Letter

Ergodic inclusions in many-body localized systems

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We investigate the effect of ergodic inclusions in putative many-body localized systems. We consider the random field Heisenberg chain, which is many-body localized at strong disorder and we couple it to an ergodic bubble, modeled by a random matrix Hamiltonian. Recent theoretical work suggests that localized systems are unstable to ergodic bubbles, driving the delocalization transition. We tentatively confirm this by numerically analyzing the response of the on-site purities to the insertion of the bubble. For a range of intermediate disorder strengths, this response decays very slowly, or not at all, with increasing distance to the bubble. This suggests that at those disorder strengths, the system is actually delocalized in the thermodynamic limit. However, the signal is quite weak and artefacts in the numerics cannot be ruled out conclusively.

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Introduction. The discovery that noninteracting particles in a disorder potential can become completely immobile by Anderson in 1958 [1] has created a new field of study, which became enormously active in the last decade after it was predicted that this localized phase could persist in the presence of interactions, leading to a perfect insulator at any temperature [2–7]. While it is difficult to realize such systems in condensed matter experiments due to the presence of phonons, many-body localized (MBL) systems were realized in synthetic quantum matter [8–10]. It was first suggested that the interplay of interaction and disorder gives rise to a nonequlibrium phase transition between a thermal phase at weak disorder, which satisfies the eigenstate thermalization hypothesis (ETH) [11–14] and a many-body localized phase at strong disorder. Such a transition would be unparalleled in equilibrium [15–22]. A large body of theoretical work now supports the picture that the many-body localized phase is characterized by an emergent complete set of quasilocal integrals of motion [23,24], which are fully consistent with the observed phenomenology of area law entanglement in all many-body eigenstates [25–27], as well as with the logarithmic post-quench entanglement production [28–31].

Recently, however, the stability of the insulating phase in the thermodynamic limit has been put into question, generating a debate on MBL as a phase of matter [32–45]. Some works suggest that the critical disorder might be way higher than what was expected [37,38,46–48], others even predict an infinite critical disorder in the thermodynamic limit [35,39,40,43,49]. As a matter of fact, quantum simulations have shown MBL signatures in systems with a few tens of particles [8,10,50,51], thus today's experiments can only access a MBL regime that asymptotically, in time and system size, may or may not thermalize. One central aspect in this debate is the delocalization transition mechanism and the crossover behavior in finite systems, whose understanding is still incomplete. In the vicinity of the transition, anomalously slow dynamics was observed [52–56], which was related to anomalous thermalization behavior [26,57–60], and a theoretical description based on rare insulating inclusions was proposed [16,61,62]. It remains, however, unclear how this picture can be reconciled with the observation of slow dynamics in quasiperiodic potentials [19,55,63]. It was also proposed that the many-body resonances are driving the slow dynamics in this regime [64].

It was pointed out in Ref. [65] that many-body localized systems are unstable under certain conditions toward thermal inclusions by a mechanism dubbed "avalanche" [66,67], and such a transition as a function of the localization length was confirmed numerically in idealized models [68,69], also tailored to address implications for the instability of MBL in higher dimensions [65,70,71]. Current activities now focus on the identification of this mechanism driving the transition in more realistic models [40,72–75] and experiments [76,77]. This is challenging and so far direct evidence for the avalanche mechanism in standard MBL models is still lacking. In this work, we directly address the issue of avalanches in a standard

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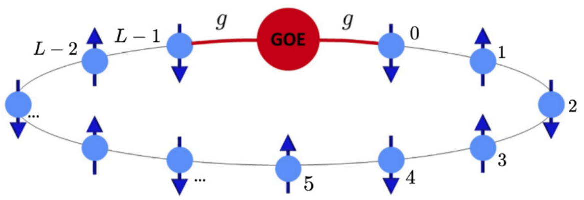

FIG. 1. System setup: A spin chain (blue) is coupled to an ergodic bubble (red). The spins at site 0 and $L-1$ are both coupled to the bubble with coupling strength g, such that both "ends" of the chain are symmetric. In order to make the bubble perfectly thermal, its Hamiltonian is given by a random GOE matrix, while the spin chain corresponds to a XXZ spin 1/2 model in the presence of a random field coupled to local $\hat{S}_i^z$ operators and nearest neighbor interactions.

