Letter

Transverse quantum fluids

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Motivated by the remarkable properties of superfluid edge dislocations in ^{4}He, we discuss a broad class of quantum systems—boundaries in phase-separated lattice states, magnetic domain walls, and ensembles of Luttinger liquids—that can be classified as transverse quantum fluids (TQFs). After introducing the general idea of a TQF, we focus on a coupled array of Luttinger liquids forming an incoherent TQF. This state is a long-range ordered quasi-one-dimensional superfluid, topologically protected against quantum phase slips by the tight binding of instanton dipoles, that has no coherent quasiparticle excitations at low energies. An incoherent TQF is a striking example of the irrelevance of the Landau quasiparticle criterion for superfluidity in systems that lack Galilean invariance. We detail its phenomenology, to motivate a number of experimental studies in condensed matter and cold atomic systems.

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Introduction and motivation. In the last two decades, there has been much attention on systems where nontrivial gapless physics is confined to a surface of a bulk material, with such prominent examples as topological insulators [1] and their symmetry-protected topological generalizations [2]. In a broader sense, these are examples of a large class of low-dimensional systems in which the surrounding bulk host is fundamentally important for understanding their properties. Such systems include randomly pinned surface Goldstone modes [3,4], an edge state of a confinement-free quantum Hall droplet [5], a superfluid interface between two checkerboard-solid domains [6], as well as open spin/particle chains coupled to a dissipative bath [7–16].

A particularly interesting system studied recently is an edge dislocation with a superfluid core [17–19] in solid ^{4}He where the low-temperature motion of the dislocation transverse to its core and Burgers vector (the so-called climb) requires boson number flow onto the core. The corresponding superfluid state of the edge dislocation has been dubbed as a transverse quantum fluid (TQF), a new one-dimensional state of bosons qualitatively distinct from a Luttinger liquid (LL) with (i) a quadratic spectrum of excitations, (ii) off-diagonal long-range order at $T = 0$, (iii) exponential dependence of the phase slip probability on the inverse flow velocity, and (iv) nonapplicability of the Landau criterion. The key ingredient responsible for all these features is the translational invariance with respect to the core motion in the climb direction, necessarily accompanied by atom transfer to the ^{4}He crystal bulk. Most importantly, this implies *infinite compressibility*, which is solely responsible for properties (i)–(iii).

In this Letter, we argue that similar—infinite-compressibility-driven—phenomenology emerges in other interesting examples and discuss four specific systems that we predict to host a TQF. These are (A) a self-bound droplet of hard-core bosons on a two-dimensional (2D) lattice, (B) a Bloch domain wall in an easy-axis ferromagnet, (C) a phase separated state of two-component bosonic Mott insulators with the boundary in the countersuperfluid phase [20] (or in a phase of a two-component superfluid) on a 2D lattice, as illustrated in Fig. 1, and (D) a 1D bosonic liquid, Josephson coupled to a collection of transverse LLs, which are otherwise decoupled from each other—a setup in Fig. 2, similar to the one considered in Ref. [13], and also related closely to a number of other setups and models discussed in the past [8–11]. While A, B, and C are naturally forming edge systems that share the same low-energy description with the superclimbing dislocation [17,18], system D is rather an "engineered" state with infinite compressibility distinguished by its lack of well-defined elementary excitations altogether; yet, it is a superfluid by other hallmark properties—(ii), (iii), and (iv) above.

In the context of these TQF systems, we note that historically the stability of a homogeneous superflow with velocity v was linked to the Landau criterion $v < \min \epsilon(k)/k$, where $\epsilon(k)$ is the dispersion of elementary quasiparticle excitations. In the absence of Galilean invariance, this criterion can be strongly violated and, in particular, does not apply to models A, B, and C, which feature $\epsilon(k) \propto k^2$. This observation is even more striking for model D lacking well-defined elementary excitations, precluding the applicability of the Landau criterion.

