Letter

Theory of weak localization in graphene with spin-orbit interaction

L. E. Golub
Terahertz Center, University of Regensburg, 93040 Regensburg, Germany

(Received 13 October 2023; accepted 29 February 2024; published 19 March 2024)

A theory of weak localization in graphene with Rashba splitting of the energy spectrum is developed. Anomalous magnetoresistance caused by weak localization is calculated accounting for inter- and intravalley, and spin-orbit and spin-valley scattering processes. It is shown that the anomalous magnetoresistance is described by an expression different from the traditional Hikami-Larkin-Nagaoka formula. The reason is that the effect of Rashba splitting gives rise to the spin-orbit vector potential which is not reduced to a spin dephasing only. The developed theory can be applied to heterostructures of graphene with transition metal dichalcogenides.

DOI: 10.1103/PhysRevB.109.L121407

Introduction. Weak localization (WL) is an interference phenomenon based on the wave properties of particles. WL mainly consists in the interference of waves backscattered from groups of defects. In conductors WL results in anomalous negative magnetoresistance in classically weak magnetic fields at low temperatures. The study of the magnetic-field and temperature dependence of anomalous magnetoresistance allows determining various system parameters such as dephasing rates rarely accessible in other experiments [1]. In spin-orbit coupled systems, WL is much richer due to the constructive and destructive character of the interference in different spin channels described by additional spin-related phases acquired by electrons at backscattering. This results in sign-alternating magnetoresistance with a positive part at the lowest fields and is referred to as weak antilocalization (WAL). Studies of WAL in two-dimensional semiconductors give access to spin-orbit splittings and spin relaxation times [2]. Weak localization in graphene is specific due to the Berry phase π acquired by Dirac fermions at backscattering [3]. However, this does not always result in WAL due to effective intervalley scattering [4,5].

In graphene-based systems the spin-orbit effects are important when the spin-orbit coupling is induced by proximity effects [6,7]. A theory for graphene heterostructures predicts the Rashba [8] spin-orbit splitting from 0.1 to a few meV in different graphene heterostructures with transition metal dichalcogenides (TMDCs) [9–12] and topological insulators [13]. Experiments on the anomalous magnetoresistance in graphene with spin-orbit coupling demonstrate WAL in different single- and bilayer graphene/TMDC heterostructures [14–21]. The determination of spin-orbit and dephasing parameters from experimental data is always performed by the theoretical expressions of Ref. [22] containing the Hikami-Larkin-Nagaoka (HLN) function [23]. However, it is known from studies of WAL in two-dimensional semiconductors with Rashba spin-orbit splitting that the HLN expression does not describe anomalous magnetoresistance. Another formula derived by Iordanskii, Lyanda-Geller, and Pikus should be used for a description of the experimental data [24,25]. In this Letter we show that the same situation takes place in graphene with spin-orbit coupling and derive an expression for the WL-induced anomalous magnetoresistance.

Theory. The Hamiltonian of graphene with spin-orbit coupling has the following form in the basis of eight-component Bloch functions of electrons on two sublattices in two valleys and with one of two spin projections [5,22,26],

$$\begin{aligned}\mathcal{H} = v\boldsymbol{\Sigma}\cdot\boldsymbol{p} + \gamma[\boldsymbol{\Sigma}\times\boldsymbol{s}]_z + \lambda\Sigma_z s_z + \Delta_s\Pi_z\Sigma_z \\ -\mu\Pi_z\big[\Sigma_x\big(p_x^2-p_y^2\big) - 2\Sigma_y p_x p_y\big].\end{aligned} \tag{1}$$

Here, $\boldsymbol{p}$ is momentum, v is the Dirac fermion velocity, z is a coordinate normal to the graphene layer, the terms $\propto\gamma$ and $\propto\lambda$ are Rashba and Kane-Mele (enhanced by phonons [27]) spin-orbit couplings, Δ_s is an orbital gap due to staggered sublattice potential, and the term $\propto\mu$ describes trigonal warping. We use three sets of Pauli matrices to describe spin $\boldsymbol{s}$, sublattice "isospin" $\boldsymbol{\Sigma}$, and matrices $\boldsymbol{\Pi}$ acting in the valley space [5]. They are related to the Pauli matrices $\boldsymbol{\sigma}$ acting in the sublattice space by $\Sigma_{x,y} = \Pi_z\otimes\sigma_{x,y}$, $\Sigma_z = \sigma_z$. Disorder is described by a sum of the spin-independent and spin-dependent terms [22]

$$\begin{aligned}V = uI + \sum_{a,l=x,y,z} u_{al}\Sigma_a\Lambda_l \\ + \sum_{j=x,y,z} s_j\left(\sum_{a=x,y,z}\alpha_{aj}\Sigma_a + \sum_{l=x,y,z}\beta_{lj}\Lambda_l\right),\end{aligned} \tag{2}$$

where valley "pseudospin" matrices are $\Lambda_{x,y} = \Pi_{x,y}\otimes\sigma_z$, $\Lambda_z = \Pi_z$. The terms with s_z and with $s_{x,y}$ describe the spin-orbit scattering due to $z\to -z$ symmetric and asymmetric perturbations, respectively [22].