MBL model and consider a thermal inclusion coupled to a disordered spin chain.

Model. We study the random field XXZ chain, modeled by the Hamiltonian H_{XXZ} ("the chain") coupled to an ergodic (or "thermal") bubble with Hamiltonian $\hat{\mathsf{R}}_0$. The setup for the model is illustrated in Fig. 1. The total Hamiltonian is of the form

$$\hat{H} = \hat{H}_{\text{XXZ}} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{\mathsf{R}}_0 + gH_{\text{coupling}}. \tag{1}$$

Here the first factor of the tensor product refers to the chain and the second factor refers to the thermal bubble. The XXZ Hamiltonian acts on L spins labeled by $i = 0, \dots, L-1$;

$$\hat{H}_{\text{XXZ}} = J\sum_{i=0}^{L-2}\left[\tfrac{1}{2}\left(\hat{S}_i^+\hat{S}_{i+1}^- + \hat{S}_i^-\hat{S}_{i+1}^+\right) + \Delta\hat{S}_i^z\hat{S}_{i+1}^z\right] + \sum_{i=0}^{L-1} h_i\hat{S}_i^z, \tag{2}$$

where we have taken the random fields $h_i \in [-W, W]$ drawn from a box distribution. We consider the isotropic point $\Delta = 1$. All parameters are measured in $J = 1$ units. The coupling term reads

$$\hat{H}_{\text{coupling}} = \left(\hat{S}_0^z \otimes \hat{\mathbb{1}}\right) \cdot \left(\hat{\mathbb{1}} \otimes \hat{\mathsf{R}}_1\right) \cdot \left(\hat{S}_{L-1}^z \otimes \hat{\mathbb{1}}\right) + [(\hat{S}_0^+ \otimes \hat{\mathbb{1}}) \cdot (\hat{\mathbb{1}} \otimes \hat{\mathsf{R}}_2) \cdot (\hat{S}_{L-1}^- \otimes \hat{\mathbb{1}}) + \text{H.c.}], \tag{3}$$

where the matrices $\hat{\mathsf{R}}_l, l = 0, 1, 2$ are independent random matrices from the Gaussian Orthogonal Ensemble (GOE), with a scaled bandwidth defined by

$$\mathsf{R}_l = \frac{\beta}{2}(\mathsf{A} + \mathsf{A}^T) \in \mathbb{R}^{n_{\text{GOE}} \times n_{\text{GOE}}}, \quad \mathsf{A}_{ij} = \text{norm}(0,1), \tag{4}$$

where norm(0,1) are normal random variables with zero mean and unit variance. The dimension n_{GOE} of the random matrices controls the power of the ergodic bubble; here we use $n_{\text{GOE}} = 3, 4, 5, 6, 8$. β is a real variable that controls the band width of the random matrices and is chosen such that the level mixing (reflected in the overall gap ratio) is maximal for the largest possible range of parameters (see the Supplemental Material (SM) in [78] for more details).

Note that the ergodic bubble is coupled to spins 0 and $L-1$, thus closing the chain into a ring. Therefore, both ends of the chain are symmetric and the longest distance from the bubble corresponds to the spin situated at the middle of the chain $i = \lfloor L/2 \rfloor$. We also note that the coupling in Eq. (3) of the bubble to the chain is chosen such that all terms preserve the total spin $\hat{S}_z = \sum_{i=0}^{L-1} \hat{S}_i^z$ in the chain. This allows us to restrict our study to the largest sector of the Hilbert space, given by $S_z = 0$ for L even and $S_z = 1$ for L odd, with a Hilbert space of dimension $d \equiv d(L, n_{\text{GOE}}) = n_{\text{GOE}}\binom{L}{\lfloor L/2 \rfloor}$. We perform massively parallel shift-invert diagonalization [79,80] for computing eigenstates close to the exact spectral center of each sample given by $(E_{\max} + E_{\min})/2$, with $(E_{\min})E_{\max}$ being the (anti)ground state energy. We confront results for our system with a thermal bubble, Eq. (1) with the isolated XXZ chain Eq. (2).

