Letter

Dissipation-driven dynamical topological phase transitions in two-dimensional superconductors

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We induce and study a topological dynamical phase transition between two planar superconducting phases. Using the Lindblad equation to account for the interactions of Bogoliubov quasiparticles among themselves and with the fluctuations of the superconducting order parameter, we derive the relaxation dynamics of the order parameter. To characterize the phase transition, we compute the fidelity and the spin-Hall conductance of the open system. Our approach provides crucial information for experimental implementations, such as the dependence of the critical time on the system-bath coupling.

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Introduction. Phase transitions (PTs) emerge as an effect of fluctuations, both thermal [1] or quantum [2], involving collective degrees of freedom in many-body systems. Typically, a PT is characterized by the divergence of the correlation length as the temperature and/or another control parameter is tuned from the outside and with the corresponding power-law scaling of the physical quantities, with "universal" critical exponents. The usual approach to PTs in systems at equilibrium involves methods, such as looking for singularities and for critical scaling in the free-energy functional describing the system, as the control parameters are tuned close to their critical values.

A continuously increasing interest, both on the theoretical as well as on the experimental side, has been recently gained by "dynamical" PTs (DPTs) in closed many-particle quantum systems, prepared in a nonequlibrium state and then evolving in time with a pertinent Hamiltonian [3–5]. In a DPT, it is the time t that drives the system across criticality and the PT is evidenced by singularities in the matrix elements of the time evolution operator. To date, DPTs have been theoretically predicted and experimentally seen in various isolated quantum systems, in which nonequilibrium is induced by quenching some parameter(s) of the system Hamiltonian [6–15]. In a closed system, the DPT is analyzed by looking at the singularities in the Loschmidt echo (LE) $\mathcal{L}(t) = |\langle\psi(0)|\psi(t)\rangle|^2$, with $|\psi(t)\rangle$ being the state of the system at time t and $|\psi(0)\rangle$ the prequench initial state [3–6,16]. While this approach has been also extended to the case in which the system has not been prepared in a pure state [17–19], it does not apply to DPTs in open systems. The latter are described by a time-dependent density matrix $\rho(t)$, whose dynamics is determined by solving the pertinent evolution equation.

In the context of superconducting electronic systems, nonequilibrium dynamics can be induced, for instance, by suddenly quenching the interaction parameter in a BCS Hamiltonian, and by encoding the following dynamical evolution of the system into an explicit dependence of the superconducting order parameter on t, by means of a time-dependent generalization of the self-consistent BCS mean-field (MF) approach [20].

In this Letter we substantially extend and generalize the approach of Ref. [20], so as to induce a DPT between different superconducting phases realized in a planar, interacting fermionic model with an attractive interaction. In particular, we allow for the coexistence of $x^2 - y^2$ (d-wave) and xy (id-wave) superconducting gaps. Then, after quenching the interaction strength(s) at time $t = 0$, we let the superconductor behave as an open system by exchanging Bogoliubov quasiparticles with the bath. In this way, we account for the dissipative dynamics induced by the interactions between quasiparticles not captured by the BCS approximation and/or by the coupling between the fluctuations of the order parameter and the quasiparticle continuum [9,21–23] and/or for the coupling to an external metallic contact [24] [a detailed discussion is provided in the Supplemental Material (SM) [25] as well as Refs. [21,23,24,26]]. In particular, following Refs. [26,27], we do so within the Lindblad master equation (LME) approach to the time evolution of the superconductor density matrix $\rho(t)$. Our systematic approach naturally emerges from the microscopic model of Ref. [24]. Moreover, it is perfectly consistent with the one introduced in Ref. [23] on phenomenological grounds, as discussed in Ref. [28], where we also estimate typical experimental values of the coupling between the system and the bath.

