Letter

Dynamical quantum phase transitions following a noisy quench

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We study how time-dependent energy fluctuations impact the dynamical quantum phase transitions (DQPTs) following a noisy ramped quench of the transverse magnetic field in a quantum Ising chain. By numerically solving the stochastic Schrödinger equation of the mode-decoupled fermionic Hamiltonian of the problem, we identify two generic scenarios: Depending on the amplitude of the noise and the rate of the ramp, the expected periodic sequence of noiseless DQPTs may either be uniformly shifted in time or else replaced by a disarray of closely spaced DQPTs. Guided by an exact noise master equation, we trace the phenomenon to the interplay between noise-induced excitations which accumulate during the quench and the near-adiabatic dynamics of the massive modes of the system. Our analysis generalizes to any one-dimensional fermionic two-band model subject to a noisy quench.

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Introduction. Dynamical quantum phase transitions (DQPTs) have become one of the focal points in the study of quantum matter out of equilibrium [1,2], spurred by the prospect of performing high-precision tests using quantum simulators [3,4]. DQPTs appear at critical times at which the overlaps between initial and time-evolved states vanish. As a result, the rate function which plays the role of a dynamical free energy density [5] becomes nonanalytic in the thermodynamic limit. With time replacing the usual notion of a control parameter, DQPTs are different from ordinary phase transitions, requiring new ideas and concepts for their understanding. Progress has come thick and fast, with an expanding literature on theory [5–33], modeling, and experimentation [34–47].

Most research so far, theoretical as well as experimental, has considered DQPTs triggered by a quantum quench where an isolated system is forced out of equilibrium by a change of its Hamiltonian. The quench may be modeled as sudden, or more realistically, as having a finite duration with a Hamiltonian parameter being swept from an initial to a final value, also known as a "ramp." While the quench is usually assumed to be governed by a well-defined Hamiltonian, its realization in an experiment is always imperfect. As a result, when energy is transferred into or out of an otherwise isolated system via a quench in the laboratory, there will inevitably be time-dependent fluctuations ("noise") in this transfer. Examples include noise-induced heating caused by amplitude fluctuations of the lasers forming an optical lattice [48] and fluctuations in the effective magnetic field applied to a system of trapped ions [49]. This raises the important issue about the robustness of DQPTs following a *noisy* quench. Do the DQPTs survive? If so, what is the effect from noise on the dynamical critical behavior?

We address these questions in the setting of the transverse field Ising (TFI) chain, arguably the simplest benchmark model for this purpose. The model has served as a paradigm for exploring quantum phase transitions in and out of equilibrium and is also the first [5] and best studied model exhibiting DQPTs. The availability of platforms for well-controlled experimental probes of DQPTs in TFI-like chains [34–36,43,44] is yet another reason why we choose it for our study. Quantitative reliable results for the simple TFI chain, amenable to experimental tests, should prepare the ground for a comprehensive theory of DQPTs following a noisy quench.

Representing the noise by a dynamical stochastic variable added to the TFI Hamiltonian, we numerically study the stochastic Schrödinger equation of the corresponding mode-decoupled fermionic Hamiltonian that governs the dynamics of a single quench. In addition, we construct and solve an exact master equation for the quench dynamics averaged over the noise distribution. This allows us to highlight the interplay between the near-adiabatic quench dynamics of the gapped modes of the system and the accumulation of noise-induced excitations. As suggested by our analysis, the competition between adiabaticity and noise-induced excitations underlies the sometimes surprising outcome of a noisy quench. While

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a small ratio between noise amplitude and rate of energy transfer at most results in a shift of the expected periodic sequence of noiseless DQPTs, a larger ratio may have a dramatic effect: The periodic sequence can now get scrambled, resulting in a disarray of closely spaced DQPTs.

Noiseless ramped quench: background. To set the stage, we write down the Hamiltonian of the Ising chain with periodic boundary conditions and subject to a noiseless transverse magnetic field $h_0(t)$:

$$H_0(t) = -J\sum_{n=1}^{N}\sigma_n^x\sigma_{n+1}^x - h_0(t)\sum_{n=1}^{N}\sigma_n^z. \tag{1}$$

When the field is time independent, $h_0(t) = h$, and with J set to unity, the ground state is ferromagnetic for $|h| < 1$, otherwise paramagnetic, with the phases separated by equilibrium quantum critical points at $h = \pm 1$ [50]. Here, and in what follows, $\hbar = 1$.