We begin by introducing the TQF in the A, B, and C systems as sketched in Fig. 1. At the level of low-energy description, these three systems are equivalent. The boundary of a bulk Mott insulator (model A) can be superfluid, provided the bulk is close to the superfluid transition. A domain wall in a magnet (model B) formed between domains with opposite easy-axis magnetization, can undergo a transition from the Ising type to the Bloch type characterized by the easy-plane magnetization, free to choose any direction. The description

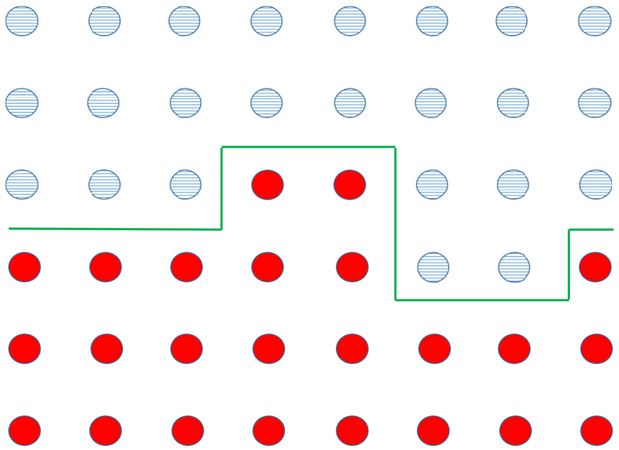

FIG. 1. A sketch of models A, B, C, where the domain line (green) is proposed to realize a TQF. In cases A and C, the solid (red) circles represent bosons forming a Mott insulating phase, while the rest of the sites shown by patterned circles remain empty (model A), or represent particles of the second bosonic component also forming a Mott insulating phase when the mixture is immiscible (model C). Particle motion by exchange along the phase separation (green) line—the domain wall—corresponds to the deformation of the domain wall in direct analogy with atom redistribution along the edge dislocation core under its climb. For model B, different circles represent opposite orientations of magnetization across the Bloch domain wall [21].

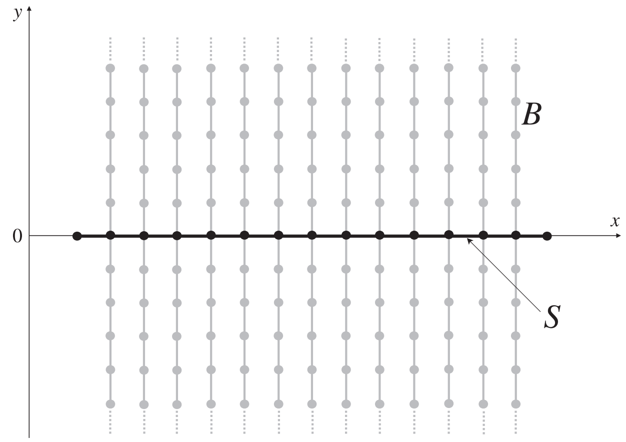

FIG. 2. A proposed realization of the incoherent TQF in a bosonic system on a square lattice, where vertical "bath" LLs are coupled to the horizontal "system" LL at $y = 0$, but are otherwise decoupled. The setup is similar to the one proposed in Ref. [13] and can also be viewed as a coupled chain of Kane-Fisher dots [28].

of the classical transition between these two phases in a 3D ferromagnet was introduced more than 60 years ago in Ref. [21]. The Bloch-type domain walls also occur in the liquid ^{3}He-*A* phase [22].

Model C maps onto a ferromagnet, as described in Refs. [20,23], thus making its domain wall equivalent to that in model B. At the microscopic level, the easy-axis magnetization in model C describes counterflow superfluidity of the components. For each of these proposed realizations of TQF the domain line must be in the quantum-rough state. This state occurs naturally in the vicinity of the phase separation transition or close to the transition from the Ising- to the Bloch-type domain wall. Conversely, if the domain line is even weakly pinned to be quantum smooth, at sufficiently long scales, the TQF crosses over to a conventional LL state.

