Janus graphene: A two-dimensional half-auxetic carbon allotrope with a nonchemical Janus configuration

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(Received 17 July 2023; revised 23 December 2023; accepted 22 January 2024; published 11 March 2024)

We propose a nonchemical intrinsic Janus configuration and identify it in a pure sp^2 hybridized carbon monolayer, termed Janus graphene. The spontaneous unilateral growth of carbon atoms drives the nonchemical Janus configuration in Janus graphene, which is totally different from the chemical effect in common Janus materials such as MoSSe. A structure-independent half-auxetic behavior is mapped in Janus graphene, that the structure maintains expansion whether stretched or compressed, which lies in the key role of the p_z orbital. With the unique half-auxeticity that emerged in the nonchemical Janus configuration, Janus graphene enriches the functional carbon family as a promising candidate for micro/nanoelectronic device applications.

DOI: 10.1103/PhysRevB.109.L121402

Introduction. The carbon element has an extremely strong bonding ability and abundant bonding forms in chemical reactions, such as linear alkyne (sp), planar olefin (sp^2) , and tetrahedral alkane (sp^3) bonds. The superior performance of graphene, such as superconducting [1–3] and quantum Hall effect [4,5], motivate researchers to extend their interest from three-dimensional to two-dimensional (2D) carbon crystals. For example, experimental preparations have been conducted for monolayer amorphous carbon [6], biphenylene [7], and monolayer fullerene [8]. Furthermore, the initial prediction of bulk T-carbon was made in 2011 [9], followed by its experimental synthesis in 2017 [10]. The synthesis and discovery of these promising carbon configurations has encouraged the designing and screening of multifunctional carbon monolayers with specific technological interests.

As an emerging class of nanomaterials, Janus nanostructures are functional structures with different properties composed of two materials (or different elements) on the same object, driven by chemical effects. "Janus bead" was first proposed by the French scientist Casagrande in 1989 to describe the unique structure of semihydrophobic and semihydrophilic materials on the surface of spherical particles [11]. Then, in 1991, the name "Janus" of the two-faced god in Greek mythology was first proposed by the French physicist Gennes to describe particles with dual properties in his Nobel Prize speech [12]. Subsequently, Janus materials have been widely used in many fields, such as biology, energy, chemistry, and physics, due to their unique asymmetry [13-21].

2D Janus atomic crystals are monolayer materials with structural symmetry breaking caused by chemical effects. Tremendous effort has been devoted to designing and synthesizing the 2D Janus material with chemisorption [22,23], atomic substitution [17], and stacked heterojunctions [24], facilitating the generation of many 2D nanomaterials with unique physical properties, such as MoSSe [17]. Note that the Janus properties of current Janus materials depend on chemical effects, i.e., different adatoms, functional groups, or elemental effects. Currently, the preparation of Janus materials relies on chemical interactions, including atomic substitution [17,25,26], chemisorption [22,23,27,28], and heterojunction [24,29,30], which divides Janus materials into three types as shown in Fig. S1 of the Supplemental Material [31]: (1) The element-type Janus structures, where different elements are arranged along the out-of-plane direction, such as 2H-MoSSe [17]; (2) the adatom-type Janus structures, which are formed by the adsorption of different atoms on both sides along the out-of-plane sides, such as graphenelike Janus materials [32]; (3) the heterojunction-type Janus structures, which are van der Waals layered compounds formed by different types of monolayers along out-of-plane directions, such as graphene/MgX (X=S, Se), and graphene/BN [33–36]. However, intrinsic carbon materials, where only carbon atoms exist, can hardly achieve Janus properties without extrinsic chemical effects. Such a problem inspires us to design 2D Janus carbon materials with unique symmetry breaking using advanced crystal design methods.

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FIG. 1. Janus graphene and its evolution. (a) Two-dimensional carbon structure gene bank. (b) Two-dimensional carbon structure constructed by carbon gene recombination. (c) Comparison of Janus and non-Janus phases. (d) The phonon dispersion and phonon density of states (DOS) of Janus graphene.

This Letter proposes an intrinsic nonchemical Janus configuration in purely sp^2 hybridized carbon monolayer, named as Janus graphene, which is totally different from the previously reported Janus structures driven by chemical effects. An emerging half-auxetic property is found in the pure sp^2 hybrid Janus graphene, which breaks the notion that pure sp^2 -hybridized carbon monolayers cannot exhibit intrinsic negative Poisson's ratio (NPR) behavior. The successfully designed Janus graphene exhibits its unique Janus structural and unconventional half-auxeticity, which would be a strong candidate for the applications of carbon materials in micro/nano electronic devices.

