Letter

Generation of gauge magnetic fields in a kagome spin liquid candidate using the Dzyaloshinskii-Moriya interaction

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The recent discovery of magnetization oscillations in a kagome spin liquid candidate motivates us to examine the origin of the gauge magnetic field term that can give rise to quantum oscillations of fermionic spinons. We find that in the presence of the Dzyaloshinskii-Moriya interaction and an average spin polarization, the spin permutation operator around the unit cell acquires an imaginary part, and a net gauge flux is generated through the unit cell of the kagome lattice. This mechanism of gauge field generation can account for the strength of the gauge magnetic field needed to explain the experiment.

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Introduction. The quantum spin liquid is an exotic state of matter that has generated intense theoretical interest since the proposal by Anderson in 1973 [1], and a great effort has gone into searching for its realization in nature [2]. While the initial proposal was for a system with antiferromagnetic interactions which fails to order due to quantum fluctuations in frustrated lattices, it is now recognized that the spin liquid state is a prime example of the notion of emergence, where new degrees of freedom which are absent in the microscopic Hamiltonian emerge at low energy and low temperature [3,4]. For example, starting with a spin-1/2 Heisenberg model where excitations are $S = 1$ spin flips, spinons which carry spin-1/2 and no charge emerge, together with an internal gauge field coupled with spinons. If the spinons are fermions, it may have a Fermi surface and the gauge field may be a U(1) gauge field. Much focus has been on two-dimensional (2D) systems and the U(1) gauge field is a 2D version of the electromagnetic field in our world. The spinons coupled to the gauge magnetic field form Landau levels and may exhibit quantum oscillations. Indeed, a proposal was made by Motrunich [5] that an insulator near the Mott transition may be a spin liquid candidate and there is a linear coupling between the physical magnetic field perpendicular to the plane, B_c, and the gauge magnetic field b. Near the Mott transition, the ratio $\alpha = b/B_c$ was found to be of order unity [5].

Recently Zheng *et al.* [6] reported magnetization oscillations in a kagome spin liquid candidate $YCu_3(OH)_6Br_2[Br_{1-y}(OH)_y]$ (YCOB) which was interpreted as originating from an emergent fermionic spinon coupled to the gauge magnetic field b. For a fixed magnetic field, the period of the oscillation is found to be proportional to $\cos(\theta)$, where θ is the angle between the applied B field and the axis perpendicular to the kagome plane. This demonstrates the orbital origin of the effect. Furthermore, the analysis found that α is of the order of unity or even larger [6]. This large value of α is unexpected because YCOB is a robust insulator with a charge gap of several volts. In a Hubbard model described by hopping t and repulsion U, the linear coupling between b and B_c found by Motrunich [5] is of order t^3/U^2. The quadratic restoring force is estimated to be of the order of the exchange energy $J = 4t^2/U$ and we expect α to be of the order of t/U and small in the large-U limit [6]. This motivates us to search for another mechanism for generating the gauge magnetic field where the Dzyaloshiskii-Moriya (DM) interaction is the source of the gauge field generation. While our study is mainly motivated by the recent experimental observations made in Ref. [6], our results are quite general and can be applied to a general class of materials with spin liquid behavior, whenever the DM interaction exists together with other interactions. Since the gauge flux appears in other spin liquids, such as chiral spin liquid, our construction also applies to spin liquid phases besides the U(1) spin liquid.

Generally speaking, the orbital signature of the oscillation suggests that spin-orbit coupling may be playing a role. In YCOB, as in the better-known kagome system Herbertsmithite, there is a DM interaction in addition to the Heisenberg exchange term:

$$H = \sum_{\langle i,j \rangle} (J\, \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \boldsymbol{D}_{i,j} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j), \tag{1}$$