Numerical protocol. We consider two types of observables. First, we compute the r_n parameter of adjacent energy gaps in the middle of the spectrum given by $r_n = \min(\delta_n, \delta_{n+1})/\max(\delta_n, \delta_{n+1})$ [5], with $\delta_n = E_{n+1} - E_n$. Here E_{n-1}, E_n, E_{n+1} are consecutive energy levels of the XXZ chain, and the system with a bubble, respectively. This parameter is useful to distinguish the ergodic and localized phase in a simple way.

Second, we use the single site purity $\gamma_i = \text{Tr}(\rho_i^2)$, where $\rho_i = \text{Tr}_{\{L-i\}}(|n\rangle\langle n|)$ is the reduced density matrix on a single site i for a given eigenvector $|n\rangle$. Due to the $U(1)$ symmetry of the model, γ_i can be conveniently expressed via the matrix element $\langle n|S_i^z|n\rangle$ as (a brief analysis of these matrix elements is presented in the SM in [78])

$$\gamma_i = 2\langle n|S_i^z|n\rangle^2 + \tfrac{1}{2}. \tag{5}$$

In a fully ergodic system, the ETH and random matrix theory predict that $\gamma_i - 1/2 \sim d^{-1/2}$ where d is the total Hilbert space dimension $d \sim \mathcal{O}(e^L)$, for states $|n\rangle$ chosen at maximal entropy [14]. The matrix elements $\langle n|S_i^z|n\rangle$ by themselves show qualitative signatures of thermalization, see the SM in [78] for a short discussion.

In models where ergodicity is induced from boundary effects, as it presumably happens in the case investigated here, one should be more careful and write $\gamma_i - 1/2 \sim d_{\text{eff},i}^{-1/2}$ where $d_{\text{eff},i}$ is the effective dimension, see [26] and the SM in [78]. In contrast, in a localized system, we expect γ_i to depend substantially on the state $|n\rangle$ and the disorder realization, and we expect the average $\bar{\gamma}_i$ to be given by a volume-independent value, tending to one as $W \to \infty$. For each disorder realization, we use 50,...,100 eigenstates $|n\rangle$ and for each set of model parameters at least 2000 disorder realizations of the fields h_i and bubble Hamiltonian $\hat{R}_l$.

Shift of MBL transition in the interacting chain. In an infinitely long disordered chain, it should not matter whether we add a thermal bubble or not, because thermal Griffiths regions acting as ergodic patches are expected to be present anyhow. In a chain of moderate size, we expect that a substantial fraction of samples appears to be localized simply by lack of ergodic regions. This would lead to a shift of the apparent critical disorder value W_c for short chains if one compares the isolated chain to our model where we add a bubble by hand. In Fig. 2, we compare the disorder averaged gap ratio $\bar{r}$ of adjacent energy gaps in the middle of the spectrum given of the isolated XXZ chain (right) to the XXZ chain-bubble system (left) as a function of disorder strength W for different system sizes L. Using the crossing of $\bar{r}$ of size L and $L+2$ as a proxy for the apparent critical point at this length scale, the inset

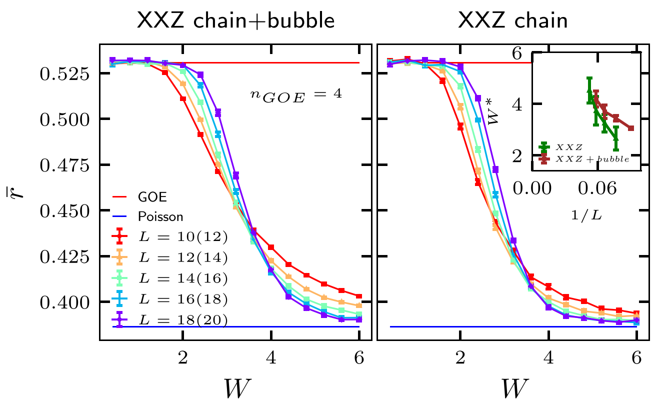

FIG. 2. Average gap ratio [5] $\overline{r}$ for the disordered field XXZ spin chain of size L coupled to a thermal bubble of size $n_{\rm GOE} = 4$ (left) and the same system without bubble and chain length $L+2$ (right). Inset: Pairwise crossing of gap ratio $\overline{r}(W)$ as function of $1/L$ for both systems.