The regime of the self-pinned lines has been studied in numerous publications (see Refs. [24–26] and references therein). In contrast, a quantum-*rough* Bloch domain wall is a 1D quantum fluid that is qualitatively distinct from a LL, with a low-energy description given by a TQF Hamiltonian [18]

$$H[\phi, n] = \int \left[\frac{\chi}{2}(\partial_x n)^2 + \frac{n_s}{2}(\partial_x \phi)^2\right] dx, \tag{1}$$

expressed in terms of the superfluid phase (or, equivalently, the angular orientation of the easy-axis magnetization) $\phi(x)$ and the canonically conjugate 1D projected density $n(x)$, proportional to the domain wall transverse (vertical, y in Fig. 1) displacement. In (1), n_s is the superfluid/spin stiffness, and χ is the domain wall line tension (energy per unit length). For edge dislocations, χ is fixed by the lattice shear modulus; for a domain wall in models A, B, and C, the χ stiffness can be tuned by proximity of the bulk material to the corresponding critical point. The key feature distinguishing TQF from LL is its divergent compressibility, i.e., absence of the leading interaction term $\sim n^2$ in (1). This leads to (i) a quadratic dispersion, $\epsilon_k = \sqrt{\chi n_s}\, k^2$, for elementary excitations with linear momentum k, (ii) off-diagonal long-range order, and (iii) exponential suppression of the phase slip events [18].

An intriguing aspect of a 1D superfluid interface between two insulating ground states is the emergent link between two seemingly unrelated properties—superfluidity and roughness. Depending on the type of insulating state(s), superfluidity and roughness can be mutually exclusive or inevitably linked. The latter situation takes place in the vicinity of the superfluid-checkerboard solid quantum critical point when the deconfinement of spinons converts the smooth insulating domain wall into a rough superfluid [6].

Finite compressibility κ may be induced in TQF (and thus transforming it into LL) either by an external potential or through lattice pinning, as discussed in Ref. [18]. In the presence of thereby generated interaction energy $\kappa^{-1} n^2$ in (1), on a length scale beyond $\xi = \sqrt{\chi\kappa}$ the system exhibits a TQF-to-LL crossover, with the familiar low-energy linear spectrum and algebraically decaying off-diagonal correlations. This crossover can be suppressed if the domain wall is incommensurately tilted (by pinning the end points) and has a finite concentration of kinks larger than ξ^{-1}, as discussed in Ref. [27].

Incoherent superfluid. We now focus on a qualitatively distinct and most interesting model D—a particularly simple microscopic representative of a family of long-ranged ordered dissipative models [8–11]—where infinite compressibility and TQF properties emerge due to Josephson coupling to the "transverse" bulk array of LLs, rather than transverse domain wall fluctuations. As illustrated in Fig. 2, this model can be realized either with cold atoms or superconducting wires, and consists of a "system" LL (running along x), with strong Josephson links to a transverse array of identical and independent "bath" LLs labeled by index i. All LLs are taken to be in a superfluid regime, characterized by a Luttinger parameter $K > 1$. Coupling between the bath LL must be negligible to avoid a global superfluid phase or gapped bulk insulator. (This setup was proposed and numerically simulated in Ref. [13], where, however, the TQF superfluidity of the system along the x direction was neither recognized nor explored.)

For model D, the Euclidean action,

$$S[\phi(\tau,x),\varphi_i(\tau,y)] = \int d\tau dx(\mathcal{L}_s + \mathcal{L}_{\text{int}}) + \int d\tau dy \mathcal{L}_b,$$

is based on three Lagrangian densities,

$$\mathcal{L}_s = \frac{\kappa_s}{2}(\partial_\tau\phi)^2 + \frac{n_s}{2}(\partial_x\phi)^2, \tag{2}$$

$$\mathcal{L}_{\text{int}} = \frac{g}{2}\sum_i [\phi(x) - \varphi_i(y=0)]^2\delta(x-x_i), \tag{3}$$

$$\mathcal{L}_b = \sum_i \left[\frac{\kappa_b}{2}(\partial_\tau\varphi_i)^2 + \frac{n_b}{2}(\partial_y\varphi_i)^2\right], \tag{4}$$