where J is the antiferromagnetic coupling constant and $\boldsymbol{D}_{i,j}$ is the DM vector. The ratio $|\boldsymbol{D}_{i,j}|/J$ is about 0.1, which produces a gauge flux large enough to be consistent with the experimental value [6]. To proceed, let us focus on the z component of the DM vector $\boldsymbol{D}_{i,j}$. The sign of the vector depends on the convention of ordering i and j. This is indicated by arrows in Fig. 1(a), where we choose the convention that the arrows run counterclockwise around each triangle in the kagome lattice, and we order i and j in Eq. (1) from the tail to the head of the arrow. With this convention, the z components of the $\boldsymbol{D}_{i,j}$ vector, which we denote by D_z, are all equal in magnitude and have the same sign [7]. It is convenient for us for work with the Pauli operators $\boldsymbol{\sigma} = 2\boldsymbol{S}$ in the rest of the Letter. For each triangle, the scalar chirality operator is defined by

$$\hat{C}_{ijk} = \boldsymbol{\sigma}_i \cdot (\boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k), \tag{2}$$

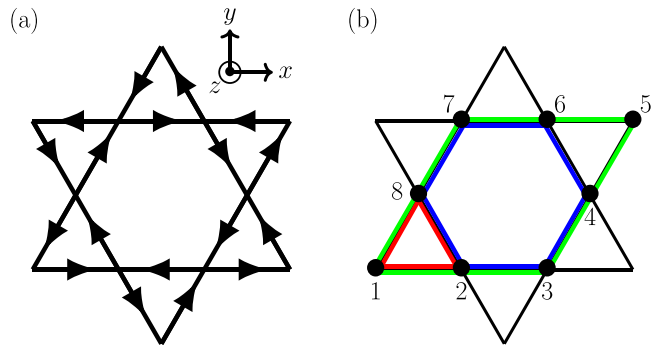

FIG. 1. (a) Convention of the ordering of vertices defining $\boldsymbol{D}_{i,j}$ in Eq. (1). (b) The kagome unit cell is highlighted in green, together with the counterclockwise numbering of sites in the unit cell. An isolated triangle and hexagon in the kagome unit cell are also highlighted in red and blue, respectively.

where i, j, and k run counterclockwise around the triangle. In the presence of the DM interaction, $D_0 = \langle(\boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k)_z\rangle$ is nonzero. Lee and Nagaosa [8] made use of this fact to show that fluctuations in σ_z couple linearly to fluctuations of the chirality operator, thus providing a way to measure chirality fluctuations through the triangles using neutron scattering.

Gao and Chen [9] took this one step further and argued that, in the presence of a finite magnetic field B_c along the c axis, $\langle\sigma_z\rangle$ is nonzero. Then the average chirality on the triangle $C_{123} = \langle\hat{C}_{123}\rangle \propto D_0\langle\sigma_z\rangle$ is also nonzero. As shown by Wen *et al.* [10], the gauge magnetic flux through the triangle is proportional to the scalar chirality C_{123}. This is a mechanism to produce a gauge flux through the triangles. However, Gao and Chen [9] stated that a negative gauge flux is generated through the hexagon in the kagome lattice which exactly cancels the flux through the triangles. So they concluded that there is no net flux through the unit cell; however, this conclusion was reached without providing any details. Here, we give a thorough derivation of the gauge flux through the unit cell. We reach the conclusion that there is finite gauge flux through the kagome unit cell that is proportional to $\langle S_z\rangle$. Hence the DM term can indeed generate the average gauge magnetic field b that is needed to explain the observed magnetization oscillations.

Spin permutation and chirality. For a robust insulator, we would like to restrict ourselves to spin space and ignore any charge fluctuations. We begin by reviewing the various connections between operators in spin space to chirality and gauge flux as described by Wen *et al.* [10]. We introduce the spin permutation operator $\hat{P}_{1\ldots n}$, which maps the state $|s_1, \ldots, s_n\rangle$ to $|s_n, s_1, \ldots, s_{n-1}\rangle$, i.e., shifts the spins forming a periodic array in an anticlockwise order, where $s_i = \pm 1$ denotes the spin state on site i. The permutation operator can be decomposed into a product of pair exchanges $\hat{P}_{i,j}$ which in turn equals $\frac{1}{2}(1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$. Thus, we arrive at the following expression for the expectation value of the permutation operator:

$$P_{1\ldots n} = \langle\hat{P}_{1\ldots n}\rangle = \frac{1}{2^{n-1}}\langle(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(1 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)\cdots (1 + \boldsymbol{\sigma}_{n-1} \cdot \boldsymbol{\sigma}_n)\rangle. \tag{3}$$