The Hamiltonian $H_0(t)$ in Eq. (1) can be mapped onto a model of spinless fermions with operators $c_n, c_n^\dagger$ using a Jordan-Wigner transformation [51]. Performing a Fourier transformation, $c_n = (\mathrm{e}^{i\pi/4}/\sqrt{N})\sum_k \mathrm{e}^{ikn}c_k$ (with the phase factor $\mathrm{e}^{i\pi/4}$ added for convenience), $H_0(t)$ gets expressed as a sum over decoupled mode Hamiltonians $H_{0,k}(t)$:

$$H_0(t) = \sum_k C_k^\dagger H_{0,k}(t) C_k, \quad k = \frac{(2m-1)\pi}{N}, \tag{2}$$

with $m = 1, 2, \ldots, N/2$. Here N and the fermion parity $\exp(i\pi\sum_{n=1}^{N} a_n^\dagger a_n)$ are taken to be even [52]. $C_k^\dagger = (c_k^\dagger \ c_{-k})$ are Nambu spinors, and

$$H_{0,k}(t) = h_{0,k}(t)\sigma^z + \Delta_k\sigma^x, \tag{3}$$

with $h_{0,k}(t) = 2[h_0(t) - \cos(k)]$ and $\Delta_k = 2\sin(k)$ when $J = 1$. The instantaneous eigenstates and eigenvalues of $H_{0,k}(t)$ are given by

$$|\chi_k^\pm(t)\rangle = \alpha_k^\pm(t)|\alpha\rangle + \beta_k^\pm(t)|\beta\rangle, \tag{4}$$

$$\varepsilon_k^\pm(t) = \pm\varepsilon_k(t) = \pm\sqrt{h_{0,k}^2(t) + \Delta_k^2}, \tag{5}$$

where $|\alpha\rangle = (1\,0)^T$, $|\beta\rangle = (0\,1)^T$, and $\alpha_k^\pm(t) = [h_{0,k}(t) \mp \varepsilon_k(t)]/N_k^\pm(t)$, $\beta_k^\pm(t) = \Delta_k/N_k^\pm(t)$, with $N_k^\pm(t)$ normalization constants. Note that for large N, the gap between the two levels vanishes in the limit $k \to \pi$ ($k \to 0$) when reaching the critical points $h_0(t) = -1$ ($h_0(t) = +1$). Also note that the Pauli matrices in Eq. (3) are not to be mixed up with the spin operators in Eq. (1).

As a preliminary, let us briefly review DQPTs in case of a noiseless ramp with sweep velocity v, $h_0(t) = h_f + vt$, from an initial value h_i at time $t = t_i < 0$ to a final value h_f at $t = t_f = 0$. The Hamiltonian in Eq. (3) for each mode has the form $H_{0,k}(t) = \frac{1}{2}v\tau_k\sigma_z + \Delta_k\sigma_x$, so transition rates can be calculated by the Landau-Zener formula [53,54]. Here $\tau_k = 2h_{0,k}(t)/v$ defines a mode-dependent time, which changes sign when an avoided level crossing occurs [55,56]. As expected from the adiabatic theorem [57], a quasiparticle mode with wave number k remains in its instantaneous eigenstate in the limit $v\Delta_k/2\varepsilon_k^3(t) \to 0$ [58] [with $2\varepsilon_k(t)$ the gap of the mode at time t; cf. Eq. (5)]; hence $\{|\chi^\pm(\tau_k)\rangle\}$ spans the adiabatic basis, with $\{|\alpha\rangle, |\beta\rangle\}$ the diabatic basis.

Starting with $h_i \ll -1$ in the ground state of the paramagnetic phase, all modes initially reside in the lower level $|\chi_k^-(t_i)\rangle$. After a ramp across the critical field $h = -1$ to some final value h_f in the ferromagnetic phase, the probability to find mode k in the upper level $|\chi_k^+(t_f)\rangle$ will depend on the value of k, and we denote this probability by p_k. Modes close to $k = 0$ show no sign change of τ_k, so they mostly remain in the lower level $p_k < 1/2$, while modes close to the gap-closing limit $k = \pi$ will be excited to the upper level with probability $p_k > 1/2$ [5,15]. Given these two cases, continuity of the spectrum as a function of k in the thermodynamic limit implies that there exists a "critical mode" k^* with equal probabilities $p_{k^*} = 1/2$ for occupation of the lower and upper levels after the ramp, corresponding to a maximally mixed state. This is the mode that triggers the appearance of DQPTs at critical times [1,59],

$$t_c^n = (2n+1)\frac{\pi}{2\varepsilon_{k^*,f}}, \qquad n = 0, 1, \ldots, \tag{6}$$

with $\varepsilon_{k^*,f} = \varepsilon_{k^*}(t_f)$ the energy in Eq. (5). Note that the ramp occurs at negative times, $t < t_f = 0$, while the DQPTs take place at positive times.