The low-temperature transport involves only the electrons at the Fermi level, which is assumed to be far enough from the Dirac point. Therefore it is useful to pass to the new basis of electron states characterized by the momentum $\boldsymbol{p}$, which belong to a valley $K_\pm$. If we consider the Hamiltonian $\mathcal{H}_0 = v\boldsymbol{\Sigma}\cdot\boldsymbol{p}$ only, then we obtain that the energy of these states is equal to vp and the eigenfunction is $|K_\pm,\boldsymbol{p}\rangle = [1,\pm\exp(i\varphi_{\boldsymbol{p}})]^T/\sqrt{2}$, where $\exp(i\varphi_{\boldsymbol{p}}) = (p_x+ip_y)/p$. In this new basis, the conduction-band Hamiltonian has the following

TABLE I. Dephasing rates Γ_j^l for different valley and spin channels with $l, j = s, t_0, t_1$. Here, $\Gamma_{\rm KM} = \frac{2\pi}{\hbar} g \sum_{l=x,y,z} \alpha_{lz}^2 + \frac{2\pi}{\hbar} g (u\lambda/\epsilon_{\rm F})^2$, $\Gamma_\alpha = \frac{2\pi}{\hbar} g \sum_{l=x,y,z} \alpha_{lx}^2$, $\Gamma_z = \frac{2\pi}{\hbar} g \sum_{l=x,y,z} u_{lz}^2 + \Gamma_w + \frac{2\pi}{\hbar} g (u\Delta_s/\epsilon_{\rm F})^2$, where $\Gamma_w = (\mu\epsilon_{\rm F}^2/\hbar v^2)^2 \tau_{\rm tr}$, $\Gamma_{iv} = \frac{2\pi}{\hbar} g \sum_{l=x,y,z} u_{lx}^2$, $\{\Gamma_{zv,e}, \Gamma_{iv,e}, \Gamma_{zv,o}, \Gamma_{iv,o}\} = \frac{\pi g}{\hbar}\{\beta_{zz}^2, \beta_{xz}^2, \beta_{zx}^2, \beta_{xx}^2\}$, $\Gamma_* = \Gamma_z + \Gamma_{iv}$, and $\Gamma_{\rm SO} = \Gamma_{\rm asy} + \Gamma_{\rm sym}$, where $\Gamma_{\rm asy} = \Gamma_\alpha + 2\Gamma_{zv,o} + 4\Gamma_{iv,o}$ and $\Gamma_{\rm sym} = \Gamma_{\rm KM} + 2\Gamma_{zv,e} + 4\Gamma_{iv,e}$.

	Valley		
Spin	t_1	t_0	s
t_1	$\Gamma_* + \Gamma_{\rm KM} + 2\Gamma_{iv,e} + \Gamma_\alpha + 2\Gamma_{zv,o} + 4\Gamma_{iv,o}$	$2\Gamma_{iv} + \Gamma_{\rm KM} + 2\Gamma_{zv,e} + \Gamma_\alpha + 2\Gamma_{zv,o} + 4\Gamma_{iv,o}$	$\Gamma_{\rm SO}$
t_0	$\Gamma_* + 2(\Gamma_\alpha + \Gamma_{zv,e} + \Gamma_{iv,e} + 2\Gamma_{iv,o})$	$2(\Gamma_{iv} + \Gamma_\alpha + 2\Gamma_{zv,o} + 2\Gamma_{iv,e})$	$2\Gamma_{\rm asy}$
s	$\Gamma_* + 2(\Gamma_{zv,e} + 2\Gamma_{zv,o} + \Gamma_{iv,e} + 2\Gamma_{iv,o})$	$2(\Gamma_{iv} + 2\Gamma_{iv,e} + \Gamma_{iv,o})$	0

form [28],

$$\mathcal{H}_c = vp + \frac{\gamma}{p}[\boldsymbol{p} \times \boldsymbol{s}]_z - \mu p^2 \cos 3\varphi_{\boldsymbol{p}} \Pi_z, \tag{3}$$

and the matrix elements of disorder scattering read

$$V_{\boldsymbol{p}',\boldsymbol{p}} = {\rm e}^{-i\theta/2}\Bigg[\cos(\theta/2)\Bigg(u + \sum_{l,j=x,y,z} \beta_{lj}\Pi_l s_j\Bigg) + \sum_{i,i'=x,y,z} \varkappa_i(\alpha_{ii'}s_{i'} + u_{ii'}\Pi_{i'}) + u\frac{\varkappa_z}{\epsilon_{\rm F}}(\lambda s_z + \Delta_s \Pi_z)\Bigg]. \tag{4}$$

Here, $\epsilon_{\rm F}$ is the Fermi energy, $\theta = \varphi_{\boldsymbol{p}'} - \varphi_{\boldsymbol{p}}$ is the scattering angle, and we introduced a vector $\boldsymbol{\varkappa}(\boldsymbol{p}', \boldsymbol{p}) = [\cos\Phi, -\sin\Phi, i\sin(\theta/2)]$ where $\Phi = (\varphi_{\boldsymbol{p}'} + \varphi_{\boldsymbol{p}})/2$. The Kane-Mele term $\propto\lambda$ and the staggered-potential term $\propto\Delta_s$ in Eq. (1) mix the conduction- and valence-band states in each valley, which results in spin- and valley-dependent scattering corrections.

