental Material. For the uniform magnetic field and local impurity cases we find $|G_u(t)| \approx t^{-Ah^2 c_{h1}} \exp[-Ah^2(c_{h0} + \mathcal{O}(1/t))]$ and $|G_l(t)| \approx t^{-\lambda^2 c_{l1}} \exp[-\lambda^2(c_{l0} + \mathcal{O}(1/t))]$, where c_{h0} c_{h1}, c_{l0}, and c_{l1} are constants depending on $\mathbf{J}$. At long times we observe that in contrast to the gapped case, gapless systems have algebraically decaying fidelity $|G(t)| \sim t^{-\beta}$. Similar algebraic decay is found in 1D systems under a quench [48]. In the uniform magnetic field case we see that for larger Ah^2 the decay is faster—that is, the fidelity diminishes more rapidly for larger systems and stronger fields. For the impurity quench the result depends only on the impurity strength and the value of c_{l1}, not on system size.

Next, we study the asymptotic behavior of the fidelity under noisy magnetic field and noisy impurity quenches. Explicit expressions for $F(t)$ are found using the semianalytical asymptotic methods discussed in Methods. The leading-order behavior in all noisy cases is found to be exponential decay—this can be seen readily via a cumulant expansion. Specifically, we find $F(t) \sim Ce^{-\alpha t}t^{-\beta}$, where the coefficients α and β depend on features of the system such as impurity or field strength as well as couplings $\mathbf{J}$. The dependence on these physical parameters is given in Table I. This time-dependence (particularly the leading exponential decay appearing for a noisy quench) mirrors 1D results for the out-of-equilibrium Loschmidt echo and transport properties [48,49]. Results for two characteristic choices of parameters (one gapless and one gapped) are given in the Supplemental Material (Sec. IV) [42].

To ensure we capture the decay of off-diagonal elements of the density matrix, we additionally study $\mathrm{Tr}[\rho^2]$, noting that $\mathrm{Tr}[\rho^2] < 1$ corresponds to a mixed state. We find that $\mathrm{Tr}[\rho(t)^2] = \tilde{F}(t)^2$ (again to second order in a cumulant expansion; see Sec. VIII of the Supplemental Material), where $\rho(0) = |g\rangle\langle g|$ is a pure state and $\tilde{F}(t)$ is a second-order cumulant expansion of the fidelity with the second cumulant doubled. Hence, $\mathrm{Tr}[\rho^2]$ decays faster than the fidelity, signaling that while the state becomes mixed, the initial state retains finite probability in the ensemble, allowing a nonzero stability.

We stress that in the case of a noisy quench $F(t)$ does not depend sensitively on whether the system is gapped. However, it does depend on physical parameters. Under a noisy field, stability depends on system size A, noise strength κ, and magnetic field strength h through the parameter $Ah^2\kappa^2$. We find that, for example, increasing system size diminishes long-time stability exponentially. This has implications for quantum device design subject to environmental noise: in the presence of noise, the fidelity is maintained for longer in a device that is smaller, or more weakly coupled to the environment through better shielding. In contrast, stability under a noisy impurity quench has no dependence on system size—the strength of the exponential and logarithmic decay are set only by the strength of the noisy impurity. Finally, we note that the general form $F(t)$ in Table I has also been found for an Ising chain in a noisy magnetic field, highlighting that this form is not special to either the Kitaev model or to 2D systems [24].

Excited States. The basic excitations of the Kitaev model are spinons and flux excitations. The study of excitations has technological and theoretical relevance: flux excitations, being related to anyons, are particularly interesting in the context of topological quantum computing. Spinons, on the other hand, have potential relevance to the field of spintronics with coherent spin excitations having the potential to transmit energy through transport effects such as the spin Seebeck effect [50,51].