Noisy ramped quench: formalism. To approach the problem with a noisy quench we add a random variable $\eta(t)$ to the magnetic field, writing $h(t) = h_0(t) + \eta(t)$. We shall assume the noise distribution to be Gaussian with vanishing mean, $\langle\eta(t)\rangle = 0$, and with canonical Ornstein-Uhlenbeck two-point correlations [69]:

$$\langle\eta(t)\eta(t')\rangle = \frac{\xi^2}{2\tau_n}e^{-|t-t'|/\tau_n}. \tag{7}$$

Here τ_n is the noise correlation time and ξ the noise amplitude for fixed τ_n. The frequently employed white-noise limit is obtained by letting $\tau_n \to 0$.

As before, the probabilities p_k for nonadiabatic transitions will change continuously with k in the thermodynamic limit, but it is *a priori* unclear if the special value $p_k = 1/2$ occurs at all, or maybe even for several k values. The inequality $p_{k,\max} > 1/2$ close to $k = \pi$ is ensured by the Kibble-Zurek mechanism (KZM), which predicts a breakdown of adiabaticity when approaching gap closing [70,71]. On the other hand, noise will in general facilitate additional transitions, so it is uncertain if modes with $p_{k,\min} < 1/2$ remain, which is the required condition for DQPTs [1,59]. While there are closed expressions for finite-time transition probabilities in the adiabatic basis with no noise [58], there are no known such results when noise is present. Could it be that noise may increase the probability for nonadiabatic transitions, corrupting the inequality $p_{k,\min} < 1/2$? Or maybe instead drive oscillations of the p_k function across $1/2$, causing additional DQPTs?

To find out we numerically solve the stochastic Schrödinger equations (SSEs) [72–74],

$$[H_{0,k}(t) + \eta(t)H_1]|\psi_k(t)\rangle = i\frac{\partial}{\partial t}|\psi_k(t)\rangle, \tag{8}$$

for the allowed values of k [cf. Eq. (2)] and for single realizations of the noise function $\eta(t)$ in the quench interval $t \in [t_i, 0]$, with $H_1 = 2\sigma^z$ [cf. Eq. (3) with $h_{0,k}(t) \to h_{0,k}(t) + \eta(t)$]. Having obtained the solution