It follows from Eqs. (3) and (4) that the Hamiltonian in both valleys has the same form as that for spin-split massive electrons with angle-dependent scattering, so the only difference is the factor ${\rm e}^{-i\theta/2}$ in the scattering amplitude. Therefore the problem of WL in graphene with spin-orbit splitting is reduced to the problem of WL of massive electrons with Rashba splitting solved in Ref. [24]. For the Hamiltonian (3), the equation for the cooperon $\mathcal{C}(\boldsymbol{q})$ has the form [29]

$$\big\{Dq^2 + \Gamma_\phi + \Gamma + \Gamma_{\rm R}\big(\boldsymbol{S}^2 - S_z^2\big) + \sqrt{2\Gamma_{\rm R}\tau_{\rm tr}}[\boldsymbol{S}\times\boldsymbol{q}]_z v\big\}\mathcal{C}(\boldsymbol{q}) = 1. \tag{5}$$

Here, $\boldsymbol{q}$ is the generalized momentum of a double charge in the magnetic field, Γ_ϕ is the spin- and valley-independent dephasing rate, $D = v^2\tau_{\rm tr}/2$ is the diffusion coefficient, where $\tau_{\rm tr}^{-1} = \pi g u^2/(2\hbar)$ is the transport relaxation rate and $g = \epsilon_{\rm F}/(2\pi\hbar^2 v^2)$ is the density of states at the Fermi energy per spin per valley, Γ is the dephasing operator including the effect of warping from the last term in Eq. (3) but independent of the Rashba splitting, $\Gamma_{\rm R} = 2(\gamma/\hbar)^2\tau_{\rm tr}$ is the Rashba-term induced Dyakonov-Perel spin relaxation rate, and $\boldsymbol{S}$ is the operator of the total spin of two interfering particles. It is crucial that the bilinear in momentum and spin term $\propto[\boldsymbol{S}\times\boldsymbol{q}]_z$ is present in the cooperon equation which mixes different spin interference contributions into the conductivity. The $\Gamma_{\rm R}$-related terms in Eq. (5) mean that the spin-orbit splitting results in a spin-orbit vector potential $\boldsymbol{A}_{\rm SO} = (\gamma/ev)[\hat{\boldsymbol{z}} \times \boldsymbol{S}]$ which is not reduced to spin dephasing only [29].

There are valley and spin-singlet (s) and triplet (t_0, t_1) interference channels of WL where t_0 and t_1 correspond to spin/pseudospin z projections equal to zero or ±1, respectively [30]. Projecting the operator Γ in Eq. (5) onto these states [29], we obtain, in addition to Γ_ϕ, nine dephasing rates Γ_j^l for valley l and spin j channels with $l, j = s, t_0, t_1$ given in Table I [31]. Note that, by contrast with Ref. [22], $\Gamma_{\rm R}$ is not added to Γ_α because it enters into the cooperon equation (5) not only as quadratic but also as linear in $\boldsymbol{S}$ terms, and is an independent parameter of the theory.

Solving Eq. (5) in magnetic field $\boldsymbol{B}$ normal to the graphene layer we obtain the WL induced magnetoconductivity $\Delta\sigma = \sigma(B) - \sigma(0)$ in the following form [29],

$$\frac{\Delta\sigma}{\sigma_0} = -\sum_{l=t_1,t_0,s} c_l\left[\mathcal{F}_t\left(\frac{\mathcal{B}_\phi}{B}, \frac{\mathcal{B}_{\rm R}}{B}, \frac{\mathcal{B}_{t_1}^l}{B}, \frac{\mathcal{B}_{t_0}^l}{B}\right) - F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_s^l}\right)\right]. \tag{6}$$

Here, $\sigma_0 = e^2/(2\pi h)$, the common negative sign is caused by the Berry phase π of Dirac fermions, $\{\mathcal{B}_{\rm R}, \mathcal{B}_\phi, \mathcal{B}_j^l\} = \{\Gamma_{\rm R}, \Gamma_\phi, \Gamma_j^l\}\hbar/(4|e|D)$, and $c_{t_1} = 2$, $c_{t_0} = 1$, $c_s = -1$. The singlet contribution is expressed via the HLN function $F(x) = \psi(1/2 + 1/x) + \ln x$ with the digamma function $\psi(y)$. By contrast, the triplet contribution is given by a four-parametric function,

$$\mathcal{F}_t(b_\phi, b_{\rm R}, b_1, b_0) = \sum_{m=0,\pm}\big[u_m\psi(1/2 + b_\phi + \bar{b} - v_m) - u_m^{(0)}\ln\big(b_\phi + \bar{b} - v_m^{(0)}\big)\big] + \frac{1}{(b_\phi + b_{\rm R} + b_1)^2 - 1/4}. \tag{7}$$

