Letter

Large-scale universality in quantum reaction-diffusion from Keldysh field theory

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We consider the quantum reaction-diffusion dynamics in d spatial dimensions of a Fermi gas subject to binary annihilation reactions $A + A \to \emptyset$. These systems display collective nonequilibrium long-time behavior, which is signalled by an algebraic decay of the particle density. Building on the Keldysh formalism, we devise a field theoretical approach for the reaction-limited regime, where annihilation reactions are scarce. Combining a perturbative expansion of the dissipative interaction with Euler-hydrodynamic scaling limit, we derive a description in terms of a large-scale universal kinetic equation. Our approach shows how the time-dependent generalized Gibbs ensemble assumption, which is often employed for treating low-dimensional nonequilibrium dissipative systems, emerges from systematic diagrammatics. It also allows us to exactly compute—for arbitrary spatial dimension—the decay exponent of the particle density. The latter is based on the large-scale description of the quantum dynamics and it differs from the mean-field prediction even in dimension larger than one. We moreover consider spatially inhomogeneous setups involving an external potential. In confined systems the density decay is accelerated towards the mean-field algebraic behavior, while for deconfined scenarios the power-law decay is replaced by a slower nonalgebraic decay.

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Introduction. Reaction-diffusion (RD) systems [1–3], where particles diffuse and react upon meeting, are ideal systems for the investigation of dynamical universal behavior. For example, for binary annihilation reactions $A + A \to \emptyset$, the late time decay of the particle density takes a universal power-law form. In the "diffusion-limited" regime [4–12], where diffusion is weak, the origin of this dynamical behavior are spatial density fluctuations. Here, mean-field approaches cannot be applied and field-theoretical and renormalization group analyses [13–22] correctly predict the observed power law. The mean-field approximation is, however, valid in more than one dimension and/or in the "reaction-limited" regime of fast hopping mixing [2,3,6,23,24].

In quantum many-body systems large-scale properties are even harder to uncover than in the classical realm, already in the one-dimensional case, since they entail the simulations of large sizes and long times [25–31]. In this regard, quantum RD systems have moved into the focus of attention. They follow simple dynamical rules [32–45], which connect to cold-atomic experiments [46–56], and they allow for novel forms of particle-density decay beyond mean field. This has been shown in Refs. [34–36,39,40,42,45] for one-dimensional systems in the reaction-limited regime. In this limit, analytical predictions can be obtained under the assumption that the systems relaxes to a time-dependent generalized Gibbs ensemble (TGGE) [57–60]. The connection between the TGGE assumption and diagrammatics techniques, and the study of universal decay in generic spatial dimensions d, requires, however, the development of a field theory.

In this Letter, we accomplish this by exploiting the Keldysh path integral representation of the quantum master equation [61–68]. We investigate as a paradigmatic example the Fermi gas in d spatial dimensions subject to binary annihilation reactions $A + A \to \emptyset$. From the Keldysh field theory, we perform a diagrammatic expansion of the dissipative interaction vertices. In the Euler-scaling limit of hydrodynamics [38,69–72], when space-time derivatives are kept at leading order, this expansion acquires the universal form of a kinetic Boltzmann equation. In $d = 1$, this analysis provides results equivalent to that of the TGGE ansatz for the reaction-limited regime and therefore provides a connection between the much-employed TGGE relaxation assumption, hydrodynamic scaling limits, and diagrammatic expansions in dissipative systems. Crucially, from the field-theory description, we exactly compute the density decay exponent in arbitrary dimensions, which is found to deviate from the mean-field prediction even in $d > 1$. This result is in contrast with the classical case and it is rooted into the large-scale universal description of the underlying quantum dynamics. We also consider the case of inhomogeneous systems where we study quenches of a trapping potential confining the fermions. For a quench from a double to single well potential, we find an acceleration of the particle decay, which diverts the decay exponent towards the mean-field one. For a trap-release quench, we, instead, find a qualitatively different scenario: the algebraic decay first slows down on an intermediate time window, and then it gets replaced at long times by a slower nonalgebraic decay.

