Letter | Editors' Suggestion

Non-Hermitian Hamiltonians violate the eigenstate thermalization hypothesis

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The eigenstate thermalization hypothesis (ETH) represents a cornerstone in the theoretical understanding of the emergence of thermal behavior in closed quantum systems. The ETH asserts that expectation values of simple observables in energy eigenstates are accurately described by smooth functions of the thermodynamic parameters, with fluctuations and off-diagonal matrix elements exponentially suppressed in the entropy. We investigate to what extent the ETH holds in non-Hermitian many-body systems and come to the surprising conclusion that the fluctuations between eigenstates are of equal order to the average, indicating no eigenstate thermalization. We support this conclusion with mathematically rigorous results in the Ginibre ensemble and numerical results in other random matrix ensembles, the non-Hermitian Sachdev-Ye-Kitaev model, and a local non-Hermitian spin chain, indicating universality in a broad class of chaotic non-Hermitian quantum systems.

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Introduction. Thermalization in quantum systems describes the dynamical process of out-of-equilibrium states settling down to quasiequilibrium states that are well described by quantum statistical mechanics and thermodynamics. Understanding which systems thermalize and how they do so is a foundational question in quantum physics that has received tremendous attention in the past decades [1].

In recent years, the field of *non*-Hermitian quantum physics has emerged as a generalization of the standard paradigm of Hermitian physics to describe systems with dissipation [2]. It is natural to ask if and how thermalization manifests in these systems. This direction goes under the name of *dissipative quantum chaos* and has recently enjoyed several interesting results [3–24].

The eigenstate thermalization hypothesis (ETH) is the archetypal description or definition of thermality in closed quantum systems. The ETH asserts that for chaotic quantum systems, the matrix elements of simple [25] operators are smoothly varying on the diagonal in the energy eigenbasis $\{|E_i\rangle\}$ with entropically suppressed fluctuations

$$\langle E_i|\,\mathcal{O}\,|E_j\rangle = f_{\mathcal{O}}(\overline{E})\delta_{ij} + e^{-S(\overline{E})/2}\omega_{\mathcal{O}}(\overline{E},\Delta E)R_{ij}. \quad (1)$$

Here, $f_{\mathcal{O}}$ and $\omega_{\mathcal{O}}$ are smooth $O(1)$ functions, $\overline{E} = \frac{E_i+E_j}{2}$, $\Delta E = |E_i - E_j|$, and R_{ij} are pseudorandom numbers with unit variance. This describes thermalization because it implies that expectation values of simple operators can be accurately approximated using only the thermodynamic parameters such as energy. This statement applies to late-time states following time evolution and the energy eigenstates themselves.

In this Letter, we ask if there is an analog of (1) for a large class of non-Hermitian Hamiltonians that characterizes dissipative quantum chaos. Certain features of Hermitian quantum chaos, such as level repulsion of eigenvalues [26], remain when relaxing Hermiticity [27], while others, such as volume-law entanglement [28], do not [12]. Answering whether non-Hermitian Hamiltonians obey ETH is an open question, requiring different techniques from the Hermitian case. Non-Hermitian operators generally have distinct left and right eigenvectors, $\{\langle L_i|\}$ and $\{|R_i\rangle\}$, that form a biorthonormal basis

$$\langle L_i|\,R_j\rangle = \delta_{ij}. \quad (2)$$

Surprisingly, we observe that the eigenstates of the non-Hermitian version of the Sachdev-Ye-Kitaev (SYK) model [21–23], the canonical model of many-body quantum chaos [29–31], do *not* satisfy the ETH; the fluctuations, both on and off the diagonal, are just as large as the mean for expectation values of simple operators. The stark contrast between matrix elements for the Hermitian and non-Hermitian SYK (nSYK) models is shown in Fig. 1. In the Supplemental Material [32] we also investigate ETH in a local non-Hermitian spin chain observing a similar behavior. The N_ψ Majorana fermion Hamiltonians are of the form

$$H_{\text{nSYK}} = \sum_{i_1<i_2<i_3<i_4}^{N_\psi} (J_{i_1i_2i_3i_4} + iM_{i_1i_2i_3i_4})\psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4}, \quad (3)$$

where $J_{i_1i_2\ldots i_q}$ and $M_{i_1i_2\ldots i_q}$ are independent and identically distributed (i.i.d.) real Gaussian random variables with zero mean, variance $\frac{2}{N_\psi^3}$, and $\{\psi_i,\psi_j\} = 2\delta_{ij}$. $M_{i_1i_2\ldots i_q} = 0$ in the Hermitian case.

We will explain this striking phenomenon analytically by modeling non-Hermitian quantum chaotic Hamiltonians by random matrices. The connection between chaotic quantum systems and random matrix theory for both Hermitian and non-Hermitian systems is well known [27,33,34] and provides a simple way to motivate (1). Indeed, it is widely believed

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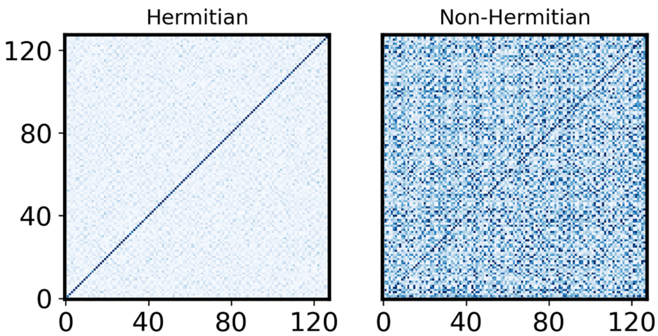

FIG. 1. The absolute value of the matrix elements of the single-site number operators, $\hat{n}_{ij} := |\langle L_i|\,\hat{n}\,|R_j\rangle|$, are shown for a single realization of the Hermitian SYK model (left) and non-Hermitian SYK model (right) with $N_\psi = 14$. The x and y axes label the left and right eigenvectors, respectively. While $\hat{n}_{ij}$ is almost exclusively supported on the diagonal in the Hermitian case, there are large fluctuations across the entire matrix in the non-Hermitian case.

that nonintegrable quantum systems exhibit the same spectral statistics as Hermitian random matrices. In the context of random Hamiltonians, "simple" operators are deterministic operators, independently chosen from the Hamiltonian. For $N \times N$ Wigner matrices, (1) has been proven rigorously [35,36].