Here,

$$\bar{b} = (b_0 + 2b_1)/3, \quad b_- = (b_0 - b_1)/3, \tag{8}$$

and the coefficients u_m, v_m are found by the method of Punnoose [32,33],

$$v_m = 2\delta\cos\left(\varphi + \frac{2\pi}{3}m\right), \quad \delta = \sqrt{-C/3},$$
$$\varphi = \frac{1}{3}\arccos\left(-\frac{G}{\delta^3}\right) - \frac{2\pi}{3}, \quad u_m = \frac{3v_m^2 + 4b_{\rm R}v_m + A}{\prod_{m'\neq m}(v_m - v_{m'})}, \tag{9}$$

where

$$A = 5b_{\mathrm{R}}^2 + 4b_{\mathrm{R}}(b_\phi + \bar{b}) - 1 - b_-(3b_- + 2b_{\mathrm{R}}),$$
$$C = A - 4b_{\mathrm{R}}(b_{\mathrm{R}} - b_-),$$
$$G = 2b_{\mathrm{R}}(b_\phi + \bar{b})(b_{\mathrm{R}} - b_-) + b_{\mathrm{R}}^3 - b_-(b_{\mathrm{R}}b_- - b_-^2 + b_{\mathrm{R}}^2 + 1). \tag{10}$$

The coefficients $v_m^{(0)}$ and $u_m^{(0)}$ are calculated by Eqs. (9) with the zero-field asymptotes of A, C, and G given by $A^{(0)} = A + 1$, $C^{(0)} = C + 1$, $G^{(0)} = G + b_-$. The coefficients v_m depend on magnetic field, and hence their zero-field asymptotes $v_m^{(0)}$ are different from v_m. The same is true for the coefficients u_m. Therefore Eq. (7) does not reduce to the HLN-like expression.

Discussion. In the absence of Rashba splitting, $\mathcal{B}_{\mathrm{R}} = 0$, we have $A = C = -1 - 3b_-^2$ and $G = b_-(b_-^2 - 1)$. This yields $u_{0,\pm} = 1$, $\{v_0, v_+, v_-\} = \{b_- - 1, b_- + 1, -2b_-\}$, hence the triplet contribution is simplified to $\mathcal{F}_t(b_\phi, 0, b_1, b_0) = 2F[(b_\phi + b_1)^{-1}] + F[(b_\phi + b_0)^{-1}]$, and we obtain the HLN-like expression

$$\left.\frac{\Delta\sigma}{\sigma_0}\right|_{\mathcal{B}_{\mathrm{R}}=0} = -\sum_{l=t_1,t_0,s} c_l\left[2F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_{t_1}^l}\right) + F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_{t_0}^l}\right) - F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_s^l}\right)\right]. \tag{11}$$

Let us consider a limit of fast valley-triplet relaxation: $\Gamma_j^{t_{1,0}} \gg \Gamma_j^s$. Then the anomalous magnetoresistance is described by just four dephasing rates, Γ_{R}, Γ_ϕ, Γ_{SO}, and Γ_{asy},

$$\left.\frac{\Delta\sigma}{\sigma_0}\right|_{\Gamma_j^{t_{1,0}} \gg \Gamma_j^s} = \mathcal{F}_t\left(\frac{\mathcal{B}_\phi}{B}, \frac{\mathcal{B}_{\mathrm{R}}}{B}, \frac{\mathcal{B}_{\mathrm{SO}}}{B}, \frac{\mathcal{B}_{\mathrm{asy}}}{B}\right) - F\left(\frac{B}{\mathcal{B}_\phi}\right), \tag{12}$$

where $\mathcal{B}_{\mathrm{SO}} = \Gamma_{\mathrm{SO}}\hbar/(4|e|D)$ and $\mathcal{B}_{\mathrm{asy}} = \Gamma_{\mathrm{asy}}\hbar/(2|e|D)$. If, in addition, $\mathcal{B}_{\mathrm{R}} = 0$, then, combining two previous equations, we obtain the expression of Ref. [22],

$$\left.\frac{\Delta\sigma(B)}{\sigma_0}\right|_{\mathcal{B}_{\mathrm{R}}=0,\Gamma_j^{t_{1,0}} \gg \Gamma_j^s} = 2F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_{\mathrm{SO}}}\right) + F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_{\mathrm{asy}}}\right) - F\left(\frac{B}{\mathcal{B}_\phi}\right). \tag{13}$$