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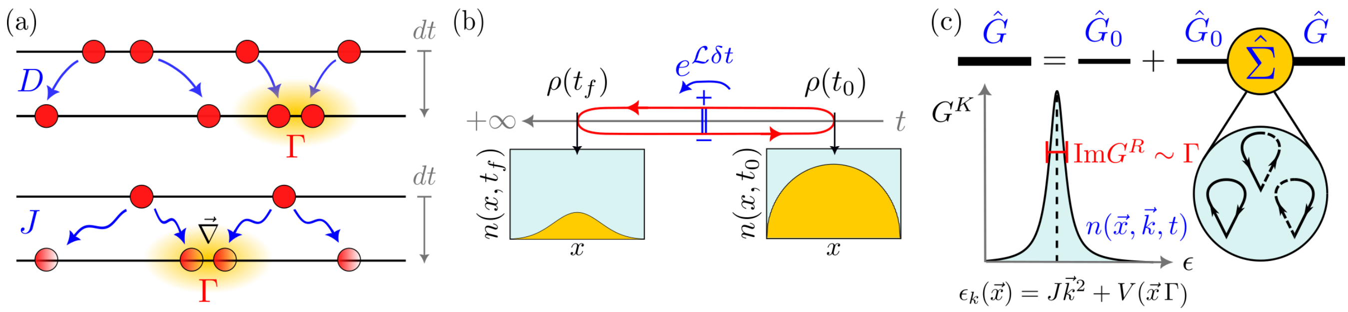

FIG. 1. Quantum RD dynamics via Keldysh field theory. (a) Comparison of classical and quantum RD dynamics: classical incoherent diffusion (D, top, blue solid lines) is replaced by quantum coherent ballistic motion (J, bottom, blue wiggly lines), while in both cases annihilation, $A + A \to \emptyset$, is irreversible. (b) Time evolution of the density matrix $\rho(t)$ along the closed-time contour associated to the Lindblad map $\mathcal{L}$. Two time branches, forward (+) and backward (−), are required. The particle number decreases during time evolution, as can be seen by comparing the initial $n(x, t_0)$ and final $n(x, t_f)$ density profiles. (c) Annihilation interaction vertices define the self-energy Keldysh matrix $\hat{\Sigma}$, which dresses the Green's functions $\hat{G}$ with respect to their bare values $\hat{G}_0$. In the reaction-limited regime $\hbar n(x,t)\Gamma/J \ll 1$, the quasiparticle dispersion relation ϵ_k is not modified, while quasiparticles acquire a large-finite lifetime $\sim\Gamma^{-1}$, given by the energy width ϵ of $G^K(\vec{x}, t, \vec{k}, \epsilon)$ (sketched in red). This lifetime is given by tadpole Feynman diagrams (sketched in the light blue inset). In the Euler-scaling limit, these diagrams reproduce the TGGE predictions for $n(\vec{x}, \vec{k}, t)$ (spectral integral in ϵ of $G^K(\vec{x}, t, \vec{k}, \epsilon)$ depicted in light blue).

Quantum RD Keldysh action. The dynamics of the considered Fermi gas in d spatial dimensions is governed by the quantum master equation [73–76] with Lindblad map $\mathcal{L}$

$$\dot{\rho}(t) = \mathcal{L}[\rho(t)] = -\frac{i}{\hbar}[H, \rho(t)] + \mathcal{D}[\rho(t)]. \tag{1}$$

The Hamiltonian H describes coherent free motion in the presence of an external trapping potential $V(\vec{x})$:

$$H = \int d^d\vec{x}\, \psi^\dagger(\vec{x})[-J\nabla^2 + V(\vec{x})]\psi(\vec{x}). \tag{2}$$