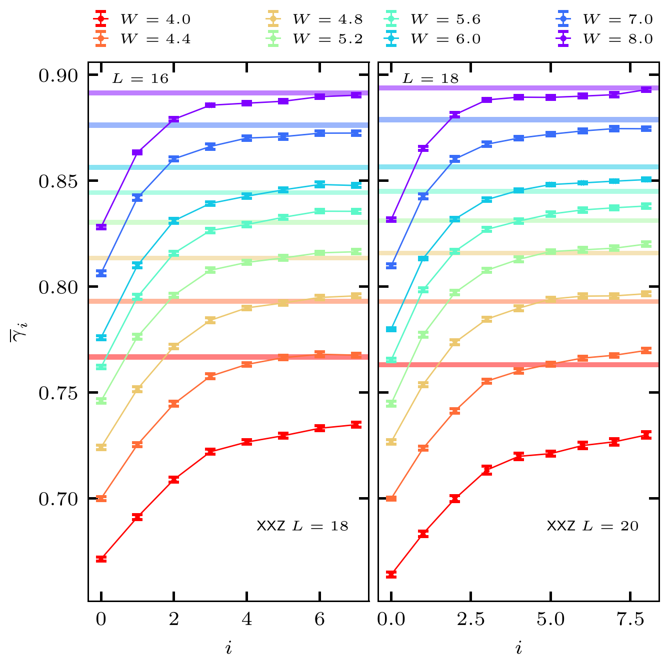

FIG. 3. Purity γ_i as function of the distance from the bubble for different disorder strengths W. Left (Right) panel shows system size $L = 16$ ($L = 18$) and $n_{\rm GOE} = 4$. Horizontal shaded areas correspond to the $\gamma_{\rm XXZ}$ in the absence of the bubble with $L+2$ chain length compared to the bubble-chain system.

suggests that it is indeed the case that the crossings appear at slightly larger disorder strengths in the presence of the bubble. However, due to large statistical uncertainty, it is far from clear whether this is a significant signal. Even if it were significant, finite-size effects render the crossing analysis an unreliable way to pin down the critical point precisely.

Change of local ergodicity in interacting chains. We investigate the one-site purities γ_i as a measure of local ergodicity. Throughout this paper, we consistently use the labels "localized" and "thermal" as inferred from the crossing point of $\overline{r}$ (see Fig. 2), i.e., with the critical point at $W \approx 3.5$–4, even though recent works locate the transition at larger W [37,38,43,47]. Inspecting the average purities in the presence and absence of the thermal bubble (see Fig. 3) is clear that the effect of the bubble at long distance is rather small and subtle. Therefore, it is useful to first map out what the theory of quantum avalanches predicts.

Theoretical background. For a simple avalanche model of our setup, we assume the existence of an apparent localization length ξ, describing the system in absence of the bubble, and a number p giving the probability that the bubble can kickstart a thermalization process in its near vicinity, see Ref. [69] for a detailed discussion and arguments why $p \ll 1$ in our setup. A simple model, based on an unrealistic dichotomy between ergodicity and localization, leads (see the SM in [78]) to the following very rough prediction:

$$(\bar{\gamma}_i - 1/2) \approx (1-p)(1-e^{-|i|/\xi})(\bar{\gamma}_{\rm XXZ-1/2}) + p d_{{\rm eff},i}^{-1/2}, \quad (6)$$

with $\bar{\gamma}_i$ the average purity at site i and $\bar{\gamma}_{\rm XXZ}$ the average purity in the bubbleless system. For $p = 0$, we recover a perfectly localized system, whereas for $p = 1$, the system is ergodic but the average purity still increases with i, because the effective dimension $d_{{\rm eff},i}$ depends on the distance to the bubble. This effective dimension is defined as

$$d_{{\rm eff},i} = e^{-2i/\xi} d_{\rm therm}, \quad (7)$$

with $d_{\rm therm}$ the total Hilbert space dimension of the thermal region, given by

$$d_{\rm therm} = \begin{cases} n_{\rm GOE}^{\frac{1}{1-\xi/\xi_*}} & \text{if } \xi < \xi_* \\ 2^L n_{\rm GOE} & \text{if } \xi > \xi_*, \end{cases}$$

where $\xi_* = 1/\log 2$ is the critical localization length. According to the above formula Eq. (6), there are two regimes in which $\bar{\gamma}_{\rm XXZ} - \bar{\gamma}_i$ does not decay to zero, or only very slowly.