Physically, $P_{1\ldots n}$ describes the motion of a spin around a loop. If $P_{1\ldots n}$ has an imaginary part, the motion picks up a Berry's phase which we can associate with the flux through the loop. By writing $P = |P|e^{i\Phi}$, Φ follows from the real and imaginary parts of P. In particular, we show that the imaginary part of P is nonzero for a loop that encloses a unit cell of the kagome lattice. We identify $\Phi/2\pi$ as the gauge magnetic flux per unit cell seen by the spinon and calculate Φ to first order in the DM interaction. This calculation forms the core of this Letter.

Before presenting the full details, let us mention another method of computing the gauge flux introduced by Wen *et al.* [10]. They introduced a second operator $\hat{\chi}$ to describe the gauge flux, which is given by the product

$$\hat{\chi}_{1\ldots n} = \hat{\chi}_{1,2}\hat{\chi}_{2,3}\cdots\hat{\chi}_{n,1}, \tag{4}$$

where

$$\hat{\chi}_{i,j} = f^{\dagger}_{i,\sigma} f_{j,\sigma} \tag{5}$$

and $f_{j,\sigma}$ is the annihilation operator of a fermion, which satisfies the constraint that the occupation number is unity on each site. To get a physical meaning, suppose we create a particle at site 1 with spin σ by applying $f^{\dagger}_{i,\sigma}$ on a spin liquid background. Then $\hat{\chi}_{i,i+1}$ appearing in $\hat{\chi}_{1\ldots n} = \hat{\chi}_{1,2}\hat{\chi}_{2,3}\cdots\hat{\chi}_{n,1}$ successively destroys a particle at site $i+1$ and creates a particle at site i starting from site 1. This results in the particle at site 1 coming back to its original position after traversing the loop formed by sites $1, \ldots, n$. Due to the background gauge field in spin liquid, the particle experiences the gauge flux associated with the loop as a Berry phase. Therefore, the operator $\hat{\chi}_{1\ldots n}$ describes the motion of the fermion around a loop in the restricted subspace and its mean value $\chi_{1\ldots n} = \langle\hat{\chi}_{1\ldots n}\rangle$ can again be used to characterize the gauge flux seen by the spinon. We discuss the second way of determining the gauge flux in Sec. III.

Permutation operator. To see how the real and imaginary parts of the expectation value of the loop operator $P_{1\ldots n}$ Eq. (3) are related to Heisenberg and chirality terms, let us first consider the simplest example, which is a triangle [10]:

$$\begin{aligned}\hat{P}_{123} &= \tfrac{1}{4}(1 + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma}_2)(1 + \boldsymbol{\sigma_2} \cdot \boldsymbol{\sigma}_3)\\ &= \tfrac{1}{4}[1 + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma_2} \cdot \boldsymbol{\sigma}_3 + (\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\sigma_2} \cdot \boldsymbol{\sigma}_3)]\\ &= \tfrac{1}{4}(1 + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma}_3 + \boldsymbol{\sigma_2} \cdot \boldsymbol{\sigma}_3 - i\hat{C}_{123}).\end{aligned} \tag{6}$$

The real part of $P_{123} = \langle\hat{P}_{123}\rangle$ is determined by the expectation value of the Heisenberg terms and the imaginary part is determined by the chiralities. Based on this observation, it seems natural to guess the formula for the expectation value of an arbitrary permutation operator as being a sum of Heisenberg terms acting on all possible pairs of sites and a sum of chiralities acting on all possible triples of sites. However, such a naive guess fails immediately in the square case

$$\hat{P}_{1234} = \frac{1}{8}\Bigg[1 + \sum_{1\leqslant a<b\leqslant 4}\hat{h}_{a,b} + \hat{h}_{1,2}\hat{h}_{3,4} + \hat{h}_{1,4}\hat{h}_{2,3} - \hat{h}_{1,3}\hat{h}_{2,4} - i(\hat{C}_{123} + \hat{C}_{124} + \hat{C}_{134} + \hat{C}_{234})\Bigg], \tag{7}$$

where we introduced $\hat{h}_{a,b} \equiv \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$ to simplify the equations. In the equation, products of Heisenberg terms appear. So for larger n, the expression becomes more complicated and we find terms involving higher order in Heisenberg and chiralities in the imaginary part as well.