Here $J = \hbar^2/(2m)$, while ψ, $\psi^\dagger$ are fermionic field operators satisfying canonical anticommutation relations $\{\psi(\vec{x}), \psi^\dagger(\vec{x}')\} = \delta(\vec{x} - \vec{x}')$. It is important to note that Eq. (2) describes coherent-ballistic motion, differently from the classical case, which features diffusion, see Fig. 1(a). The dissipator $\mathcal{D}[\rho]$ embodies irreversible reaction processes

$$\mathcal{D}[\rho(t)] = \sum_\alpha \int d^d\vec{x} \Big[L_\alpha(\vec{x})\rho(t)L_\alpha^\dagger(\vec{x}) - \frac{1}{2}\{L_\alpha^\dagger(\vec{x})L_\alpha(\vec{x}), \rho(t)\} \Big], \tag{3}$$

with $\alpha = 1, 2, \dots, d$. We focus on binary annihilation reactions $A + A \to \emptyset$, see Fig. 1(a), modeled by the jump operators

$$L_\alpha(\vec{x}) = \sqrt{\Gamma}\, \psi(\vec{x})\, \partial_{x_\alpha} \psi(\vec{x}). \tag{4}$$

The constant Γ (units: length^{d+2}/time) characterizes the annihilation reactions. In the Supplemental Material [77], the jump operator (4) is obtained by taking the continuum limit of nearest-neighbors annihilation. The latter is the natural annihilation decay to consider for fermionic particles, where on-site reactions are forbidden.

Nonequilibrium universal behavior manifests in the power-law decay of the density $n(x, t) = \langle \psi^\dagger(\vec{x})\psi(\vec{x})\rangle_t$ in time. Power-law decay is a general consequence of the nonlinearity of the binary annihilation process (4), and it can therefore be present both for ballistic [as in the case of (2)] and diffusive transport of particles. Here, we characterize this decay in the reaction-limited regime of weak dissipation. This regime amounts to considering weak dissipative perturbations (4) $\sim\Gamma$ to the integrable (noninteracting) Hamiltonian (2). In particular, we take $\hbar n(x,t)\Gamma/J \ll 1$, so that reactions are weak and the density slowly changes in time. We further allow for weak spatial inhomogeneities due the presence of the trapping potential $V(\vec{x}\,\Gamma)$, which we assume to vary on macroscopic length scales $\vec{x} \sim \Gamma^{-1}$. In this limit, the quasiparticle dispersion relation $\epsilon_k(\vec{x}) = J\vec{k}^2 + V(\vec{x}\,\Gamma)$ of H in Eq. (2), with $\vec{k}$ the momentum, is locally modified by the external potential.

In this regard, we study the quantum reaction-limited regime in the Euler-scaling limit [38,69–72,78–84]. The Euler scale is the largest scale of hydrodynamics, where space-time observation points are large keeping their ratio finite: $\vec{x}, t \to \infty$, $\Gamma \to 0$ with $\bar{\vec{x}} = \Gamma\vec{x}$ and $\bar{t} = \Gamma t$ fixed. In the different context of Hamiltonian integrability-breaking perturbations, similar scaling limits have been studied in Refs. [58,78]. In the ensuing "Boltzmann regime", it has been shown [58] that the slow dynamics of the weakly broken charges of the unperturbed Hamiltonian is governed by the instantaneous GGE of the integrable Hamiltonian. In the context of dissipative systems, in the Euler-scaling limit the similar TGGE assumption has been put forward [57,59,60], but a derivation of that is missing. In this Letter, we aim at connecting the TGGE assumption to diagrammatics techniques showing which assumptions in the latter eventually allow us to reobtain the former.