Here, we focus onto the DPT between a (topologically trivial) id and a $d + id$ planar superconducting phase, the latter of which is known to describe the class C of topological planar superconductors, characterized by particle-hole conjugation and broken time-reversal symmetry [29–34]. Therefore, we realize a topological DPT (TDPT), to characterize which we first of all look at the transition in time of the superconducting order parameter, between the asymptotic values

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(at $t = 0$ and $t \to \infty$), respectively corresponding to the *id* and to the $d + id$ phase. Then, rather than the LE, we approach the DPT by using the fidelity $\mathcal{F}(t)$ between the initial pure state and the one described by $\rho(t)$, which is more suitable for an open system [28]. Finally, to make a rigorous statement on the topological properties of the phases separated by the DPT, we compute the spin-Hall conductance of the system as a function of t, $\sigma(t)$. In a stationary state, $\sigma(t)$ is proportional to the topological invariant whose nonzero value is a signal of a nontrivial topological phase [32,35]. Furthermore, it is well defined even when the system goes through the DPT [35]. In addition to defining a protocol to monitor a DPT in an open system, a topic that has recently become of the utmost relevance [36], our approach provides remarkable results of practical interest in a possible experimental realization of the system that we study (e.g., in ultracold atom lattices), such as the variation of the "critical time" t_* as a function of the system-bath coupling.

Model Hamiltonian. Our main reference Hamiltonian H_{MF} stems from the self-consistent mean-field (SCMF) approximation of the Hamiltonian describing interacting spinful fermions on a two-dimensional (2D) square lattice introduced in Ref. [28], with a nearest-neighbor and a next-to-nearest neighbor density-density interaction, both attractive in the spin singlet channel, respectively, with interaction strengths V and Z (both >0). We therefore set [20,28]

$$H_{\mathrm{MF}} = \sum_{\mathbf{k}}[c^\dagger_{\mathbf{k},\uparrow}, c_{-\mathbf{k},\downarrow}]\begin{bmatrix} \xi_{\mathbf{k}} & -\Delta_{\mathbf{k}} \\ -[\Delta_{\mathbf{k}}]^* & -\xi_{\mathbf{k}} \end{bmatrix}\begin{bmatrix} c_{\mathbf{k},\uparrow} \\ c^\dagger_{-\mathbf{k},\downarrow} \end{bmatrix}, \quad (1)$$

with $\mathbf{k}$ summed over the full Brillouin zone and with $c_{\mathbf{k},\sigma}$ and $c^\dagger_{\mathbf{k},\sigma}$ being the fermion operators in $\mathbf{k}$ space. The $\mathbf{k}$-dependent gap in Eq. (1) takes, in general, a nonzero imaginary part. The two parameters take the form

$$\begin{aligned}\xi_{\mathbf{k}} &= -2[\cos(k_x) + \cos(k_y)] - \mu, \\ \Delta_{\mathbf{k}} &= 2\Delta_{x^2-y^2}\{\cos(k_x) - \cos(k_y)\} - 4i\Delta_{xy}\sin(k_x)\sin(k_y),\end{aligned} \quad (2)$$

with the chemical potential $\mu = 0$ (half filling) and $\Delta_{x^2-y^2}$ and Δ_{xy} determined by the SCMF equations [28]

$$\begin{aligned}\Delta_{x^2-y^2} &= \frac{V}{4\mathcal{N}}\sum_{\mathbf{k}}\frac{[\cos(k_x) - \cos(k_y)]\Delta_{\mathbf{k}}}{\epsilon_{\mathbf{k}}}, \\ \Delta_{xy} &= \frac{iZ}{2\mathcal{N}}\sum_{\mathbf{k}}\frac{\sin(k_x)\sin(k_y)\Delta_{\mathbf{k}}}{\epsilon_{\mathbf{k}}},\end{aligned} \quad (3)$$

$\mathcal{N}$ being the number of lattice sites, $\epsilon_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$ and the lattice constant = 1. Our model calculation encompasses all the relevant features that should characterize a TDPT, both in solid-state [24], as well as in quantum-optical open systems [36–38]. In Fig. 1(a) we plot the phase diagram of our system in terms of V and Z (which we regard as our physically tunable parameters) as determined by Eqs. (3). For small, though finite, values of V and Z the system lies within a normal phase (N). Keeping Z (V) small and increasing V (Z), our system undergoes a phase transition, with a gap $\Delta_{x^2-y^2}$ (Δ_{xy}) continuously developing a nonzero value for $V > V_c$ ($Z > Z_c$), with $V_c \approx 0.35$ ($Z_c \approx 0.7$), and with the gaps increasing with V and Z, according to Eqs. (3). At large enough values of both V and Z, the system undergoes a topological phase transition, at which a $d + id$ phase opens, where both $\Delta_{x^2-y^2}$ and Δ_{xy} are $\neq 0$. The latter phase exhibits nontrivial topological properties. We now show how to realize a TDPT between the *id* and the $d + id$ phase, along the dissipative dynamics of the nonequilibrium superconductor.