To find the correct expression for the permutation operator, we first recall the following identities of spin operators, where we defer the derivation to Supplemental Material [11]:

$$(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3) = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 - i\hat{C}_{123}, \tag{8}$$

$$i\hat{C}_{123}\,(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4) = -(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_4)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_4) + i\hat{C}_{124}. \tag{9}$$

Then, let us note that the permutation operator $\hat{P}_{1\ldots n}$ acting on n sites is related to $\hat{P}_{1\ldots n-1}$ via

$$\hat{P}_{1\ldots n} = \hat{P}_{1\ldots n-1}\hat{P}_{n-1,n} = \hat{P}_{1\ldots n-1}\frac{1+\boldsymbol{\sigma}_{n-1}\cdot\boldsymbol{\sigma}_n}{2}. \tag{10}$$

Finally, starting from the expression for $\hat{P}_{1\ldots n-1}$, the expression for $\hat{P}_{1\ldots n}$ can be obtained from Eq. (10) together with Eqs. (8) and (9). The resulting expression of the permutation operator on six sites (hexagon) is given by

$$\hat{P}_{123456} = \frac{1}{32}\Bigg(1 + \sum_{1\leqslant a<b\leqslant 6}\hat{h}_{a,b} + \sum_{\substack{(a,b),(c,d)\\ a<c}}{}' \mathrm{sign}(abcd)\hat{h}_{a,b}\hat{h}_{c,d} + \sum_{\substack{(a,b),(c,d),(e,6)\\ b\leqslant 3,\, 4\leqslant d<5}}{}' \mathrm{sign}(abcde6)\hat{h}_{a,b}\hat{h}_{c,d}\hat{h}_{e,6}$$
$$-\hat{C}_{123}\hat{C}_{456} - i\sum_{1\leqslant a<b<c\leqslant 6}\hat{C}_{abc} - i\sum_{\substack{(a,b),(c,d,e)\\ b\leqslant 4,\, b<d\\ a>=4,\, a>e}}{}' \mathrm{sign}(abcde)\hat{h}_{a,b}\hat{C}_{cde}\Bigg), \tag{11}$$

where $\hat{h}_{a,b} = \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$ as before, $\mathrm{sign}(ab\ldots f)$ equals (-1) the power of the total number of exchanges in order to make the tuple $(a, b, \ldots, f)$ into an ascending order and $\sum'_{I_1,\ldots,I_k}$ for tuples $I_1 = (a_1, \ldots, a_i), \ldots, I_k = (b_1, \ldots, b_j)$ denotes the summation over all possible assignments of $\{1, \ldots, n\}$ into tuples $I_1, \ldots, I_k$, satisfying the following rules: (i) the element of $\{1, \ldots, n\}$ appears at most once, (ii) each tuple is in ascending order, and (iii) each tuple satisfies any one set of the constraints specified in the summation. Each line denotes a different set of constraints to be satisfied and only one set of constraints (appearing in one of the lines) needs to be satisfied. For example, $I_1 = (1, 2)$, $I_2 = (3, 4, 5)$ is a valid assignment in the last summation satisfying the first among two sets of constraints in the sum while $I_1 = (1, 3)$, $I_2 = (3, 2, 4)$ is not a valid assignment since I_2 is not ordered and 3 appears twice. Note also, for example, $\sum'_{I_1=(a,b,c)} = \sum_{1\leqslant a<b<c\leqslant n}$. $P_{12345678}$ is given by