where $\phi \equiv \phi(x,\tau)$ and $\varphi_i \equiv \varphi_i(y,\tau)$ are the local superfluid phases of the system and bath LLs, respectively, $g > 0$, and κ_s, κ_b and n_s, n_b are, respectively, compressibilities and superfluid stiffnesses. In (3) the Josephson coupling $-g\cos[\phi(x,\tau) - \varphi_i(y=0,\tau)]$ has been approximated by the quadratic form, valid for strong coupling g in the LLs' superfluid regime, $K > 1$, where g flows to infinity [28]. Since we are interested in the long-wavelength limit, we go to the continuum by replacing $\sum_i$ with $\int dx/a$ and $\varphi_i(y)$ with $\varphi(x,y)$, and take the lattice constant a as the unit of length, or, equivalently absorbing it into the definition of model parameters.

The bath phase φ can be straightforwardly integrated out of the quadratic Lagrangian, decoupled in the momentum space,

$$S = \frac{1}{2}\sum_{\omega,k_x}\left[\left(\kappa_s\omega^2 + n_s k_x^2\right)\left|\phi_{\omega,k_x}\right|^2 + g\left|\phi_{\omega,k_x} - \tilde{\varphi}_{\omega,k_x}\right|^2\right] + S_b, \tag{5}$$

$$S_b = \frac{1}{2}\sum_{\omega,k_x,k_y}\left[\kappa_b\omega^2 + n_b k_y^2\right]\left|\varphi_{\omega,k_x,k_y}\right|^2, \tag{6}$$

with $\tilde{\varphi}_{\omega,k_x} = L_y^{-1/2}\sum_{k_y}\varphi_{\omega,k_x,k_y}$. We thereby get the effective 1D system action

$$S = \frac{1}{2}\sum_{\omega,k_x}\left[\left(\kappa_s\omega^2 + n_s k_x^2\right) + \frac{gK_b|\omega|}{g + K_b|\omega|}\right]|\phi_{\omega,k_x}|^2, \tag{7}$$

with $K_b \equiv 2\sqrt{n_b\kappa_b}$ and $g \to \infty$. We dub the low-energy long-wavelength limit of this action as the "incoherent TQF" (iTQF),

$$S_{\text{iTQF}} = \frac{1}{2}\sum_{\omega,k_x}\left[K_b|\omega| + n_s k_x^2\right]\left|\phi_{\omega,k_x}\right|^2, \tag{8}$$

where $\omega \ll \omega_b = K_b/\kappa_s$. With Wick's rotation to the real time/frequency this action describes the diffusive dynamics of the system's superfluid phase, with $\omega = -\mathrm{i}Dk_x^2$ with $D = n_s/K_b$. The corresponding real-time action can be equivalently obtained using Feynman-Vernon and Schwinger-Keldysh double-time contour methods [29–31].

Next, we show that despite lacking well-defined elementary excitations and being characterized by diffusive dynamics, iTQF exhibits a 1D off-diagonal long-range order and exponentially suppressed probability of the phase slip at a small superflow velocity v, i.e., it is a robust superfluid. This contrasts qualitatively with the ideal Bose gas and the 1D LL, that are, respectively unconditionally and power-law unstable at nonzero v.

Off-diagonal long-range order. The single-particle density matrix at $T = 0$ can be straightforwardly evaluated for the Gaussian action (8) as

$$\langle e^{i\phi(x,0)}e^{-i\phi(0,0)}\rangle = \exp\left[-\int\frac{d\omega dk_x}{(2\pi)^2}\frac{1-\cos(k_x x)}{K_b|\omega| + n_s k_x^2}\right]. \tag{9}$$

The integral in the exponent saturates to a constant at large separations x, i.e., the system is phase ordered and exhibits a nonzero condensate fraction. In contrast, at nonzero temperature T the integral over frequency is replaced with a discrete Matsubara sum $T\sum_{\omega_n}$ and $\omega \to \omega_n = 2\pi Tn$, controlled by $\omega_n = 0$ classical contribution, thereby leading to the asymptotic exponential decay of the density matrix with exponent $\propto -(T/2n_s)x$. This law is generic to equilibrium classical phase fluctuations associated with 1D classical Hamiltonian density $(\partial\phi/\partial x)^2$, insensitive to the nature of quantum dynamics.