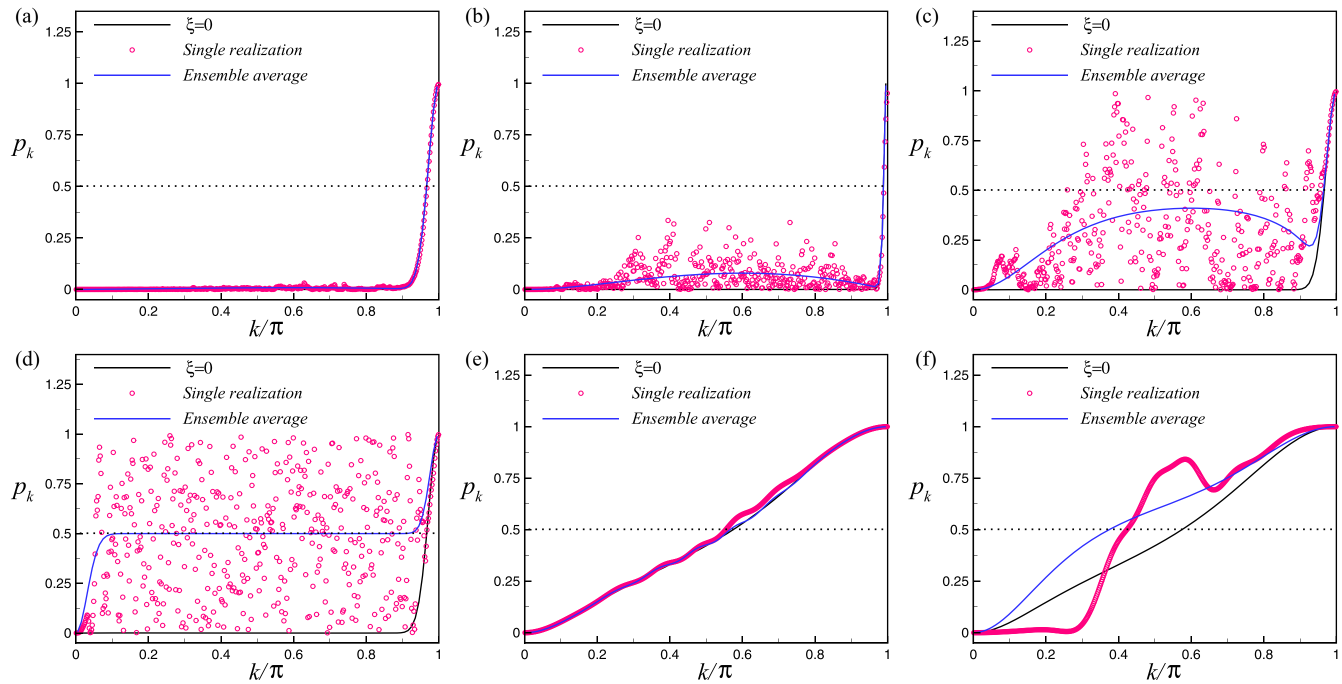

FIG. 1. Probabilities for finding a mode with momentum k in the upper level after a ramp across the single quantum critical point $h_c = -1$ ($h_i = -50, h_f = 1/2$) for system size $N = 1000$ and different noise amplitudes ξ, sweep velocities v, and noise correlation times τ_n: (a) $\xi = 0.01$, $v = 0.1$, $\tau_n = 0.01$; (b) $\xi = 0.01$, $v = 0.01$, $\tau_n = 0.01$; (c) $\xi = 0.1$, $v = 0.1$, $\tau_n = 0.01$; (d) $\xi = 1$, $v = 0.1$, $\tau_n = 0.01$; (e) $\xi = 0.1$, $v = 10$, $\tau_n = 0.01$; (f) $\xi = 1$, $v = 10$, $\tau_n = 0.01$. The probabilities p_k for single noise realizations are displayed in red, with the ensemble averages $\langle p_k \rangle$ in blue. For comparison, the probabilities p_k for noiseless cases ($\xi = 0$) are shown in black.

$|\psi_k(t)\rangle = u_k(t)|\chi_k^+(t)\rangle + v_k(t)|\chi_k^-(t)\rangle$ to Eq. (8) at $t = t_f = 0$, one reads off $p_k = |u_k(0)|^2$ [59].

In addition, we construct an exact noise master equation (ME) [75–78] for the averaged density matrix $\rho_k(t) = \langle \rho_{\eta,k}(t)\rangle$, with $\rho_{\eta,k}(t)$ the density matrix of the Hamiltonian in Eq. (8). Explicitly,

$$\dot{\rho}_k(t) = -i[H_{0,k}(t), \rho_k(t)] - \frac{\xi^2}{2\tau_n}\left[H_1, \int_{t_i}^{t} e^{-|t-s|/\tau_n}[H_1, \rho_k(s)]ds\right]. \tag{9}$$

By translating Eq. (9) into two coupled differential equations, the mean transition probabilities are obtained numerically as ensemble averages $\langle p_k\rangle$ over the noise distribution $\{\eta\}$. The averaged probabilities reveal features not easily seen from a single quench, and, moreover, allows us to validate the soundness of the SSE numerics. For details, see [59].

Results. Let us analyze the results predicted by Eqs. (8) and (9) for a quench across the equilibrium critical point $h = -1$, from $h_i = -50$ to $h_f = 1/2$. The effect of noise is bound to increase with the amplitude ξ but will also depend on the correlation time τ_n, as well as on the sweep velocity v. For transparency we focus on a few representative cases, displayed in panels (a)–(f) of Fig. 1.

(a) We take off from a noiseless quench that supports an extended adiabatic regime, i.e., with modes satisfying $p_k \approx 0$. As discussed above, when a quench is noiseless there appears only a single critical momentum k^* (satisfying $p_{k^*} = 1/2$). Panel (a) shows that adding noise in the velocity-weighted low-amplitude limit $\xi/v \ll 1$ does not perturb k^*. Hence, in this limit the corresponding DQPTs are robust against noise.