$$\hat{P}_{12345678} = \frac{1}{128}\Bigg(1 + \sum_{(a,b)}{}' \hat{h}_{a,b} + \sum_{\substack{(a,b),(c,d)\\ a<c}}{}' \mathrm{sign}(abcd)\hat{h}_{a,b}\hat{h}_{c,d} + \sum_{\substack{(a,b),(c,d),(e,f)\\ b\leqslant 5,\, b<d<f,\, f\geqslant 6,\, a+c+e>6}}{}'' \mathrm{sign}(abcdef)\hat{h}_{a,b}\hat{h}_{c,d}\hat{h}_{e,f}$$
$$+ \sum_{\substack{(a,b),(c,d),(e,f),(g,8)\\ b\leqslant 3,\, 4\leqslant d\leqslant 5,\, 6\leqslant f\leqslant 7}}{}' \mathrm{sign}(abcdefg8)\hat{h}_{a,b}\hat{h}_{c,d}\hat{h}_{e,f}\hat{h}_{g,8} - \sum_{\substack{(a,b,c),(d,e,f)\\ c\leqslant 5,\, d\geqslant 4,\, c<d}}{}' \mathrm{sign}(abcdef)\hat{C}_{abc}\hat{C}_{def}$$
$$- \sum_{\substack{(a,b),(c,d,e),(f,g,h)\\ b\leqslant 3,\, e=5,\, h=8\\ b\leqslant 6,\, e=3,\, h=8\\ a=7,\, e=3}}{}' \mathrm{sign}(abcdefgh)\hat{h}_{a,b}\hat{C}_{cde}\hat{C}_{fgh} - i\sum_{(a,b,c)}{}' \hat{C}_{abc} - i\sum_{\substack{(a,b),(c,d,e)\\ a\geqslant 4,\, a>e\\ b\leqslant 6,\, b<d}}{}' \mathrm{sign}(abcde)\hat{h}_{a,b}\hat{C}_{cde}$$
$$-i\sum_{\substack{(a,b),(c,d),(e,f,g)\\ b\leqslant 4\leqslant d<f,\, f\geqslant 6,\, a+c+e>6\\ b\leqslant 4\leqslant f,\, g\leqslant 6\leqslant c\\ a,c\geqslant 4\geqslant g,\, b<d}}{}' \mathrm{sign}(abcdefg)\hat{h}_{a,b}\hat{h}_{c,d}\hat{C}_{efg}\Bigg), \tag{12}$$

where the summation rule is the same as the one used in Eq. (11) except for $\sum''$ in the second line where we impose an additional constraint that, when $b = 5$, we consider only the tuples satisfying $b - a + d - c + f - e \leqslant 11$ and $d - c + f - e \leqslant 8$. Among those tuples, when $d - c + f - e \geqslant 6$, we only include tuples with $(f - 8)(b - 3) = 0$ (so $f = 8$ or $b = 3$) and $\{a, c, e\} \neq \{2, 3, 4\}$ (as a set), and $d = 7$ when $a = 1$ in the summation. These additional rules in the

summation reflect a highly nontrivial combinatorial nature of the loop operator expression. In the Supplemental Material [11], we provide an explicit algorithm to get a complete expression for an arbitrary-sized plaquette, together with the scaling of the number of terms in the number of sites in the plaquette.

As one can see, the permutation operator is expressed in terms of both linear and higher-order Heisenberg and chirality terms. Since the expectation value of the Heisenberg term is not small in the spin liquid system, it is important to count higher-order terms properly. In the following, we present the mean-field approximation of the permutation operator expectation value.

Mean-field approximation of permutation operator. Here, we use the mean-field approximation to evaluate $\langle \hat{P}_{1\ldots n} \rangle$. Even in the absence of magnetic order, the nearest-neighbor Heisenberg term has a nonzero expectation value related to the ground-state energy. We make the approximation of keeping only the nearest-neighbor terms:

$$\langle \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \rangle = \begin{cases} S_0 & \text{if (i,j) are nearest-neighbor,} \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Similarly, in the presence of the DM term, we keep only the nearest-neighbor term

$$\langle (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)_z \rangle = \mathrm{sgn}(i,j) D_0, \tag{14}$$

where $\mathrm{sgn}(i,j) = 1$ if i, j is along the arrow in Fig. 1(a), and $\mathrm{sgn}(i,j) = -1$ if it is opposed. Using this mean-field approximation, the expectation value of the chirality term for any mutually distinct sites i, j, and k is given by