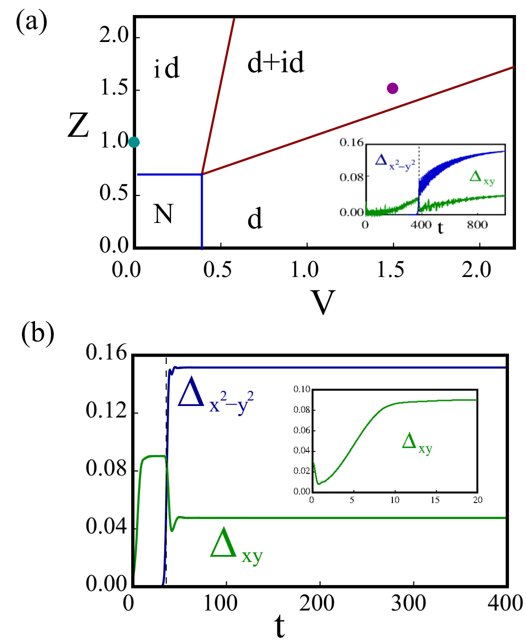

FIG. 1. (a) Equilibrium phase diagram in the V-Z plane. The cyan (magenta) dot corresponds to $(V^{(0)}, Z^{(0)})(V^{(1)}, Z^{(1)})$ (see text). [Inset: Same as in (b) but with $g = 0.002$.] (b) Time-dependent gap $\Delta_{x^2-y^2}(t)$ (blue curve) and $\Delta_{xy}(t)$ (green curve), for $V^{(1)} = Z^{(1)} = 1.5$, $g = 0.2$, and prepared, at $t = 0$, in a state with $\Delta^{(0)}_{xy} \approx 0.03$. [Inset: Zoom of the plot of $\Delta_{xy}(t)$ for $0 \leqslant t \leqslant 20$.]

Dynamical phase transition. To induce a DPT in our system, we prepare it in the ground state of H_{MF} with $\Delta^{(0)}_{x^2-y^2} = 0$ and $\Delta^{(0)}_{xy} \neq 0$. At time $t = 0$, we quench the interaction strengths to $(V^{(1)}, Z^{(1)})$, corresponding to both $\Delta^{(1)}_{x^2-y^2}$ and $\Delta^{(1)}_{xy}$ being $\neq 0$. The induced nonequilibrium dynamics makes $\Delta_{\mathbf{k}}$ to explicitly depend on t. To describe the time evolution of the open system, we extend the time-dependent SCMF approach of Ref. [20] by allowing the system to exchange Bogoliubov quasiparticles with an external bath, thus resorting to the LME approach to the density matrix dynamics (see Sec. III of SM [25]). Following Refs. [9,21–23,26,27], we therefore write the LME for $\rho(t)$ as

$$\begin{aligned}\frac{d\rho(t)}{dt} = &-i[H_{\mathrm{MF}}(t), \rho(t)] + g\sum_{\lambda=\pm}\sum_{\mathbf{k}} \\ &\times [f(-\lambda\epsilon_{\mathbf{k}}(t))(2\Gamma_{\mathbf{k},\lambda}(t)\rho(t)\Gamma^\dagger_{\mathbf{k},\lambda}(t) \\ &- \{\Gamma^\dagger_{\mathbf{k},\lambda}(t)\Gamma_{\mathbf{k},\lambda}(t), \rho(t)\}) \\ &+ f(\lambda\epsilon_{\mathbf{k}}(t))(2\Gamma^\dagger_{\mathbf{k},\lambda}(t)\rho(t)\Gamma_{\mathbf{k},\lambda}(t) \\ &- \{\Gamma_{\mathbf{k},\lambda}(t)\Gamma^\dagger_{\mathbf{k},\lambda}(t), \rho(t)\})]. \end{aligned} \quad (4)$$