(b) Increasing ξ/v by lowering the sweep velocity v as compared to (a), one enters a crossover region with $\xi/v \sim \mathcal{O}(1)$. In this region the impact of noise depends on its nonweighted amplitude ξ. Panel (b) shows that the noiseless critical momentum remains unperturbed for a sufficiently small ξ [here with the same value as in (a)]. Thus, the corresponding DQPTs stay robust against noise.

(c) Boosting the amplitude ξ in the crossover region $\xi/v \sim \mathcal{O}(10)$ [here by a factor of 10 compared to (b)] causes the p_k function for a single noise realization to cross the value $1/2$ for several k values. The convergence of p_k to a continuous function of k in the thermodynamic limit $N \to \infty$ is now extremely slow, reflecting that the large-amplitude noise variability morphs into a finite-N p_k-function with occasional large jumps between neighboring modes. Going to larger values of N will eventually smoothen the graph, implying a set of randomly distributed critical momenta $\{k_i^*\}$ in the thermodynamic limit where p_k becomes continuous. By inserting $\{k_i^*\}$ into Eq. (6), one obtains an *aperiodic* sequence of densely spaced DQPTs. Figure 2 shows how such DQPTs are signaled by cusps in the dynamical free energy $g(t) = (1/N)\ln|\mathcal{G}(t)|$, being finite-size precursors of the nonanalyticities in the thermodynamic limit. Here

$$\mathcal{G}(t) = \prod_k \langle\psi_k(0)|\exp(-iH_{0,k}(0)t)|\psi_k(0)\rangle \tag{10}$$

is the Loschmidt amplitude for the time-evolved postquench state [59].

As seen in both panels (b) and (c), the blue graphs for the ensemble-averaged transition probabilities $\langle p_k\rangle$ are concave away from the gap-closing region at $k = \pi$. This suggests an intriguing interplay between noise-induced excitations and the

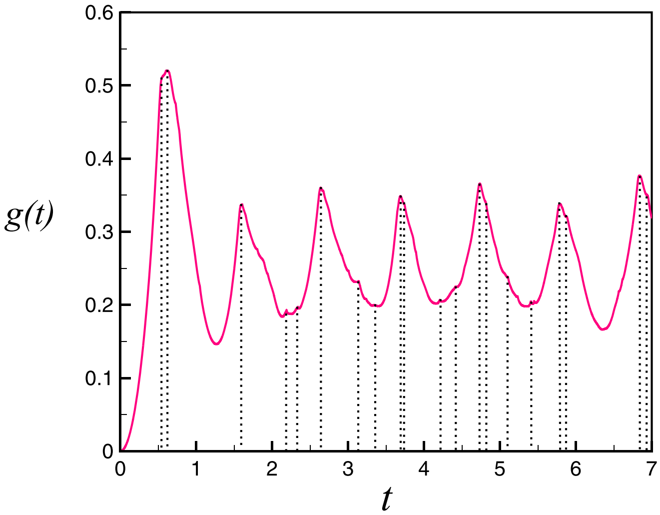

FIG. 2. The dynamical free energy $g(t)$ of the model for the noisy quench corresponding to Fig. 1(c). The vertical dotted lines mark the times for the finite-size ($N = 1000$) precursors of DQPTs.

dynamics of the gapped modes driven by the slow noiseless ramp: Deep in the adiabatic regime where the averaged instantaneous gap is large (with the average taken over the duration of the quench), noise has a negligible effect. For intermediate-sized gaps, noise excitations become effective but then level off as one approaches the neighborhood of $k = \pi$. Here the KZM takes over, dominating the nonadiabatic dynamics and making the presence of noise largely irrelevant.

(d) Increasing ξ/v further, entering the velocity-weighted large-amplitude regime $\xi/v \gg 1$ (still with the noiseless quench supporting an adiabatic regime), the number of critical momenta in the thermodynamic limit proliferate. Similar to the case displayed in panel (c), this is spelled out by the finite-size plot of p_k in panel (d), which exhibits repeated jumps of p_k across the value $1/2$. As an aside, let us remark that the number of critical momenta increase also when the correlation time τ_n decreases: A smaller τ_n implies a larger noise variability ξ/τ_n which gets inherited by the p_k function in the guise of a larger transition variability. Referring to the correlation time τ_n, we also note that noise effects are conditioned by the inequality $\tau_n < 1/v$, with $1/v$ the ramp time.