Carbon gene recombination. Based on 2D carbon geometry features, the carbon gene recombination strategy can be proposed here to design 2D carbon with different configurations, which maps the geometric properties into quotient maps for recombination and screening of 2D carbon structures. Figure S2(a) demonstrates a schematic diagram of the structure identification process, and the initial idea of the strategy is to build a carbon gene bank according to the bonding characteristics of carbon materials. According to the number of atoms, 2D carbon genes can be divided into monoatomic chain, diatomic chain, triangular ring, tetragonal ring, pentagonal ring, hexagonal ring, etc., as shown in Fig. 1(a). In the xy plane, 2D carbon can be broken down into these carbon genes, and likewise, it can also be reconstituted to form other carbon monolayers. Among them, four hexagonal rings are reconstructed along the tetragonal rings, and then octagonal rings are introduced by periodic mirroring in the tetragonal lattice [Fig. S2(b)], forming a carbon monolayer with sp^2 hybridization, i.e., Janus graphene, as shown in Figs. 1(b) and 1(c). Such evolution can also be used to reconstruct other 2D carbon configurations [37,38] with specific carbon genes

[Fig. 1(b)]. Phonon dispersion visualizes lattice vibrations in the Brillouin zone, reflecting the dynamic stability of Janus graphene due to all positive frequencies as shown in Fig. 1(d). More computational details as well as structural and stability insights can be found in Supplemental Notes S1, S2 and Figs. S1, S2, and S3 [31].

Nonchemical Janus structure. The primitive cell of the optimized structure contains 12 atoms with the space group of P4mm (99). Interestingly, this 2D carbon allotrope protrudes along one side of out of plane but exhibits planar features on the other side, which symbolizes the two-faced god Janus in ancient Roman mythology, hence the name "Janus graphene." In carbon material, graphene can be formed with the above-mentioned Janus carbon materials through asymmetric chemistry [22,23,27,28], but usually leads to some irreversible changes in physical and chemical properties, such as the destruction of the Dirac cone [22]. Unlike chemically driven Janus materials, the Janus configuration of Janus graphene is natural and intrinsic, as it purely consists of carbon atoms. Thus, it is considered as a undiscovered class of intrinsic Janus structure type. Unlike the Janus material summarized in Fig. S1, the intrinsic Janus structure relies on an uneven distribution of the structure in space, i.e., the carbon atoms break the plane symmetry through an inhomogeneous arrangement to form the Janus phase, which leads to bifunctionality on both sides in the out-of-plane direction.

Emerging half-auxeticity. The unique Janus configuration brings extraordinary structural evolution performance, especially the auxetic behavior. Diverse auxetic behaviors are found in Janus graphene compared to typical graphene and penta graphene. Graphene exhibits a total positive Poisson's ratio (Total-PPR) behavior [Fig. 2(a)] in that it contracts (stretches) in orthogonal directions when stretched (compressed), consistent with previous study [39]. Unlike graphene, a total negative Poisson's ratio (Total-NPR) behavior is found in penta graphene [Fig. 2(b)], i.e., it exhibits stretch (compression) behavior when stretched (compressed) in orthogonal directions [40]. Unusually, Janus graphene [Fig. 2(c)] always expands regardless of whether it is stretched or compressed, i.e., half-negative Poisson's ratio (Half-NPR) behavior. Schematic representations of diverse Poisson's ratio behaviors are further plotted in Figs. 2(d) and 2(e) to understand their differences, i.e., total-PPR, total-NPR, and half-NPR. The in-plane half-NPR behavior was first proposed by Ma et al., but so far has only been found in borides [41,42]. The scarcity and magic of the half-NPR behavior in Janus graphene contrasts with its mono [Fig. 2(g)], binary [Fig. 2(h)], and ternary [Fig. 2(i)] counterparts. Unlike traditional auxetic behavior, that the material expands (shrinks) when stretched (compressed), half-auxetic behavior reveals a mechanical phenomenon whereby materials always expand, whether stretched or compressed. The current auxetic behavior is only found in the sp^3 (or sp^2-sp^3)-hybridized configuration in carbon materials [43]. We searched the 108 carbon structures included in the 2D carbon database [44] and the 1114 sp^2 -hybridized 2D carbon structures screened by Shi et al. [45], but found no auxetic behavior reported in the pure sp^2 -hybridized carbon configuration. The reason lies in that the strong sp^2 hybridization favors the graphenelike planar structure, which hinders the generation of the reentry



FIG. 2. Auxetic behavior of two-dimensional carbon. Strain curves and schematic diagrams representing different Poisson's ratio behaviors for (a) graphene with a (d) total positive Poisson's ratio (Total-PPR) behavior, (b) penta graphene with a (e) total negative Poisson's ratio (Total-NPR) behavior, and (c) Janus graphene with a (f) half-negative Poisson's ratio (Half-NPR) behavior. Comparison of Poisson's ratio behavior of Janus graphene with other two-dimensional Janus compounds, including (g) unary, (h) binary, and (i) ternary phases.

mechanism that induces NPR behavior. Here, the unique geometric evolution enables the NPR behavior to be extended to pure sp^2 -hybridized carbon configurations, and this in-plane half-auxetic behavior is revealed in carbon materials. In addition, we also show the stress-strain curve diagram of Janus graphene under uniaxial strain in Fig. S4 in the Supplemental Material.