$$\begin{aligned} \langle \hat{C}_{ijk} \rangle &= \langle \epsilon_{abc} (\boldsymbol{\sigma}_i)_a (\boldsymbol{\sigma}_j)_b (\boldsymbol{\sigma}_k)_c \rangle \\ &= \langle (\boldsymbol{\sigma}_i)_z (\boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k)_z \rangle + \langle (\boldsymbol{\sigma}_j)_z (\boldsymbol{\sigma}_k \times \boldsymbol{\sigma}_i)_z \rangle \\ &\quad + \langle (\boldsymbol{\sigma}_k)_z (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)_z \rangle \\ &\approx \langle (\boldsymbol{\sigma}_i)_z \rangle \langle (\boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k)_z \rangle + \langle (\boldsymbol{\sigma}_j)_z \rangle \langle (\boldsymbol{\sigma}_k \times \boldsymbol{\sigma}_i)_z \rangle \\ &\quad + \langle (\boldsymbol{\sigma}_k)_z \langle (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)_z . \end{aligned} \tag{15}$$

In the first two lines we used operator identities and the last line is a mean-field factorization. We set $\langle (\boldsymbol{\sigma}_i)_z \rangle = h$ to be site independent. Note that i, j, and k can be any 3 sites, not just the equilateral triangle formed out of the nearest neighbor. Only pairs of j and k need to be nearest neighbors in order for $\langle (\boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k)_z \rangle$ to be nonzero to contribute to Eq. (15). To simplify the expression, we introduce $C_0 = D_0 h$ in the following.

So far, our calculation works for an arbitrary cluster with arbitrary DM vectors. In our case, the DM vectors shown in Fig. 1(a) respect the lattice translational symmetry, which is also the experimentally relevant situation [6] and we focus on this case in the following. We note that this is not the case for the hexagonal lattice, which we revisit at the end of the section.

Using the mean-field approximation, the expectation values of the flux operators for an isolated triangle and hexagon in the kagome unit cell and the expectation value of the flux operator for the kagome unit cell described in Fig. 1(b) are given by

$$P_{123} = \tfrac{1}{4}(1 + 3S_0 - i3C_0), \tag{16}$$

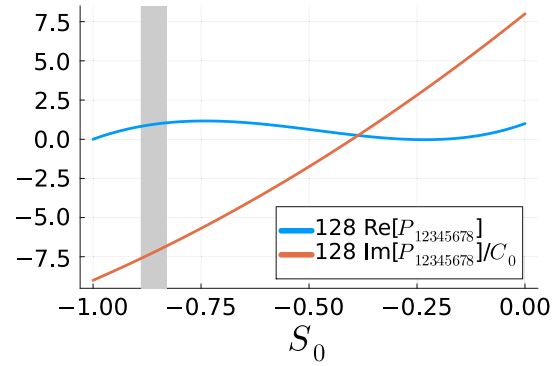

FIG. 2. Mean-field estimate of the real and imaginary part of the loop operator Eq. (18) for the kagome unit cell shown in Fig. 1(b) as functions of S_0. We have multiplied a prefactor 128 for better readability. We ignore the $S_0 C_0^2$ term in the real part since it is relatively negligible, and we plot the linear coefficient of C_0 for the imaginary part. We highlighted the region around $S_0 \approx -0.86$, the estimated value of S_0 for the Heisenberg model on the kagome lattice. In this region, the real part $\approx 1.0/128$ and the imaginary part $\approx -(7.2/128)C_0$.

$$P_{234678} = \tfrac{1}{32}\big[\big(1 + 6S_0 + 9S_0^2 + 2S_0^3 - 4C_0^2\big) + i\big(24C_0 + 23S_0C_0\big)\big], \tag{17}$$

$$P_{12345678} = \tfrac{1}{128}\big[\big(1 + 10S_0 + 29S_0^2 + 22S_0^3 + 2S_0^4 + 2S_0C_0^2\big) + i\big(8C_0 + 22S_0C_0 + 5S_0^2C_0\big)\big], \tag{18}$$

where the numbers appearing in the subscripts match the lattice site numbering in Fig. 1(b). As the computation of the numerical coefficients is quite involved, which can be seen from Eqs. (11) and (12), we employed a computer code to get the results [11].