In Ref. [22], McCann and Fal'ko (MF) derived an expression, which, in the absence of spin-orbit scattering but in the presence of Rashba splitting, gives

$$\frac{\Delta\sigma_{\mathrm{MF}}}{\sigma_0} = 2F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_{\mathrm{R}}}\right) + F\left(\frac{B}{\mathcal{B}_\phi + 2\mathcal{B}_{\mathrm{R}}}\right) - F\left(\frac{B}{\mathcal{B}_\phi}\right). \tag{14}$$

This expression treats the Rashba spin-orbit coupling as dephasing only and ignores the effect of the linear in $\boldsymbol{S}$ terms in the cooperon equation. Theory developed in the present Letter gives for this case Eq. (12), where $\mathcal{B}_{\mathrm{SO}} = \mathcal{B}_{\mathrm{asy}} = 0$. In Fig. 1(a) we compare these two expressions, demonstrating that they differ significantly. The MF expression is correct either in the absence of spin-orbit coupling ($\mathcal{B}_{\mathrm{R}} = 0$) or when it is very large and the triplet contribution is negligible

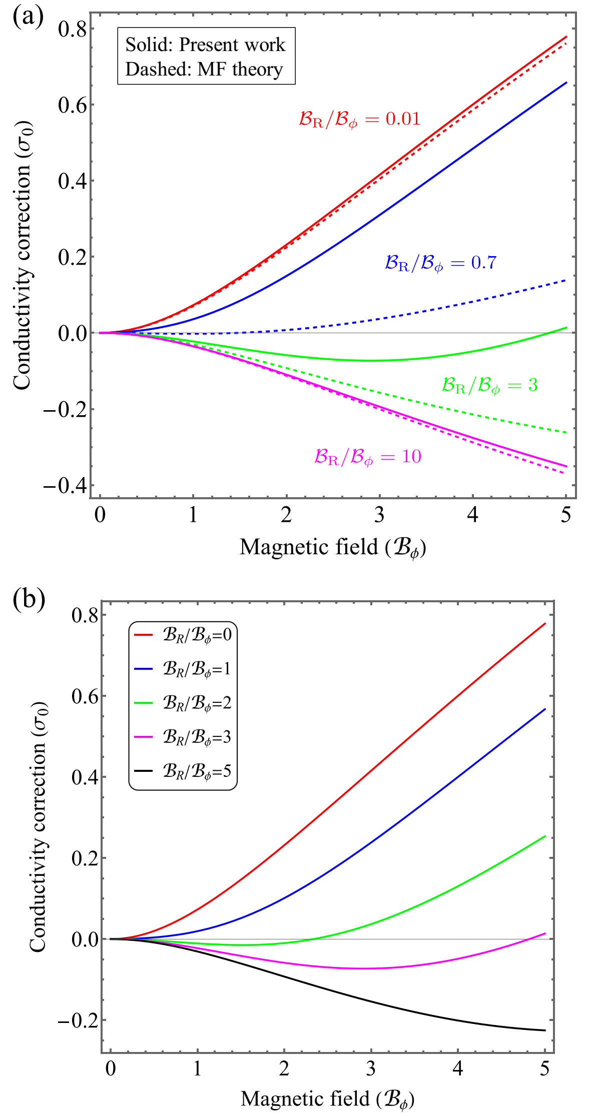

FIG. 1. Conductivity correction in the absence of spin-orbit scattering. (a) Present work, Eq. (12) (solid lines), and MF theory, Eq. (14) (dashed). (b) Equation (12) at different values of $\mathcal{B}_{\mathrm{R}}/\mathcal{B}_\phi$.

($\mathcal{B}_{\mathrm{R}} \gg \mathcal{B}_\phi$). At intermediate values of the ratio $\mathcal{B}_{\mathrm{R}}/\mathcal{B}_\phi$, the expression derived in the present Letter shows that a strong WAL effect is already present at moderate Rashba splitting when $\mathcal{B}_{\mathrm{R}} \geqslant 3\mathcal{B}_\phi$. Comparing two blue curves or two green curves in Fig. 1(a), we see that these two theories give very different results for the same Rashba splitting. Furthermore, if we compare the blue dashed and green solid curves, we see that they are close to each other. This means that they can both fit some data when the experimental points are between them. However, the extracted Rashba magnetic field $\mathcal{B}_{\mathrm{R}}$ is more than four times different when the HLN-like expression is used instead of the correct formula presented in this Letter. In Fig. 1(b) we show the magnetoconductivity in the absence of spin-orbit scattering at different values of the Rashba splitting.