In Eq. (4) g is the strength of the system-bath coupling and, consistently with the detailed balance principle, recovering the Boltzmann distribution as a stationary solution of the LME is assured by our setting of the coupling strength corresponding to the Bogoliubov quasiparticle annihilation and creation operators, $\Gamma_{\mathbf{k},\lambda}$ and $\Gamma^\dagger_{\mathbf{k},\lambda}$, to be proportional to $f(-\lambda\epsilon_{\mathbf{k}})$ and to $f(\lambda\epsilon_{\mathbf{k}})$, respectively, with $f(\epsilon)$ being the Fermi distribution function [note that we employ particle-hole symmetry to set $1-f(\epsilon)=f(-\epsilon)$] [39,40]. The time dependence of $\Delta_{\mathbf{k}}(t)$ makes $H_{\mathrm{MF}}(t)$ in Eq. (4) as well as its eigenvalues $[\pm\epsilon_{\mathbf{k}}(t)=\pm\sqrt{\xi_{\mathbf{k}}^2+|\Delta_{\mathbf{k}}(t)|^2}]$ and eigenmodes $[\Gamma_{\mathbf{k},\pm}(t)]$, to acquire an explicit time dependence, as well (see SM [25] and Refs. [20,26] for further details).

To complete the SCMF approach, we need the relation between $\Delta_{\mathbf{k}}(t)$ and $\rho(t)$. To recover it, we follow the derivation of Ref. [20] by generalizing Eqs. (3) to self-consistent relations between $\Delta_{\mathbf{k}}(t)$ and $f_{\mathbf{k}}(t)=\mathrm{Tr}[\rho(t)c_{-\mathbf{k},\downarrow}c_{\mathbf{k},\uparrow}]$, as we discuss in detail in SM [25].

In Fig. 1(b) we plot $\Delta_{x^2-y^2}(t)$ and $\Delta_{xy}(t)$ in a system prepared in the ground state $|\psi(0)\rangle$ of H_{MF} with $(V^{(0)},Z^{(0)})=(0,1.0)$, corresponding to $\Delta^{(0)}_{x^2-y^2}=0$, $\Delta^{(0)}_{xy}=0.03$. At $t>0$ we quench the interaction strengths to $(V^{(1)},Z^{(1)})=(1.5,1.5)$, corresponding to $\Delta^{(1)}_{x^2-y^2}=0.15$, $\Delta^{(1)}_{xy}=0.05$, and let the system evolve according to Eqs. (4), with $g=0.2$ (main figure) and $g=0.002$ [inset of Fig. 1(a)]. Given $(V^{(0)},Z^{(0)})$ and $(V^{(1)},Z^{(1)})$, the time evolution of the superconducting gaps is directly determined by Eqs. (4) and by the time-dependent generalizations of Eqs. (3) [see Eq. (4) of SM [25]]. We see that there is a finite interval of time $[0,t_*]$, with $t_*\approx 40$, within which $\Delta_{xy}(t)$ stays finite, and basically constant, while $\Delta_{x^2-y^2}(t)$ remains pinned at 0. As t goes across t_* (vertical, dashed line), $\Delta_{xy}(t)$ almost suddenly lowers its value, while $\Delta_{x^2-y^2}(t)$ switches from zero to a finite value, which keeps roughly constant for any $t>t_*$. t_* is determined by the finite time required for the $f_{\mathbf{k}}(t)$'s to take the appropriate threshold value to trigger the onset of the two-component order parameter. The sharp change in $\Delta_{\mathbf{k}}(t)$ across $t=t_*$ corresponds to a transition, in real time, of the system between two different phases characterized by different values of the superconducting order parameter, that is, to a DPT [3–5]. The relatively high value of g (although still quite smaller than any other energy scale in the system) induces a sharp switch in the values of the two different symmetry components of the order parameter at the DPT. To highlight the effects of varying g, in the inset of Fig. 1(a) we show the same plots as in the main figure, but with $g=0.002$. In this case, t_* becomes much larger than before and $\Delta_{xy}(t)$ oscillates and monotonically increases, starting from $\Delta^{(0)}_{xy}$, as long as $t<t_*$. At the same time, $\Delta_{x^2-y^2}(t)=0$. At $t\geqslant t_*$, both $\Delta_{xy}(t)$ and $\Delta_{x^2-y^2}(t)$ undergo a discontinuous jump, after which they start to oscillate around the values they take in the asymptotic $(t\to\infty)$ state. Again, we conclude that, at $t=t_*$, our system goes across a DPT, triggered by the mismatch between the initial and the asymptotic state of the system.