To do this, we exploit the Keldysh quantum field-theory description [63–68] of the open quantum RD dynamics (1)-(4) [77]. A general feature of Keldysh field theory is the doubling of the ψ, $\bar{\psi}$ fields into four fields—ψ_+, $\bar{\psi}_+$ and ψ_-, $\bar{\psi}_-$—evolving along a "forward" (+) and a "backward" (−) contour of the considered time interval, respectively, as sketched in Fig. 1(b). Plus and minus fields are usually rewritten in terms of Keldysh-rotated fields [62,77,85] ϕ_1, $\bar{\phi}_1$, ϕ_2, $\bar{\phi}_2$. The Keldysh partition function $Z(t) = \mathrm{tr}[\rho(t)] = \int \mathbf{D}[\phi_1, \bar{\phi}_1, \phi_2, \bar{\phi}_2] \exp\{iS[\phi_1, \bar{\phi}_1, \phi_2, \bar{\phi}_2]\}$ includes full information on the system's microscopic dynamics. The action $S = S_0 + S_{\mathcal{D}}$ is composed of two sectors. A quadratic sector S_0 describes coherent motion (2):

$$S_0 = \int d^d x \, dt' \left[\bar{\phi}_1 \left(G_0^R\right)^{-1} \phi_1 + \bar{\phi}_2 \left(G_0^A\right)^{-1} \phi_2 \right], \tag{5}$$

with $(G_0^{R/A})^{-1} = i\partial_{t'} + (J\nabla^2 - V(\vec{x}))/\hbar \pm i\delta$ the inverse retarded/advanced bare propagator, respectively. The bare Keldysh Green's function $G_0^K \equiv -i \langle \phi_1 \bar{\phi}_2 \rangle$ associated to S_0 is a regularization factor. The retarded/advanced propagator $G_0^{R/A}$ (with $V(\vec{x}) = 0$) in S_0 has a structure similar to the classical RD quadratic counterpart [14–17,20–22], the difference between the two being in the ballistic quantum motion of S_0 compared to the classical diffusive one. The second part $S_{\mathcal{D}}$ of the action contains the interaction vertices of the theory due to annihilation reactions (4):

$$\begin{aligned} S_{\mathcal{D}} = \frac{i\Gamma}{4} \int d^d x \, dt' \big[& 2(\vec{\nabla}\bar{\phi}_1 \bar{\phi}_2 + \vec{\nabla}\bar{\phi}_2 \bar{\phi}_1) \cdot (\phi_1 \vec{\nabla}\phi_2 \\ & + \phi_2 \vec{\nabla}\phi_1) + (\vec{\nabla}\bar{\phi}_1 \bar{\phi}_2 + \vec{\nabla}\bar{\phi}_2 \bar{\phi}_1) \cdot (\phi_1 \vec{\nabla}\phi_1 + \phi_2 \vec{\nabla}\phi_2) \\ & - (\vec{\nabla}\bar{\phi}_1 \bar{\phi}_1 + \vec{\nabla}\bar{\phi}_2 \bar{\phi}_2) \cdot (\phi_1 \vec{\nabla}\phi_2 + \phi_2 \vec{\nabla}\phi_1) \big]. \end{aligned} \tag{6}$$

All the interaction vertices are quartic in the fields, differently from the classical RD field theory, where both cubic and quartic interaction vertices are present. Furthermore, spatial gradients of the fields appear as a consequence of fermionic statistics.

Kinetic equation. In the reaction-limited regime $\hbar n\Gamma/J \ll 1$, the dressed Green's functions $\hat{G}$ can be rewritten in terms of the bare Green's functions $\hat{G}_0$ as the interaction vertices (6) are expanded perturbatively around the quadratic sector (5). The sum of all internal one-particle-irreducible contributions to the Feynman diagrams results in the entries of the self-energy Keldysh-space matrix $\hat{\Sigma}$ [61–68,86,87]. The ensuing Dyson equation is pictorially shown in Fig. 1(c).