(i) $\xi > \xi_*$. Here, the bubble thermalizes a fraction p of samples, resulting in a shift $\bar{\gamma}_{\rm XXZ} - \bar{\gamma}_i$ which remains finite as $i \to \infty$, even though it decreases due to the decrease of $d_{{\rm eff},i}$.

(ii) ξ approaches ξ_* from below. Then $\bar{\gamma}_{\rm XXZ} - \bar{\gamma}_i \to 0$ at large i, but the decay is arbitrarily slow when $\xi \to \xi_*$, because the decay of $d_{{\rm eff},i}$ is arbitrarily slow.

In practice, (i) and (ii) are of course hard to distinguish. *Large distance behavior.* In Fig. 3, we indeed see a sign of influence of the bubble that does not, or only very slowly, decay with distance from the bubble. We observe that the purity in the presence of the bubble seems to tend to an asymptotic value that is significantly lower than the value for the bubbleless system. This seems to be the case up to disorder strength $W = 5.6$–6, after which the signal, i.e., the difference $\bar{\gamma}_{\rm XXZ} - \bar{\gamma}_i$, becomes comparable to the error bars.

Combining this with the theoretical analysis above, we see, hence, that our results are compatible with disorder strengths up to $W = 5.6$–6 being in the delocalized regime. Since this delocalization is realized only in a small fraction of samples, i.e., $p \ll 1$, there is no surprise that our crossing analysis does not exhibit any sizable shift.

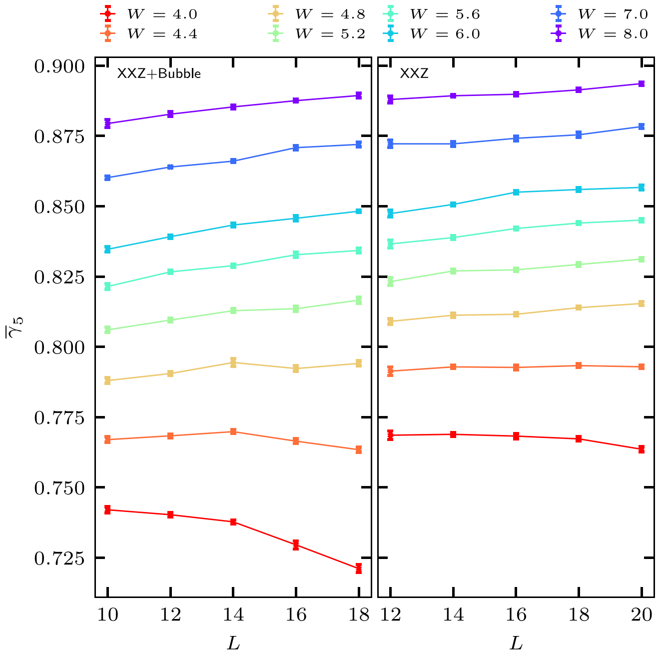

FIG. 4. Average purity $\overline{\gamma}_i$ at fixed distance $i = 5$ from the bubble plotted as function of system size L for different disorder strengths W. Left panel: XXZ spin chain of length L with bubble of size $n_{\mathrm{GOE}} = 4$. Right panel: XXZ Heisenberg chain of size L. Average is taken over disorder realizations and eigenstates.