The effect of spin-orbit scattering is demonstrated in Fig. 2. In the presence of a moderate Rashba splitting $\mathcal{B}_{\mathrm{R}} = 2\mathcal{B}_\phi$, both symmetrical and asymmetrical spin-orbit scattering processes result in a more antilocalizing behavior of the

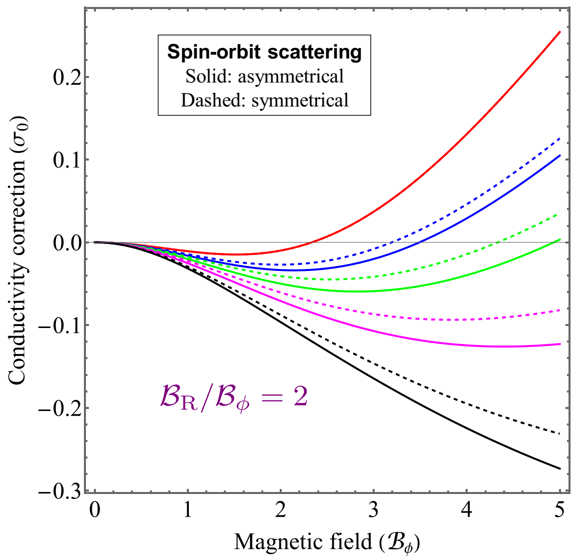

FIG. 2. Effect of spin-orbit scattering at $\mathcal{B}_R = 2\mathcal{B}_\phi$. Solid (dashed) lines: $\mathcal{B}_{asy}(\mathcal{B}_{sym}) = 0.5\mathcal{B}_\phi$ (blue), $\mathcal{B}_\phi$ (green), $2\mathcal{B}_\phi$ (magenta), and $5\mathcal{B}_\phi$ (black). Red: $\mathcal{B}_{asy} = \mathcal{B}_{sym} = 0$.

magnetoresistance. We considered asymmetrical ($\mathcal{B}_{sym} = 0$, $\mathcal{B}_{SO} = \mathcal{B}_{asy}/2$) and symmetrical ($\mathcal{B}_{asy} = 0$, $\mathcal{B}_{SO} = \mathcal{B}_{sym}$) scattering. Figure 2 shows that the effect of the asymmetrical scattering is stronger.

In an in-plane magnetic field, the Zeeman term $(\epsilon_Z/2)\boldsymbol{s}\cdot\boldsymbol{l}_\parallel$ is added to the Hamiltonian, where ϵ_Z is the Zeeman splitting and $\boldsymbol{l}_\parallel$ is a unit vector in the direction of the field. It results in a mixing of singlet and triplet spin cooperons in addition to the pure triplet mixing by the spin-orbit vector potential [34,35]. A WL conductivity correction calculation in the presence of both Rashba and in-plane Zeeman splittings [2] showed that the conductivity as a function of an in-plane magnetic field has a maximum when the Zeeman and Rashba splittings are equal, $\epsilon_Z \approx 2\gamma$. In moderate fields where the Zeeman splitting is smaller than the spin relaxation gaps, $\epsilon_Z \ll \hbar\Gamma_{R,SO}$, the effect of the parallel field mainly consists in a suppression of the spin-singlet interference channel [34,35]. For graphene with a spin-orbit interaction, this Zeeman splitting induced dephasing rate is $\Gamma_s^s = (\epsilon_Z/\hbar)^2/(\Gamma_R + \Gamma_{SO})$, and Eq. (12) yields

$$\frac{\Delta\sigma}{\sigma_0} = \mathcal{F}_t\left(\frac{\mathcal{B}_\phi}{B}, \frac{\mathcal{B}_R}{B}, \frac{\mathcal{B}_{SO}}{B}, \frac{\mathcal{B}_{asy}}{B}\right) - F\left(\frac{B}{\mathcal{B}_\phi + \mathcal{B}_s^s}\right), \quad (15)$$

where we again assumed valley-triplet channels to be suppressed, $\Gamma_j^{t_{1,0}} \gg \Gamma_j^s$. In the opposite limit of large Zeeman splitting $\epsilon_Z \gg \gamma$, Dyakonov-Perel spin relaxation is suppressed, and the magnetoconductivity is given by the expression with $\Gamma_R \to 0$, Eq. (13).

The valley-Zeeman term $\lambda_{VZ}\Pi_z s_z$ [17,26] makes a similar effect if intervalley scattering is ineffective. The spin-singlet dephasing rate $\Gamma_s \sim (2\lambda_{VZ}/\hbar)^2/(\Gamma_R + \Gamma_{SO})$ appears for each valley resulting in the magnetoconductivity given by Eq. (15) multiplied by a factor −2. In the opposite limit of comparable inter- and intravalley scattering efficiencies the valley-Zeeman splitting has no effect on WL.

The pseudospin inversion asymmetry terms in the Hamiltonian that are bilinear in spin and momentum [9,11] result in small renormalizations of the Rashba constant and the spin relaxation rate Γ_α.

The theory presented above gives WL magnetoconductivity in magnetic fields smaller than the "transport" field $\mathcal{B}_{tr} = \hbar/(4eD\tau_{tr})$. In higher fields $B \sim \mathcal{B}_{tr} \gg \mathcal{B}_\phi$, interference on ballistic trajectories with a few number of scatterers is important in WL. In this case, the nondiffusive theory can be developed accounting for both the Dirac fermion nature of carriers [36,37] and the spin-orbit coupling [2] as well as valley-Zeeman splitting of arbitrary strengths.