The Keldysh component of the Dyson equation for $G^K(\vec{x}_1, t_1, \vec{x}_2, t_1)$ determines the kinetic equation [77]. The large-scale universal limit of this equation is best extracted by performing a Fourier transform in the relative space [time] variable $\vec{x}' = \vec{x}_1 - \vec{x}_2$ $[t' = t_1 - t_2]$, $G^K(\vec{x}, t, \vec{k}, \epsilon)$,[1] i.e., the so-called "Wigner transform" [88–90]. The set of Wigner center-of-mass coordinates $\vec{x} = (\vec{x}_1 + \vec{x}_2)/2$ $[t = (t_1 + t_2)/2]$, as well as momentum $\vec{k}$ and energy ϵ, characterize the effective macroscopic evolution of Green's functions. At this point two assumptions are needed: (i) Euler-scaling limit, so slow space-time variations so as we perform the Moyal-derivative expansion of hydrodynamics [71,79,91–94] at leading order in space-time derivatives; (ii) stable quasiparticle excitations, which in the Keldysh formalism translates into a sharply peaked $G^K(\vec{x}, t, \vec{k}, \epsilon)$ around $\epsilon_k(\vec{x})$, as in Fig. 1(c). Both (i) and (ii) rely on the weak dissipative integrability breaking $\Gamma \to 0$ and this is why the kinetic equation eventually matches the TGGE prediction. This is explicitly shown via the exact relation for the equal-time Keldysh Green's function $G^K(\vec{x}, t, \vec{k}, t)$ as:

$$iG^K(\vec{x}, \vec{k}, t, t) = 1 - 2\, n(\vec{x}, \vec{k}, t). \tag{7}$$

Here, $n(\vec{x}, \vec{k}, t)$ is the one-body Wigner function, i.e., the semiclassical phase-space $(\vec{x}, \vec{k})$ occupation function [95–97]. Within the conditions (i) and (ii), the Wigner function can be identified as the emergent degree of freedom, which obeys the quantum Boltzmann-like equation ($\bar{t} = \Gamma t$ and $\bar{\vec{x}} = \Gamma \vec{x}$)

$$\begin{aligned} & [\, \partial_{\bar{t}} + \vec{v}_g(\vec{k}) \cdot \vec{\nabla}_{\bar{x}} - \vec{\nabla}_{\bar{x}} V/\hbar \cdot \vec{\nabla}_k \,] n(\bar{\vec{x}}, \vec{k}, \bar{t}) \\ & \quad = - \int \frac{d^d q}{(2\pi)^d} (\vec{k} - \vec{q})^2 \, n(\bar{\vec{x}}, \vec{k}, \bar{t})\, n(\bar{\vec{x}}, \vec{q}, \bar{t}). \end{aligned} \tag{8}$$

Crucially, in Euler-scaling limit, we find that $\hat{\Sigma}$ contributes via purely imaginary terms, which determine the right-hand side (r.h.s.), named collision integral. The latter is computed in terms of tadpole Feynman diagrams (depicted in Fig. 1(c)) at lowest order in the derivatives $\vec{\nabla}$. The appearing factor $(\vec{k} - \vec{q})^2$ stems from the fermionic statistics. Moreover, the dispersion relation $\epsilon_k(\vec{x})$ ($\vec{v}_g(\vec{k}) = 2J\vec{k}/\hbar$ is the group velocity) is not renormalized. This is a consequence of the integrability-breaking term being purely dissipative. When additional Hamiltonian integrability-breaking perturbations are introduced, the quasiparticle spectrum and the potential V can get possibly renormalized. In $d = 1$, the r.h.s. of Eq. (8) has the same form as the one derived in Refs. [34,35,40,42,45] assuming the systems relaxes to a TGGE state in between consecutive reactions. This analysis therefore shows how systems whose integrability is weakly broken due to dissipation can be equivalently studied via Keldysh diagrammatic methods. At the same time, it allows us to consider higher-dimensional systems.