Now we provide an estimate for the parameter S_0. For the Heisenberg model on the kagome lattice, the ground-state energy is close to $-0.43J$ per site [12]. Since there are two bonds per site, the energy per bond is $J\langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle \approx -0.215J$; hence $S_0 \approx -0.86$. Note this value is negative and close to unity in magnitude. Therefore, it is necessary to keep higher orders in S_0 in our calculation. In fact for the case of the triangle given in Eq. (16), the real part of P_{123} is negative. This reflects the tendency to generate a π flux due to frustration in the triangle.

We finally compute the flux of the kagome unit cell using Eq. (18) and our estimates on S_0. As can be seen from Fig. 2, the flux $\Phi_{1\ldots8}$ associated with the kagome unit cell is given by $\Phi_{1\ldots8} \approx -\tan^{-1}(7.2\,C_0)$, which is nonzero when $C_0 \neq 0$. Since C_0 is nonzero whenever the DM term exists, the kagome unit cell experiences nonzero flux upon introducing the DM term. Estimates of the size of the flux for the specific case of YCOB are given in Sec. IV.

Let us discuss the results for the triangular and the hexagonal lattices. Since the DM interaction must respect the lattice translational symmetry, a different choice of arrows should be made for the triangular lattice or the hexagonal lattice compared with that shown in Fig. 1(a). For the triangular lattice, the imaginary part of the expectation value of the

permutation operator P of a unit cell is zero. As the unit cell consists of one upward triangle and one downward triangle, we see that the chirality of the upward triangle exactly cancels the chirality of the downward triangle. For a hexagon in the hexagonal lattice, the DM interaction additionally respects a $\hat{C}_3$ symmetry, with the rotation center being a site. The real part of P for a hexagon remains the same as the real part in Eq. (17). In the imaginary part the coefficient of the leading C_0 term also vanishes. However, surprisingly, the next term proportional to S_0C_0 is not canceled and the absolute value of the coefficient is 3 (its sign depends on the convention of arrows). So even assuming $S_0 = -1$, the imaginary part of P of the elementary hexagon in the kagome lattice is $47/32$ and that in the hexagonal lattice is $3/32$; i.e., the hexagonal lattice is an order-of-magnitude smaller compared to the value from the kagome lattice.

A second route to estimate the gauge flux. In this section we comment on the second route to estimate the gauge flux using the $\hat{\chi}$ operators. Let us consider $\chi_{1\ldots n} = \langle \hat{\chi}_{1\ldots n} \rangle$, where $\hat{\chi}_{1\ldots n}$ was defined in Eq. (4). A common approximation is to consider $\chi_{1\ldots n} \approx \chi_{1,2}\chi_{2,3}\cdots\chi_{n,1}$. In the absence of the DM interaction, it is obvious that this product is real when the loop is around a unit cell, because $\chi_{i,j}$ is equal to one that is translated by a lattice vector which appears in the product as complex conjugate, e.g., $\chi_{2,3} = \chi_{6,5} = \chi^*_{5,6}$ in Fig. 1(b). Working to the first order in the DM interaction, the correction terms to $\chi_{i,j}$ are also the same under translation, so that $\chi_{i,j}$ remains equal under translation and the product $\chi_{1\ldots n}$ remains real. To find the imaginary part, we take the next level of approximation by keeping a factor $\chi_2(i, j, k)$ defined as $\langle \hat{\chi}_{i,j}\hat{\chi}_{j,k} \rangle$ and factorizing the rest into products of $\chi_{l,m}$. Then there is no longer any cancellation between pieces that were previously related by translation, e.g., compare $\chi_2(2, 3, 4)$ and $\chi_2(4, 5, 6)$ in Fig. 1(b). These two triangles have different geometries because sites 4 and 6 are nearest neighbors while sites 2 and 4 are not. Therefore, the correction due to DM terms which live only on nearest neighbors will be different. Furthermore, the contributions of $\chi_2(4, 5, 6)$ and $\chi_2(8, 1, 2)$, which come from the two opposing triangles, add because they are related by a 180° rotation. Therefore, in general $\chi_{1\ldots 8}$ will have an imaginary part to leading order in the DM interaction. Within the slave-particle mean-field theory, it is possible to compute these using diagrammatic Green's function techniques; we sketch the steps in the Supplemental Material [11].