The most striking feature in panel (d) is the plateau formation of the blue curve. Here the average transition probabilities $\langle p_k \rangle$ are numerically found to be locked to the value $0.5000 \pm 0.000\,01$, signaling the emergence of a maximally mixed state for the corresponding modes. One may understand this by noting that an Ornstein-Uhlenbeck process is stationary and therefore ergodic in the mean [69]. It follows that the long-time average of the noisy density matrix converges to that of its ensemble average. Given this, the formation of a plateau suggests that an asymptotically slow noisy quench will effectively heat the system to infinite temperature. This is supported by earlier results showing that a slow quench subject to large-amplitude white noise may lead to a maximally mixed state [79,80]. We should add that the width of a plateau increases with decreasing τ_n as well as with decreasing v.

(e) Let us finally consider a noiseless quench where, differently from the cases (a)–(d), the assumption $v\Delta_k/2\varepsilon_k^3(t) \ll 1$ is violated for most of the modes, implying that their dynamics is nonadiabatic. The nonadiabaticity is here driven by a larger value of the sweep velocity v, also giving less time for noise to become effective. As expected, and similar to the case in (a) where $\xi/v \ll 1$, panel (e) confirms that the presence of noise also now has a negligible effect when ξ/v is sufficiently small.

(f) In contrast, when ξ/v is above some threshold value, however still with $\xi/v \ll 1$, the noise may cause a noticeable shift of the single noiseless critical momentum, as displayed in panel (f). This results in a uniform shift of the sequence of noiseless periodic DQPTs; cf. Eq. (6).

It is important to note that all DQPT scenarios in panels (a)–(f) of Fig. 1 are fully determined by the p_k function. It follows that any one-dimensional fermionic two-band model subject to a noisy ramp with a behavior of the p_k function analogous to that of the TFI chain will show similar postquench dynamics. Let us also mention that the averaged p_k curves in Figs. 1(a)–1(f) obtained from the ME, Eq. (9), are well reproduced by averaging over a finite sample of solutions to the SSEs in Eq. (8), each SSE with a distinct noise realization $\eta(t)$; see Ref. [59]. This serves as a stringent check on our numerical approach.

Summary and discussion. Summing up, we have shown how the patterns of DQPTs following a noisy ramped quench of the magnetic field in the TFI chain depend on the rate of the ramp ("sweep velocity" v) and amplitude ξ of noise fluctuations. Two distinct classes of scenarios can be identified: (i) noise having a negligible or weak effect, at most shifting the expected sequence of noiseless DQPTs; and (ii) noise causing an aperiodic, closely spaced set of DQPTs. Note that the stochastic nature of noise does not allow us to delineate (i) and (ii) by a sharp phase boundary; only for a very small [large] ratio ξ/v can we predict with certainty that (i) [(ii)] materializes after a single quench.

While we have here exhibited (i) and (ii) with quench protocols where one of the TFI equilibrium quantum critical points is crossed, we expect the two scenarios to be generic. Specifically, we have checked this for a ramped quench across both TFI equilibrium quantum critical points [81]. The competition between adiabaticity and noise-induced excitations that brings about the two scenarios is known to be at play also in impacting the Kibble-Zurek scaling of defect formation when quenching across a critical point [80,82,83]. It would be interesting to pinpoint related phenomena driven by this same competition.

With the rapid advances in realizing analog quantum simulators, experimental tests of our predictions may soon be within reach. While we have focused our theoretical analysis on the underlying dynamics after a single noisy quench, an experimental followup must probably settle for ensemble averages: Real-time tracking of a single-shot outcome will most likely have to await further advances in weak measurement techniques [84,85]. On the other hand, noise-averaged (strong) measurements are expected to be fully within the realm of current experimental methods and will be highly informative (as suggested by the blue-colored graphs in Fig. 1). Ramped magnetic quenches in the presence of amplitude-controlled noise have already been achieved with trapped ions simulating the transverse-field XY chain [86]. The other backbone for an experimental exploration—detection and characterization of DQPTs—is also in place, as demonstrated on a variety of platforms for TFI-type chains with finite-range interactions: trapped ions [34,35,44], Rydberg atoms [36], and NV centers [43]. These breakthroughs, together with recent advances in quantum-circuit computations on NISQ (noisy intermediate-scale quantum) devices [87,88], hold promise for exploring DQPTs following noisy quenches also in the nearest-neighbor interacting TFI chain studied in this Letter.

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