Homogeneous decay in d dimensions. For homogeneous initial states and no trapping potential $V(\vec{x}) = 0$, the Wigner function $n(\vec{x}, \vec{k}, t)$ reduces to the momentum-occupation

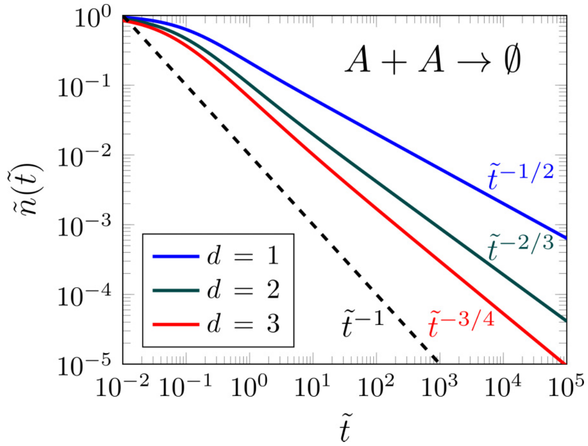

FIG. 2. Binary annihilation decay in d dimensions. Solution of the homogeneous Boltzmann equation (8) from the Fermi-sea initial state at density n_0. The rescaled density $\tilde{n} = n/n_0$ decays algebraically as a function of the dimensionless time $\tilde{t} = n_0^{1+2/d}\Gamma t$. From top to bottom, algebraic decay $\tilde{n} \sim \tilde{t}^{-\frac{d}{d+1}}$ in $d = 1, 2, 3$ (solid lines). The dashed line represents the mean-field decay exponent $\tilde{t}^{-1}$ asymptotically valid in infinite d.

[1] In the whole Letter, for lightness of notation, we use the same symbol for a function and its Wigner transform. The two functions can be distinguished from the corresponding arguments.

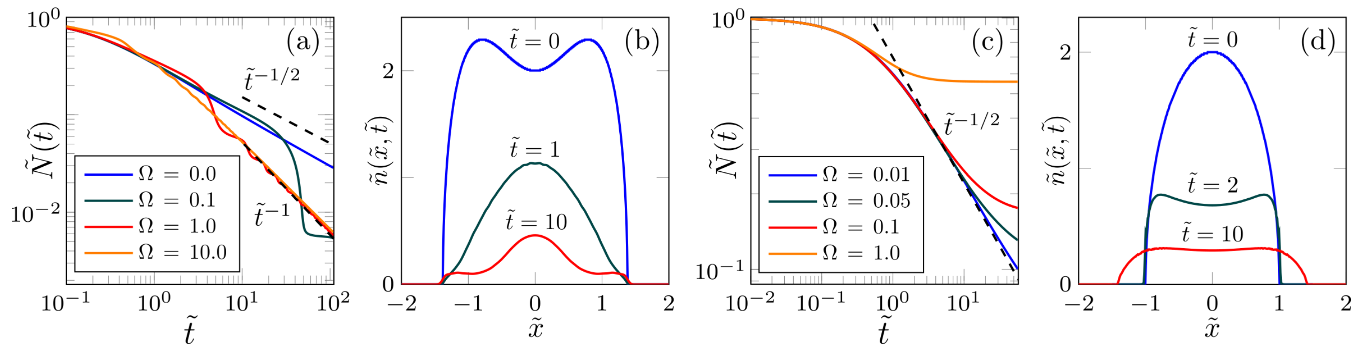

FIG. 3. Binary annihilation inhomogeneous decay in $d = 1$. (a) Double well to harmonic confinement potential quench: plot of the rescaled particle number $\tilde{N}(\tilde{t})$ as a function of rescaled time $\tilde{t}$ for increasing values of the parameter $\Omega = 2n(0,0)[2J/(8\hbar\omega N_0^3)]^{1/2}$ (from top to bottom). The algebraic time decay gets accelerated as Ω is increased towards the mean-field prediction $\tilde{N}(\tilde{t}) \sim t^{-1}$ (bottom dashed line). (b) The corresponding rescaled spatial density $\tilde{n}(\tilde{x},\tilde{t})$ profiles at increasing times (from top to bottom) are plotted as a function of space $\tilde{x}$, with $\Omega = 0.1$. For both plots $C = 0.8$ and $B = 1$. (c) Trap release quench: particle number decay $\tilde{N}(\tilde{t})$ versus time $\tilde{t}$ for increasing Ω (from bottom to top). A decay exponent $\tilde{N}(\tilde{t}) \sim \tilde{t}^{-\xi}$ is approximately observed at intermediate times, with ξ decreasing with Ω, and $\xi = 1/2$ for $\Omega = 0$ (black dashed line). At longer times a nonalgebraic slow decay sets in. (d) The associated rescaled spatial density $\tilde{n}(\tilde{x},\tilde{t})$ profiles are reported at increasing times (from top to bottom), with $\Omega = 0.1$.