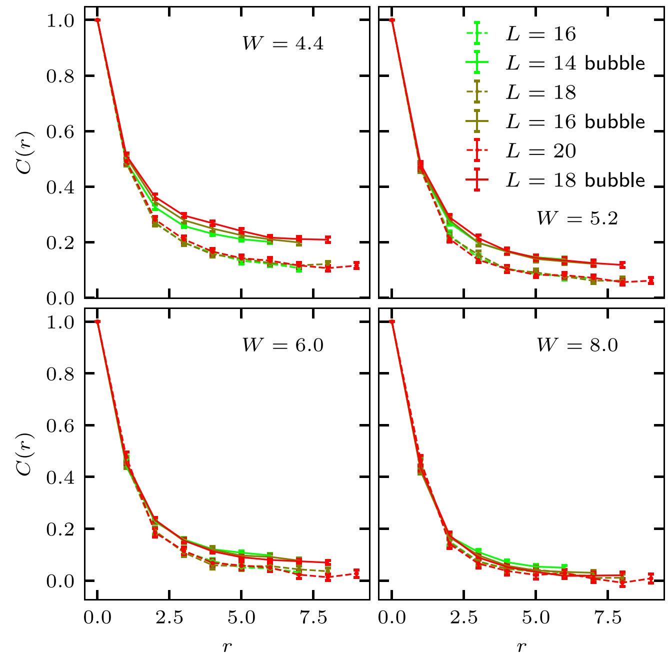

FIG. 5. Average purity correlation $C(r)$ as function of the distance from the bubble for disorder strengths $W = 4.4, 5.2, 6.0, 8.0$. Dashed lines correspond to bubbleless system of size $L + 2$ while solid lines to bubble-full system of size L.

We will analyze this further below, but let us first note that it is far from clear whether our system sizes are large enough to speculate about the limit $i \to \infty$. Indeed, in Fig. 4, we investigate the dependence of the purity at fixed distance on increasing system size L. We see that for disorder values at and above $W = 5.2$, the purity γ_5 is actually increasing with increasing system size. This is an effect that is not even accounted for in our model equation and it should be interpreted as a sign that finite-size effects are still important (see the SM in [78] for further analysis).

Another way to look for thermalization induced by the bubble is through correlations between different sites of the chain. In that spirit we look at statistical correlations between two purities γ_i and γ_j. Those are defined as

$$C(r) = \frac{\overline{(\gamma_0 - \bar{\gamma}_0)(\gamma_r - \bar{\gamma}_r)}}{\sigma_0 \sigma_r}, \tag{8}$$

where $\gamma_i - \bar{\gamma}_i$ captures the purity fluctuations around its mean and $\sigma_i^2 = \overline{(\gamma_i - \overline{\gamma_i})^2}$ is the variance of the purity at site i. The quantity $C(r)$ is the Pearson correlation coefficient between purities at sites next to the bubble ($i = 0$) and sites at distance r from that spin with $r = 0, 1, \ldots, L/2$. This quantity for different disorder and system size is shown in Fig. 5. Interestingly, for $W = 4.4$ the bubble seems to be enhancing correlations compared to the bubbleless case. This effect is even growing with system size and it persists for $W = 5.2$ and, up to some extent, for $W = 6.0$. The latter is possibly a quantitative witnessing of a small fraction p of avalanching samples. Certainly, the purity correlation $C(r)$ seems to be better in capturing the weak correlations due to avalanching samples than the bare purity $\overline{\gamma}_i$.

Since the signals that we observe, the difference in $\bar{\gamma}_{\mathrm{XXZ}} - \bar{\gamma}_i$ and the correlations in Fig. 5, are rather small; there is the concern that they might be caused by some artefact. For example, even in a well-localized system, in the absence of any avalanches, the purity-purity correlation $C(r)$ from Eq. (8) has a nonzero limit as $r \to \infty$ of order $\frac{C}{L^2}$ with $C \propto \frac{1}{W^2}$ in the high-disorder limit $W \to \infty$, see the SM for more details. While we do not see any clear mechanism how this might pollute our analysis, we find it hard to exclude it.

Conclusion. We have studied the effect of ergodic inclusions modeled by local random matrices in disordered spin chains of up to 20 sites. We report little or no drift of the thermal-to-localized transition in the average level spacing compared to the bubbleless case, which was also observed in similar settings [72]. We also investigate long distance effects of the bubble by looking at functions of the local magnetization expectation value, in this case the purity and its fluctuations. There, we have numerically witnessed a potentially divergent, long wavelength, response of an apparently localized chain (as estimated by ED studies like [79]) to an ergodic bubble. The effect is weak and not unambiguous, but it is compatible with the avalanche theory proposed in earlier works. Importantly, such weak correlations may be influenced by spurious long-range correlations due to the $U(1)$ symmetry (see the SM in [78]), although we can not confirm this from our numerics.

Recent studies also point out weak but persisting correlations produced by many-body resonances in the same disorder regime we study [37,47]. Our results suggest that the bubble is effectively enhancing such weak correlations when the system looks well localized in average. However it is yet not clear how to directly relate the observed signal with the many-body resonances.

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