Conclusion. In this Letter, we provide a general mechanism for generating gauge flux when the interaction of the material contains the DM term in addition to other interactions. While our framework works for an arbitrary lattice, we provide a detailed analysis for the kagome lattice since it in general shows a larger gauge flux compared to other lattices such as the triangular and the hexagonal lattice and therefore suits well with spin liquid states. In the following, we elaborate more on the application of these results for the case of YCOB [6].

We make some estimate of the gauge flux generated by the DM interaction in the specific case of YCOB. The gauge magnetic flux through the unit cell is given by $\Phi/2\pi$, which is given by the ratio of the imaginary part to the real part of $P_{12345678}$. Using the value $S_0 = -0.86$ obtained earlier, we read the following from Fig. 2,

$$\Phi \approx -7.2C_0 \approx -7.2\langle(\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3)_z\rangle\langle\sigma_z\rangle \tag{19}$$

for small values of C_0. Starting from Eq. (1) and treating the spins classically, we estimate the canting angle to be $\approx |D_z|/J$. Hence, very roughly, we estimate $\langle(\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3)_z\rangle \approx -D_z/J$. Furthermore $|D_z|/J \approx \Delta g/g$, where $\Delta g/g$ is the g factor anisotropy, which is roughly 0.1 in YCOB. Zorko *et al.* [13] found by neutron scattering in a related compound which has antiferromagnetic order that the ordering is a 120° antichiral state. This implies that $\langle(\boldsymbol{S}_2 \times \boldsymbol{S}_3)_z\rangle$ tends to be negative and $D_z > 0$. (Note that our convention for the sign of the DM term is opposite to that used in Refs. [7,13].) Near the 1/9 plateau, $\langle\sigma_z\rangle \approx -1/9$ for the field along the c axis. Taken together, we estimate

$$\Phi/2\pi \approx (7.2/2\pi)(D_z/J)\langle\sigma_z\rangle \approx -1.2 \times 10^{-2}. \tag{20}$$

The spinon couples to this negative gauge flux with a positive gauge charge. It is convenient to express the Berry phase in terms of an effective magnetic field b so that $\Phi = 2\pi\phi/\phi_0$, where $\phi = bA_0$, A_0 is the unit cell area, and $\phi_0 = h/e$ is the flux quantum. In YCOB, the unit-cell area $A_0 = 38.53$ Å^2 and we find the effective magnetic field that gives this flux to be $b \approx 10^4(\Phi/2\pi)$ T, which is about -120 T. This is larger than the physical magnetic field $B \approx 30$ T used in the experiment, so that $\alpha = |b|/B_c \approx 4$. In Ref. [6], α was found to be of the order of unity, but that estimate has large uncertainty because it depends quadratically on the assumed Dirac velocity which was not well determined. The important point is the flux generated by the DM interaction is large enough to give rise to the observed magnetization oscillations.

Up to now we estimated the DM contribution to the Berry phase assuming that there is no other flux through the unit cell. In the case of YCOB we need to produce an extended unit cell with nine bands in order to explain the 1/9 plateau. This can either come from breaking of translation symmetry, or by assuming $2\pi/3$ flux per unit cell as was done in Ref. [6]. This large flux produces the band structure with nine bands, and the DM contribution should be considered as a small perturbation on this band structure. In particular, the flux we estimated in Eq. (20) gives an effective uniform gauge magnetic field b which produces Landau levels in the bands near the conduction and valence band edges and is the correct one to use to compare with the experiment. In principle, we should calculate the Berry phase using a tripled unit cell, where the model has a net flux of 2π and the hopping can be taken as periodic in the absence of the DM interaction. In practice, we expect that Eq. (20) will continue to be a reasonable estimate.

Finally, we note that the mechanism of generating a net-gauge magnetic field from the DM term is quite general, and should be present as long as $\langle S_z\rangle$ is finite. For example, this will give rise to a thermal Hall effect even away from the 1/9 plateau if spinons are present. It should also be possible to use the thermal Hall effect to probe the existence of spinons in other kagome systems which often have similar DM terms.

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