function $n(\vec{k},t)$. We consider a Fermi-sea initial state, with equally populated modes up to some Fermi momentum and a total initial density n_0. The asymptotics of the particle density decay can be worked out for generic d [77]. It is convenient to introduce the adimensional rescaled density $\tilde{n} = n/n_0$ and time $\tilde{t} = n_0^{1+2/d}\Gamma t$. The long-time asymptotics for $\tilde{n}(\tilde{t})$ is given by the power law

$$\tilde{n}(\tilde{t}) \sim \left[\frac{[(\alpha_d\,\Theta_d)(d-2)!!]^2}{2^d(d+1)^d(2\pi)^{2d}} \right]^{\frac{1}{d+1}} \tilde{t}^{-\frac{d}{d+1}}, \tag{9}$$

with Θ_d the d-dimensional solid angle.[2] The solution of the homogeneous Boltzmann equation for $d = 1, 2, 3$ from the Fermi-sea initial state is shown in Fig. 2. In $d = 1$, the decay exponent is $1/2$ in agreement with the TGGE prediction [34,35,40,42,45]. We find that the algebraic decay is different from the mean-field $\tilde{t}^{-1}$ even for $d > 1$ and it approaches the latter only for $d \to \infty$. This result is surprising and fundamentally different from the classical $A + A \to \emptyset$ RD dynamics. Therein, non-mean-field algebraic decay is possible only in $d = 1$ in the diffusion-limited regime $\hbar\Gamma n/J \sim 1$ [14–17,20–22], as a consequence of spatial fluctuations. Conversely, the different dimensional dependence of the exponent in Eq. (9) does not emerge due to spatial fluctuations but from the universal large-scale limit (8) of the quantum dynamics. We remark that the power-law decay (9) is dictated by the nonlinearity of the binary annihilation reaction. In addition the decay (9) beyond mean field is not specific of the zero-temperature initial Fermi-sea state, since it also describes the dynamics ensuing from finite-temperature initial states, with the temperature changing the amplitude but not the exponent of the decay [77].

Inhomogeneous decay in one dimension. We now consider a one-dimensional quantum quench of a slowly varying trapping potential from the prequench $V_0(\varepsilon x) = A(\varepsilon x)^4/4 - m\omega^2(\varepsilon x)^2/2$ double well to the postquench harmonic $V(\varepsilon x) = m\omega^2(\varepsilon x)^2/2$ form. The small adimensional parameter $\varepsilon = \hbar n(0,0)\Gamma/J$ ensures the potentials to be slowly varying in x. This analysis is thus an example of the application of generalized hydrodynamics to the case of a weakly varying external field [80], with the additional presence here of slow dissipation. A similar setting has been consider in Ref. [35], where also the prequench potential $V_0(x)$ is harmonic. The initial condition is the local-density approximation of the ground state of the Hamiltonian H in Eq. (2) with potential $V_0(x)$. We set the initial particle number to N_0 and we perform Euler scaling according to the parameters of the harmonic potential $V(x)$: for the particle number $\tilde{N} = N/N_0$, time $\tilde{t} = \varepsilon t(2N_0)^{3/2}J/[\ell_{HO}^3\hbar n(0,0)]$, space $\tilde{x} = \varepsilon x/(\sqrt{2N_0}\ell_{HO})$, momentum $\tilde{k} = k\ell_{HO}/\sqrt{2N_0}$ and density $\tilde{n} = n(2\pi\ell_{HO}/\sqrt{2N_0})$, with $\ell_{HO} = \sqrt{\hbar/(m\omega)}$. The adimensional parameter $\Omega = 2n(0,0)[2J/(8\hbar\omega N_0^3)]^{1/2}$ quantifies the interplay between coherent motion (J) and confinement (ω). In the rescaled phase space $(\tilde{x},\tilde{k})$, the initial state is $n(\tilde{x},\tilde{k},t) = 1$ if $B - \tilde{k}^2 + \tilde{x}^2 - C\tilde{x}^4 > 0$, and zero otherwise. Here $C = A\hbar N_0/(m^2\omega^3)$ and $B = \mu/(\hbar N_0\omega)$, with the chemical potential μ fixing the initial particle number N_0.