Quantum phase slips (instantons). The superflow at zero temperature decays by quantum phase slips [32]. Within the leading exponential approximation, the probability of this tunneling process can be estimated via the Euclidean action associated with the unbinding of instantons carrying opposite topological "charges," $\pm q$ (integer multiples of 2π), which measure the phase winding around singular points in space and imaginary time. In conventional superfluid LLs biased by a chemical potential difference $\delta\mu$, phase slips lead to the power-law dependence (current-voltage, I-V, characteristic), $\delta\mu \sim |v|^\alpha$ with $\alpha = 2K - 1$, universally determined by the Luttinger parameter K [33].

Quantum phase slips in the TQF are qualitatively different from those in the LL because the instanton pairs in TQF are tightly confined. This leads to the exponential dependence of the bias $\delta\mu$ on the inverse of superflow velocity v. Despite the fact that TQF and iTQF are distinct phases, at the level of phase fluctuations [see Eqs. (1) and (8)], the dependence of the instanton action on v turns out to be the same, as we now demonstrate.

To derive the dissipation via instantons we follow the path outlined in Ref. [18] and introduce the velocity field $v_\mu = \partial_\mu\phi$ in the $(1+1)$-dimensional space-time $x_\mu = (x,\tau)$. As described above, instantons have the form of point-vortex singularities in the otherwise regular field v_μ,

$$\partial \times v = q(x_\mu) = \sum_j q_j\,\delta^2(x_\mu - x_{\mu,j}), \tag{10}$$

where $\partial \times v \equiv \epsilon_{\mu\nu}\partial_\mu v_\nu$ is a shorthand notation for the $(1+1)$D space-time curl of v_μ and $x_{\mu,j}$ is the space-time position of the jth instanton. The instanton contribution to the action (8) is given by

$$S = \frac{1}{2}\sum_{\omega,k_x}\left[\frac{K_b|v_\tau|^2}{|\omega|} + n_s|v_x|^2 - i\lambda(ik_x v_\tau - i\omega v_x - q)\right], \tag{11}$$

where λ is an auxiliary field enforcing the vorticity constraint (10) (for $|\omega| \ll \omega_b$). The probability to find a set of instantons with charges $\{q_i\}$ at locations $\{x_i\}$ is given by the integral

$$P = \int [dv_\mu][d\lambda] e^{-S}. \tag{12}$$

The Gaussian integration in the leading exponential approximation results in

$$P \propto e^{-\frac{1}{2}\sum_{j\neq i} V(x_{\mu,i}-x_{\mu,j})q_i q_j}, \tag{13}$$

where

$$V(x,\tau) = \int \frac{d\omega dk_x}{(2\pi)^2} \frac{n_s K_b}{|\omega|\big(K_b|\omega| + n_s k_x^2\big)} [e^{i(\omega\tau + k_x x)} - 1] \tag{14}$$

is the instanton interaction potential. This interaction leads to confinement in space, $V(x, \tau = 0) \sim |x| \ln |x|$ at a large separation of two instantons with opposite "charges," and weaker confinement in time, $V(x = 0, \tau) \approx -\sqrt{2K_b n_s \tau/\pi}$. Apart from the additional $\ln |x|$ with spatial separation, this behavior is similar to that of TQF instantons found in Ref. [18].

An imposed superflow v drives instantons apart, working against $V(x, \tau)$ via an additional contribution $2\pi n_s v\tau$ to the instanton-pair action. Now the corresponding action $S = -(2\pi)^2 V(x = 0, \tau) - 2\pi n_s |v|\tau$ features an extremum at $\sqrt{\tau} = \sqrt{\pi K_b n_s/2}/v$, leading to the phase slip probability $P \propto e^{-v_1/v}$ with $v_1 = \pi^2 K_b$ and predicting exponentially suppressed I-V,

$$\delta\mu \sim e^{-v_1/v}, \tag{15}$$

contrasting qualitatively with the power-law LL I-V. This prediction holds in the asymptotic limit $v_1/|v| \gg 1$. However, the optimal solution $\tau \sim 1/v^2$ breaks down at short timescales, for $\tau \approx 1/\omega_b = \kappa_s/K_b$, corresponding to a characteristic velocity $v_c \approx v_1/\sqrt{2\pi K_s}$, where $K_s = \pi\sqrt{n_s\kappa_s}$ is the effective Luttinger parameter of the system. For $K_s \gg 1$, this velocity $v_c \ll v_1$, and we expect the TQF I-V (15) to cross over to the power-law characteristic of the LL at $|v| > v_c$.