In bilayer graphene with spin-orbit coupling, the spin-orbit vector potential in Eq. (5) is quadratic in q. Therefore it has no significant effect on the anomalous magnetoresistance, and the HLN-like theory [21] is correct. The same is true for WL in TMDC layers where the spin-orbit vector potential is absent due to the lack of linear in momentum terms in the Hamiltonian [38].

Conclusion. The developed theory of WL in graphene accounts for the Rashba spin-orbit splitting, and spin-orbit and valley-dependent scattering. It is shown that the Rashba interaction affects WL in graphene not via spin dephasing but via a spin-orbit vector potential. This results in the expression for the anomalous magnetoconductivity which is not reduced to the traditional formulas with HLN functions. The importance of this difference is demonstrated. The present theory allows one to determine adequately the spin-orbit parameters of graphene with spin-orbit interaction from experimental data.

Acknowledgments. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) via Project-ID 314695032 – SFB 1277 (Subproject No. A04) and the Volkswagen Stiftung Program (97738).

[1] G. Bergmann, Weak localization in thin films: a time-of-flight experiment with conduction electrons, Phys. Rep. **107**, 1 (1984).

[2] M. M. Glazov and L. E. Golub, Spin–orbit interaction and weak localization in heterostructures, Semicond. Sci. Technol. **24**, 064007 (2009).

[3] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, The electronic properties of graphene, Rev. Mod. Phys. **81**, 109 (2009).

[4] D. V. Khveshchenko, Electron localization properties in graphene, Phys. Rev. Lett. **97**, 036802 (2006).

[5] E. McCann, K. Kechedzhi, V. I. Fal'ko, H. Suzuura, T. Ando, and B. L. Altshuler, Weak-localization magnetoresistance and valley symmetry in graphene, Phys. Rev. Lett. **97**, 146805 (2006).

[6] M. Gmitra and J. Fabian, Graphene on transition-metal dichalcogenides: A platform for proximity spin-orbit physics and optospintronics, Phys. Rev. B **92**, 155403 (2015).

[7] A. Avsar, H. Ochoa, F. Guinea, B. Özyilmaz, B. J. van Wees, and I. J. Vera-Marun, *Colloquium*: Spintronics in graphene and other two-dimensional materials, Rev. Mod. Phys. **92**, 021003 (2020).

[8] E. I. Rashba, Properties of semiconductors with an extremum loop. I. Cyclotron and combinational resonance in a magnetic field perpendicular to the plane of the loop, Sov. Phys. Solid State **2**, 1109 (1960).

[9] M. Gmitra, D. Kochan, P. Högl, and J. Fabian, Trivial and inverted Dirac bands and the emergence of quantum spin Hall states in graphene on transition-metal dichalcogenides, Phys. Rev. B **93**, 155104 (2016).

[10] A. W. Cummings, J. H. Garcia, J. Fabian, and S. Roche, Giant spin lifetime anisotropy in graphene induced by proximity effects, Phys. Rev. Lett. **119**, 206601 (2017).

[11] K. Zollner, M. Gmitra, and J. Fabian, Heterostructures of graphene and hBN: Electronic, spin-orbit, and spin relaxation properties from first principles, Phys. Rev. B **99**, 125151 (2019).

[12] Y. Li and M. Koshino, Twist-angle dependence of the proximity spin-orbit coupling in graphene on transition-metal dichalcogenides, Phys. Rev. B **99**, 075438 (2019).

[13] T. Naimer and J. Fabian, Twist-angle dependent proximity induced spin-orbit coupling in graphene/topological insulator heterostructures, Phys. Rev. B **107**, 195144 (2023).

[14] Z. Wang, D.-K. Ki, H. Chen, H. Berger, A. H. MacDonald, and A. F. Morpurgo, Strong interface-induced spin–orbit interaction in graphene on WS_2, Nat. Commun. **6**, 8339 (2015).

[15] Z. Wang, D.-K. Ki, J. Y. Khoo, D. Mauro, H. Berger, L. S. Levitov, and A. F. Morpurgo, Origin and magnitude of 'designer' spin-orbit interaction in graphene on semiconducting transition metal dichalcogenides, Phys. Rev. X **6**, 041020 (2016).

[16] T. Völkl, T. Rockinger, M. Drienovsky, K. Watanabe, T. Taniguchi, D. Weiss, and J. Eroms, Magnetotransport in heterostructures of transition metal dichalcogenides and graphene, Phys. Rev. B **96**, 125405 (2017).

[17] S. Zihlmann, A. W. Cummings, J. H. Garcia, M. Kedves, K. Watanabe, T. Taniguchi, C. Schönenberger, and P. Makk, Large spin relaxation anisotropy and valley-Zeeman spin-orbit coupling in WSe_2/graphene/h-BN heterostructures, Phys. Rev. B **97**, 075434 (2018).