The role of the p_z orbital. Before closing, the electronic properties of Janus graphene are investigated to further understand the emerging features from the intrinsic Janus structure, as shown in Fig. 3. The precise HSE06 functional identifies a wide bandgap of 2.21 eV for Janus graphene, higher than the 1.16 eV of the Perdew-Burke-Ernzerhof functional as shown

in Fig. 3(a) and Supplemental Material Note S3. Compared with graphene, the valence-band maximum (VBM) and the conduction-band minimum (CBM) mainly contributed by the p_z orbital are not at the same high-symmetry point, where the p_z orbital plays a major role for bonded states (Supplemental Material Note S3 and Fig. S5). The three-dimensional images of VBM and CBM are further revealed by Fig. 3(b). In Janus graphene, the deviation of the out-of-plane π bonds in space in the vertical direction leads to the breaking of out-of-plane symmetry. In the planar phase, the intrinsic Poisson's ratio is generally positive because the electrons of the p_z orbitals are coupled in the large ring-shaped π bonds and cannot be easily activated by the force field to generate a reentrant mechanism



FIG. 3. Electronic properties of Janus graphene. (a) Electronic bandgap of Janus graphene. (b) Three-dimensional (3D) valence-band maximum (VBM) and the conduction-band minimum (CBM) in Janus graphene. (c) The 3D and 2D electron localization function (ELF) of Janus graphene.

due to the robust π bonds and symmetric planar structure. Typically, planar graphene often needs to apply a strain field of more than 15% to activate the reentry mechanism to achieve NPR [39,46], which is usually unacceptable in practical operations. As shown in Fig. 3(c), the covalent electrons in the C-C bond in Janus graphene are unevenly distributed along the out-of-plane direction, i.e., strong asymmetry. This leads to intrinsic Janus properties in the structure and makes the reentry mechanism easier to activate due to the more isolated p_z electrons. During the structure evolution, the p_z electrons in Janus grpahene deviate from the planar state and become more sensitive to the external force field. Under a stretching force field, it is forced to couple in the in-plane direction, producing an anomalous response in geometrical angles (β response in Supplemental Material Note S4 and Figs. S6, S7 and S8), exciting potential reentry mechanisms, and finally leading to NPR. The unique orbital evolution of Janus graphene brings it nonchemical Janus properties and unconventional NPR effects in terms of geometric and mechanical responses, making it an excellent platform for studying structural and functional linkages.

Conclusion. In summary, we proposed an unprecedented nonchemical intrinsic Janus configuration concept, and engineered a pure sp^2 hybridized 2D carbon allotrope with this configuration. This carbon allotrope exhibits double-faced features similar to the Roman god Janus, hence named as Janus graphene. Different from the traditional Janus configuration, the Janus configuration of Janus graphene does not depend on chemical effects, which is formed by the carbon

atoms growth towards the out-of-plane unilateral side of the 2D plane. The unique Janus geometry maps the unconventional half-auxetic property that the Janus graphene always expands whether it is stretched or compressed. This half-auxetic property shows a structure-independence feature. We extend this auxeticity to pure sp^2 -hybridized configurations and identify its origin in the structure's diverse response to strain. The unique Janus structure and unconventional half-auxetic properties combined with widebandgap semiconducting properties endow Janus graphene with multifunction, providing a strong candidate for micronanoelectronic devices.

Acknowledgments. This work is supported by the National Natural Science Foundation of China (Grant No. 52006057), the Natural Science Foundation of Chongqing, China (Grant No. CSTB2022NSCQ-MSX0332), the Fundamental Research Funds for the Central Universities (Grant No. 531119200237), and the State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body at Hunan University (Grant No. 52175013). H.W. is supported by the National Natural Science Foundation of China (Grant No. 51906097). Z.Q. is supported by the National Natural Science Foundation of China (Grants Nos.12274374 and No. 11904324) and the China Postdoctoral Science Foundation (2018M642776).

G.Q. supervised the project. L.Y. performed all the calculations, analysis, and writing. All the authors contributed to interpreting the results. The Letter was written by L.Y. with contributions from all the authors.

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