Discussion and conclusion. While the history of supersolid experiments in ^{4}He following Ref. [34] is rather controversial, some of the ideas and concepts generated to understand the phenomenon have proved to be quite generic and apply to a broad class of physical systems. In addition to the TQF state of an edge dislocation, similar TQF physics appears in a variety of systems: superfluidity along the boundary between two-dimensional insulating droplets, domain boundaries in an easy-axis ferromagnet, and in immiscible two-component Mott insulators. The case of iTQF is more exotic and requires a special setup illustrated in Fig. 2 and discussed in Ref. [13]. However, Ref. [13] and numerous studies of magnetic domain walls (see Refs. [24–26] and references therein) failed to identify the iTQF and TQF properties of these systems.

The key ingredient defining the class of TQF and iTQF states along with their unusual properties is infinite compressibility encoded by the effective one-dimensional field theory that we derived,

$$\sigma = \lim_{\omega\to 0} \frac{1}{\omega^2} \lim_{k_x\to 0} \frac{\delta^2 S[\phi_{\omega,k_x}, \phi^*_{\omega,k_x}]}{\delta\phi_{\omega,k_x}\delta\phi^*_{\omega,k_x}} = \infty. \tag{16}$$

This property is *sufficient* to ensure superfluid long-range order and tight binding of instantons irrespective of other system details such as gapped or gapless bulk excitations, the spectrum, or the very existence of elementary excitations. The condition $\sigma = \infty$ is not only sufficient but also necessary: At $\sigma \neq \infty$, the low-energy physics of the system would correspond to that of Luttinger liquid.

To the best of our knowledge, the unusual quasi-1D superfluidity in the domain boundaries between (and surfaces of) 2D bulk-insulating phases was discussed only in the context of quantum Hall states in Ref. [5]. However, chiral, time-reversal breaking boundaries in this system—while featuring similar to TQF quadratic effective field theory and thus nonacoustic dispersion of elementary excitations—are qualitatively distinct at the fundamental (nonlinear) level. The hallmark of TQF—be it a "canonical" case captured by superclimbing dislocations [18] or by models A, B, and C in Fig. 1, or the special iTQF case—is the tight confinement of instantons with opposite charges and the resulting protection against the quantum phase slips. This aspect is absent in the chiral quantum Hall edge states, instead protected by the gapped bulk. Qualitatively, this situation is most close to the surface currents in 2D superconductors. Despite their 1D character—enforced by the Meissner effect—the currents are protected from phase slips by the bulk order. Moreover, it is the physics of instantons that justifies the "quantum" characterization of the TQF and iTQF states. Otherwise, the parabolic dispersion of TQF and the diffusive iTQF dynamics are perfectly well captured by the corresponding classical-field theories—similar to that of the edge currents in the quantum Hall and superconducting systems.

Finally, we emphasize that from the conceptual point of view, the iTQF state is a striking demonstration of the conditional character of many dogmas associated with superfluidity, such as the necessity of elementary excitations, in general, and the ones obeying the Landau criterion in particular, as well as the absence of long-range order in one-dimensional quantum superfluids. Experimental and numerical implementation of TQF and iTQF models is the crucial next step in the exploration of this interesting physics.

Note added. Recently, the authors (together with L. Pollet) performed a numerically exact simulation of the lattice realization of model D [35]. The results are in perfect agreement with the effective field theory of the present Letter. This, together with the potential experimental realizability and interest from a number of leading atomic, molecular, and optical (AMO) groups in simulating model D experimentally, provides significant motivation to our study.

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