We evidence the onset of the DPT by looking at nonanalyticities in the fidelity $\mathcal{F}(t)$ between the initial state $|\psi(0)\rangle$ and the state described by $\rho(t)$, $\mathcal{F}(t)=\langle\psi(0)|\rho(t)|\psi(0)\rangle$. Indeed, as pointed out in Refs. [41,42], at a DPT $\mathcal{F}(t)$ is expected to show nonanalyticities similar to what one would obtain in

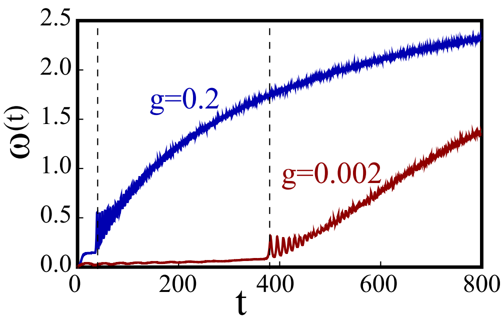

FIG. 2. $\omega(t)$ as a function of time t computed with the time-dependent MF Hamiltonian with parameters $\Delta_{x^2-y^2}(t)$ and $\Delta_{xy}(t)$, as in Fig. 1(b), for $g=0.2$ (blue curve) and $g=0.002$ (red curve). The dashed vertical lines mark the DPT.

the LE, computed in a closed system. In fact, while the LE at time t is defined if the system lies in a pure state at any t, when the system state is described in terms of a density matrix $\rho(t)$, $\mathcal{F}(t)$ shows to be the pertinent quantity to evidence the DPT. In particular, we look for nonanalyticities in the rate function $\omega(t)=-\frac{1}{N}\log[\mathcal{F}(t)]$ [3–5,7] (see Ref. [28] for the mathematical derivation). In Fig. 2 we plot $\omega(t)$ as a function of t in both cases corresponding to the plots in Fig. 1. Aside from the different scale t_* at different values of g, we note that, for $0\leqslant t<t_*$, $\omega(t)$ takes a mild dependence on t, with $\omega(t)\sim 0.1$–0.2, denoting an appreciable overlap between $|\psi(0)\rangle$ and the state at time t. This basically evidences the persistence of the system within the same phase [3–5,43,44]. At $t=t_*$, the sudden change in the slope of $\omega(t)$ demonstrates how $t=t_*$ corresponds to a nonanalyticity tied to the DPT. The subsequent rapid increase in $\omega(t)$ for $t>t_*$ corresponds to a drastic reduction in $\mathcal{F}(t)$ (by orders of magnitude), which is a clear signal that, moving across $t=t_*$, the system has gone through a DPT.

Topological phase transition. To physically ground the topological nature of the DPT, we now review the calculation of the spin-Hall conductance $\sigma(t)$ across the DPT of our system (for details, see SM [25] as well as Refs. [15,29,31,32,35,37,38,45]). In an equilibrium state $\sigma(t)$ is proportional to the Chern number $\mathcal{C}$: In the trivial phase, $\mathcal{C}=0$, while in a topologically phase, $\mathcal{C}=\pm2$ [29,30]. Apparently, $\mathcal{C}$ is ill defined across the DPT. Instead, $\sigma(t)$ is perfectly well defined and can be computed at any finite t within linear (in the applied voltage bias) response theory. Following the derivation of Ref. [25], we note that Fig. 1(b) suggests that, for $g=0.2$, $\Delta_{x^2-y^2}(t)$ and $\Delta_{xy}(t)$ can be well approximated as $\Delta_{x^2-y^2}(t)=\theta(t-t_*)\Delta^{(1)}_{x^2-y^2}$, and $\Delta_{xy}(t)=\theta(t_*-t)\Delta^{(0)}_{xy}+\theta(t-t_*)\Delta^{(1)}_{xy}$, with $\theta(t)$ being Heaviside's step function, $\Delta^{(0)}_{x^2-y^2}=0$, $\Delta^{(0)}_{xy}=0.03$, $\Delta^{(1)}_{x^2-y^2}=0.15$, and $\Delta^{(1)}_{xy}=0.05$. In Fig. 3 we plot $\sigma(t)$ computed accordingly. As expected, for $t<t_*$, $\sigma(t)=0$. Passing across the DPT at $t=t_*$, $\sigma(t)$ jumps to a finite value, and then, for $t>t_*$, it shows damped oscillations $[\sim(t-t_*)^{-1}]$ toward the asymptotic value $\sigma_\infty=\lim_{t\to\infty}\sigma(t)=2(2\pi)^{-1}$. This is exactly what is expected for a TDPT. In addition, we have also verified that σ_∞ does not change on varying $\Delta^{(1)}_{x^2-y^2}$ and $\Delta^{(1)}_{xy}$, provided we stay within the $d+id$ phase in Fig. 1(a).