[18] T. Wakamura, F. Reale, P. Palczynski, S. Guéron, C. Mattevi, and H. Bouchiat, Strong anisotropic spin-orbit interaction induced in graphene by monolayer WS_2, Phys. Rev. Lett. **120**, 106802 (2018).

[19] T. Wakamura, F. Reale, P. Palczynski, M. Q. Zhao, A. T. C. Johnson, S. Guéron, C. Mattevi, A. Ouerghi, and H. Bouchiat, Spin-orbit interaction induced in graphene by transition metal dichalcogenides, Phys. Rev. B **99**, 245402 (2019).

[20] T. Wakamura, S. Guéron, and H. Bouchiat, Novel transport phenomena in graphene induced by strong spin-orbit interaction, C. R. Phys. **22**, 145 (2022).

[21] J. Amann, T. Völkl, T. Rockinger, D. Kochan, K. Watanabe, T. Taniguchi, J. Fabian, D. Weiss, and J. Eroms, Counterintuitive gate dependence of weak antilocalization in bilayer graphene/WSe_2 heterostructures, Phys. Rev. B **105**, 115425 (2022).

[22] E. McCann and V. I. Fal'ko, $z \to -z$ symmetry of spin-orbit coupling and weak localization in graphene, Phys. Rev. Lett. **108**, 166606 (2012).

[23] S. Hikami, A. I. Larkin, and Y. Nagaoka, Spin-orbit interaction and magnetoresistance in the two dimensional random system, Prog. Theor. Phys. **63**, 707 (1980).

[24] S. V. Iordanskii, Y. B. Lyanda-Geller, and G. E. Pikus, Weak localization in quantum wells with spin-orbit interaction, JETP Lett. **60**, 206 (1994).

[25] W. Knap, C. Skierbiszewski, A. Zduniak, E. Litwin-Staszewska, D. Bertho, F. Kobbi, J. L. Robert, G. E. Pikus, F. G. Pikus, S. V. Iordanskii, V. Mosser, K. Zekentes, and Y. B. Lyanda-Geller, Weak antilocalization and spin precession in quantum wells, Phys. Rev. B **53**, 3912 (1996).

[26] M. Offidani and A. Ferreira, Microscopic theory of spin relaxation anisotropy in graphene with proximity-induced spin-orbit coupling, Phys. Rev. B **98**, 245408 (2018).

[27] H. Ochoa, A. H. Castro Neto, V. I. Fal'ko, and F. Guinea, Spin-orbit coupling assisted by flexural phonons in graphene, Phys. Rev. B **86**, 245411 (2012).

[28] S. Ilić, J. S. Meyer, and M. Houzet, Weak localization in transition metal dichalcogenide monolayers and their heterostructures with graphene, Phys. Rev. B **99**, 205407 (2019).

[29] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.109.L121407 for details of derivations and additional analysis, which includes Ref. [39].

[30] F. Sousa, D. T. S. Perkins, and A. Ferreira, Weak localisation driven by pseudospin-spin entanglement, Commun. Phys. **5**, 291 (2022).

[31] In the notations of Ref. [22], $\Gamma^l_j = \Gamma^m_n$ where $l, j = s, t_0, t_1$ correspond to $m, n = 0, z, x$.

[32] A. Punnoose, Magnetoconductivity in the presence of Bychkov-Rashba spin-orbit interaction, Appl. Phys. Lett. **88**, 252113 (2006).

[33] S. Schmult, M. J. Manfra, A. Punnoose, A. M. Sergent, K. W. Baldwin, and R. J. Molnar, Large Bychkov-Rashba spin-orbit coupling in high-mobility $GaN/Al_xGa_{1-x}N$ heterostructures, Phys. Rev. B **74**, 033302 (2006).

[34] A. G. Mal'shukov, K. A. Chao, and M. Willander, Magnetoresistance of a weakly disordered III-V semiconductor quantum well in a magnetic field parallel to interfaces, Phys. Rev. B **56**, 6436 (1997).

[35] G. M. Minkov, A. V. Germanenko, O. E. Rut, A. A. Sherstobitov, L. E. Golub, B. N. Zvonkov, and M. Willander, Weak antilocalization in quantum wells in tilted magnetic fields, Phys. Rev. B **70**, 155323 (2004).

[36] I. V. Gornyi, V. Y. Kachorovskii, and P. M. Ostrovsky, Interference-induced magnetoresistance in HgTe quantum wells, Phys. Rev. B **90**, 085401 (2014).

[37] M. O. Nestoklon and N. S. Averkiev, Nondiffusion theory of weak localization magnetoresistance in graphene, Phys. Rev. B **90**, 155412 (2014).

[38] H. Ochoa, F. Finocchiaro, F. Guinea, and V. I. Fal'ko, Spin-valley relaxation and quantum transport regimes in two-dimensional transition-metal dichalcogenides, Phys. Rev. B **90**, 235429 (2014).

[39] N. S. Averkiev, L. E. Golub, and G. E. Pikus, Weak localization in p-type quantum wells, Semiconductors **32**, 1087 (1998).