We consider the long-time fidelity of a state with m discrete spinon excitations for noiseless quenches. It is straightforward to compute the quantity $G^{ex}(t) = \langle\{k\}|e^{iH_0t}e^{-i(H_0+V)t}|\{k\}\rangle$ to second order in a cumulant expansion where $|\{k\}\rangle = \prod_{\{k\}}\gamma_k^\dagger|g\rangle$, with $\gamma_k^\dagger$ a spinon creation operator, is the excited state with an arbitrary number of modes excited, and $\{k\}$ denotes the set of excited modes. For the impurity quench ($V = \lambda\sigma_r^z$), $G^{ex}(t)$ is identical to the ground state fidelity, so that $G_I^{ex}(t) = G_I(t)$ within the cumulant expansion. This holds for any number of excitations and for any choice of parameters $\mathbf{J}$, and tells us that our results for the ground state fidelity under an impurity quench already capture the long-time stability of spin excitations.

We can similarly analyze $G^{ex}(t)$ for a magnetic field quench with $V = h\sum_r\sigma_r^z$. In this case the spinon fidelity is not identical to that of the ground state, but modulates it so that

$$|G_h^{ex}(t)| \approx |G_h(t)| \exp\left[4Ah^2\int_{\{k\}}\frac{d^2k}{(2\pi)^2}\frac{\epsilon_k}{E_k}\left(\frac{1-\cos(E_kt)}{E_k^2}\right)\right], \tag{5}$$

where the integral is over an occupied region of the Brillouin zone, $E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$ for $\epsilon_k = 2J_z - 2J_x\cos k_x - 2J_y\cos k_y$ and $\Delta_k = 2J_x\sin k_x + 2J_y\sin k_y$. The first factor is just the ground state result, and now each excitation contributes a factor $\exp[4h^2\frac{\epsilon_k}{E_k}(\frac{1-\cos(E_kt)}{E_k^2})]$. Occupied modes around Dirac points make non-negligible contributions to the fidelity, strengthening or weakening the stability considerably.

By analyzing the sign of the integrand in Eq. (5), we can specify the regions of the BZ that contribute positively and negatively to the fidelity relative to the ground state. This is shown in Fig. 2(b), where the interior region (bounded by the red curve in the figure) contains modes that reduce stability relative to the ground state calculation, while the exterior region's modes increase it. The greatest contributions lie on either side of the Dirac points at $(\pm\pi/3, \mp\pi/3)$. Selectively filling only positively contributing modes leads to substantial gain in stability for the gapless system.

Conclusions. We have studied long-time stability of the Kitaev model ground state and spin excitations under various quenches representing possible disturbances to the system. We studied four types of quenches and through a long-time asymptotic analysis of both a gapped and gapless system, we found a universal functional form for the Uhlmann fidelity: $F(t) \sim Ce^{-\alpha t}t^{-\beta}$ (see Table I). We found selectively exciting spin excitations can produce remarkably robust quantum states. Our work has relevance for the design of quantum devices. Our treatment of evolution between flux sectors could also be applied to, e.g., linear response theory, to consider response not just to an applied field but due to motion between sectors.

All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplemental Material.

Acknowledgments. We gratefully acknowledge funding from NSF Grant No. DMR-2114825. W.R. acknowledges support from the US Department of Defense through the NDSEG Fellowship. M.V. gratefully acknowledges the support provided by the Deanship of Research Oversight and Coordination (DROC) at King Fahd University of Petroleum & Minerals (KFUPM) for funding this work through exploratory research Grant No. ER221002.

W.R. developed technical tools, performed calculations, and wrote the first draft of the manuscript. M.V. devised technical problem-solving strategies. M.V. and G.A.F. conceived of the project and revised the manuscript. G.A.F supervised the project.

All authors declare that they have no competing interests.

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[42] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.109.L220406 for a pedagogical treatment of an exact solution method for the Kitaev model, which forms the basis for our work; the complete derivation of the second-order cumulant expansion for evolution under a quantum master equation; detailed calculations of the cumulants for each of the four disturbances we treat; a pedagogical discussion of the integral approximation methods developed for use throughout this work, as well as the results they yield in the present study and plots thereof; further details of calculations for the fidelity of excited states in the first flux sector, including plots; and a brief note on how a calculation of the purity is related to the fidelity at the level of a second-order cumulant expansion.

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