In Fig. 3(a), the decay of $\tilde{N}$ as a function of $\tilde{t}$ is shown. The decay accelerates periodically as Ω is increased since particles bounce off the potential walls and gather up at the center of the well, as shown in Fig. 3(b) for the density $\tilde{n}(\tilde{x},\tilde{t})$. As a consequence of such breathing motion all decay profiles converge from the short-time $\tilde{N}(\tilde{t}) \sim \tilde{t}^{-1/2}$ algebraic behavior towards the mean-field asymptotic decay $\tilde{N}(\tilde{t}) \sim \tilde{t}^{-1}$.

Next, we consider the long-time decay for the deconfinement dynamics of a harmonic trap-release quench of the Fermi gas from $V_0(x) = m\omega^2(\varepsilon x)^2/2$ to $V(x) = 0$. We rescale also in this case variables with respect to the harmonic potential $V_0(x)$ parameters. In Fig. 3(c) the decay of $\tilde{N}$ in time $\tilde{t}$ is reported. One first observes [77] an approximate algebraic decay $\tilde{N}(\tilde{t}) \sim \tilde{t}^{-\xi}$, with an exponent ξ continuously decreasing as Ω is increased (from the value $\xi = 1/2$ at $\Omega = 0$). At longer times, an unexpectedly slow decay, when compared to any power law, sets in. This slow decay is unexpected because

[2] $\alpha_d = 1$ for d even, $\alpha_d = \sqrt{\pi/2}$ for d odd.

it is not solely determined by the decrease of the density due to the expansion in free space, in Fig. 3(d), but also by the fermionic statistics. The most relevant reactions at low densities in the trap-release protocol, indeed, take place between particles with same momenta. The fermionic statistics, manifest in the factor $(\tilde{k} - \tilde{q})^2$ in Eq. (8), thereby suppresses these reactions and determine the decay of Figs. 3(c)–3(d).

Summary. We provided a Keldysh field-theory description of quantum RD dynamics of binary annihilation $A + A \to \emptyset$. We analytically derived in the Euler-scaling limit the universal large-scale Boltzmann equation in arbitrary dimension d describing the reaction-limited regime of slow reactions $\hbar n\Gamma/J \ll 1$. In $d = 1$, our results match the prediction from the TGGE ansatz, connecting the latter to field-theoretical diagrammatic expansions. For homogeneous systems, we analytically showed that the density algebraic decay exponent features an unexpected dependency on d and it deviates from mean-field value even in $d > 1$, in contrast with classical RD dynamics. In one-dimensional inhomogeneous setups involving a trapping potential, we found that the decay is either accelerated towards the mean-field value (confined systems), or severely slowed down (deconfined systems). Our results find a natural application in cold-atomic experiments involving two-body losses [32,46,55,56] in the strong-dissipation Zeno regime. From the formulation here proposed, several relevant questions can be addressed. As an example, one can assess the impact of elastic-Hamiltonian collisions on the decay exponent [55,56,98–100]. The presence of Hamiltonian-integrability breaking perturbations can, indeed, result into hydrodynamic diffusion [101], and it would be interesting to study the possible impact of diffusion on the asymptotic power-law decay of the density. Away from the reaction-limited, it is also crucial to study the quantum diffusion-limited regime $\hbar n\Gamma/J \sim 1$ via renormalization group schemes, as done for the classical RD [17,20–22].

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