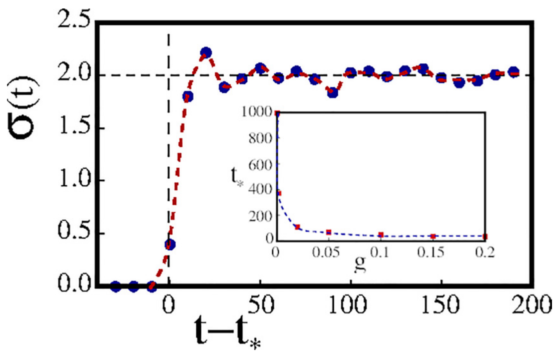

FIG. 3. $\sigma(t)$ [in units of $(2\pi)^{-1}$] computed in the sudden jump approximation (see SM [25]), with $\Delta^{(0)}_{xy} = 0.09$, $\Delta^{(1)}_{x^2-y^2} = 0.15$, $\Delta^{(1)}_{xy} = 0.05$, and $g = 0.2$. The dashed vertical line marks the DPT at $t = t_*$, and the dashed horizonal line marks the value of σ_∞. [Inset: t_* computed in the same system for different values of g (red squares) between $g = 0.0005$ and $g = 0.2$ (the dashed lines are a guide to the eye).]

While our sudden jump approximation only applies for g as large as 0.2, from the inset of Fig. 1(a) we see that, even for $g = 0.002$, the main features of Fig. 1(b) still persist, that is, a sharp reduction in $\Delta_{xy}(t)$ and a corresponding jump in $\Delta_{x^2-y^2}(t)$ from 0 to a finite value at $t = t_*$. From the qualitative point of view, we can still get some hints by enforcing the approximation of $\Delta_{\mathbf{k}}(t)$ with a piecewise function, although along small time intervals (depending, of course, on the frequency of the superimposed oscillations). This would allow us to map out the full time dependence of $\sigma(t)$ on t. Eventually, we expect that $\sigma(t)$ will asymptotically converge to the value dictated by the asymptotic value of $\Delta_{x^2-y^2}(t)$ and of $\Delta_{xy}(t)$ as $t \to \infty$. Again, these correspond to a $d + id$ superconducting state and, therefore, we find that $\sigma(t) \to_{t\to\infty} 2(2\pi)^{-1}$, even at values of g smaller than 0.2 by two orders of magnitude.

Conclusions. In this Letter we have shown that a DPT can take place in an open nonequilibrium planar superconducting system, described by a LME that is determined by the tunnel coupling to an external metallic lead [24], or mimics the residual quasiparticle interaction beyond BCS theory [23]. To monitor the system across the DPT, we synoptically looked at the self-consistently computed superconducting gap, at the fidelity, and at the spin-Hall conductance. In particular, this last quantity is crucial in evidencing the topological nature of the DPT, as it crosses over from being zero for $t < t_*$ to an asymptotic value σ_∞, as $t \to \infty$, which corresponds to a topologically nontrivial $d + id$ phase.

Within our derivation, we explicitly show clear evidence that a DPT can take place in an open solid-state system, in particular between two phases with different topological properties. In doing so, we also highlight the importance of the system-bath coupling in stabilizing a DPT transition between two selected phases with given properties. We do so by means of a combined use of the LME and of the time-dependent SCMF approach, thus defining a systematic framework to discuss the peculiar properties of a DPT in an open system. Due to our minimal set of assumptions, we believe that the range of applicability of our approach is much wider than just the one that we discuss here. For instance, it would be important to study whether the topological DPT takes place at any finite g, or whether there is a critical value g_c such that, for $g < g_c$, it is washed out by the uncontrolled oscillations in the superconducting order parameter that arise at small values of g [20]. Along this direction, in the inset of Fig. 3 we report the results of a preliminary calculation of t_* at selected values of g between $g = 0.0005$ and $g = 0.2$. We check that a power-law fit of the dependence of t_* on g such as $t_* = Ag^{-B}$, with $A \approx 7.2$ and $B \approx 0.65$ fits the data reasonably well. In addition, the strong increase of t_* as $g \to 0$ is consistent with the absence of any DPT in the closed quenched superconductor studied in Ref. [20]. However, providing a certain answer on those issues requires developing a model calculation of t_* at a given g, which is an interesting topic for future research.

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