In order to model typical chaotic non-Hermitian Hamiltonians, we study the eigenvectors of matrices drawn from the Ginibre ensemble defined by matrices with i.i.d. complex Gaussian matrix elements [37]. We compute the behavior of the matrix elements of deterministic operators, precisely characterizing the large fluctuations. These phenomena, not seen in the Hermitian case, include the aforementioned large fluctuations as well as correlations between matrix elements [38]. Moreover, we demonstrate that our rigorous results for the Ginibre ensemble are *universal*, with extended regimes of validity in highly non-Gaussian ensembles such as the complex Bernoulli and uniform ensembles.

Structure of Ginibre eigenvectors. Let H be an $N \times N$ matrix drawn from the complex Ginibre ensemble. We normalize the entries of H so that the limiting eigenvalue distribution is given by the uniform distribution on the unit disk. We denote its (nonordered) eigenvalues by $\lambda_1, \dots, \lambda_N$ and the corresponding biorthonormal left and right eigenvectors by $\langle L_i|\,, |R_i\rangle$. The distribution of H is invariant under unitary conjugation. Following Refs. [39–41], we write $H = UTU^\dagger$, with U independent of T [42] and uniformly distributed on the unitary group, and

$$T := \begin{pmatrix} \lambda_1 & T_{12} & \dots & T_{1N} \\ 0 & \lambda_2 & \dots & T_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \lambda_N \end{pmatrix}, \tag{4}$$

where T_{ij} are uncorrelated Gaussian random variables. The first two eigenvectors of T are of the form

$$|\widetilde{R}_1\rangle = (1, 0 \dots, 0)^T, \quad |\widetilde{R}_2\rangle = (a, 1, 0 \dots, 0)^T,$$
$$|\widetilde{L}_1\rangle = (b_1, \dots, b_N)^T, \quad |\widetilde{L}_2\rangle = (d_1, \dots, d_N)^T, \tag{5}$$

with $b_1 = 1, d_1 = 0, d_2 = 1, a = -b_2^*$. Additionally, using the relation $H = UTU^\dagger$, we can express the left/right eigenvectors of H as

$$|L_i\rangle = (U^\dagger)^T\,|\widetilde{L}_i\rangle\,, \quad |R_i\rangle = U\,|\widetilde{R}_i\rangle\,. \tag{6}$$

Eigenvector overlaps. A fundamental quantity in the analysis of the spectrum of H are the so-called eigenvector overlaps

$$O_{ij} := \langle R_j|\,R_i\rangle\,\langle L_i|\,L_j\rangle. \tag{7}$$

In contrast to the eigenvectors of Hermitian Hamiltonians, where $\langle R_i|\,R_j\rangle = \langle L_i|\,L_j\rangle = \delta_{ij}$, the eigenvector overlaps can be quite large and highly correlated. We point out that the study of the quantity (7) has attracted significant interest in recent years [39–41,43–47].

From now on, even if not stated explicitly, the left/right eigenvectors $|L_i\rangle\,, |R_i\rangle$ depend on U and T through the relation (6). We will only need to consider overlaps involving one or two eigenvectors, which will be denoted by O_{11}, O_{12}, O_{22}, i.e., we drop the i, j dependence. These overlaps can be expressed in terms of the quantities appearing in (5) as

$$O_{11} = \sum_{i=1}^{N} |b_i|^2, \quad O_{12} = -b_2^* \sum_{i=2}^{N} b_i d_i^*,$$
$$O_{22} = (1 + |b_2|^2) \sum_{i=2}^{N} |d_i|^2. \tag{8}$$

Critical to our analysis are Theorems 1.1 and 1.4 of Ref. [43], where the overlaps are evaluated at large N (see also Ref. [44] for the diagonal overlaps). The full distribution of the diagonal overlaps converges to

$$O_{11} \to \frac{N(1 - |\lambda_1|^2)}{\gamma_2}, \tag{9}$$

with γ_2 being a gamma random variable with density xe^{-x}. The other overlaps that we will make use of are only known in expectation and in second moment (for two typical eigenvalues λ_1, λ_2 in the bulk of the spectrum),

$$\mathbb{E}_{U,T} O_{12} \to -\frac{1 - \lambda_1 \lambda_2{}^*}{N|\lambda_1 - \lambda_2|^4}, \tag{10}$$

$$\mathbb{E}_{U,T} |O_{12}|^2 \to \frac{(1 - |\lambda_1|^2)(1 - |\lambda_2|^2)}{|\lambda_1 - \lambda_2|^4}, \tag{11}$$

$$\mathbb{E}_{U,T} O_{11} O_{22} \to \frac{(1 - |\lambda_1|^2)(1 - |\lambda_2|^2)}{|\lambda_1 - \lambda_2|^4} \times \frac{1 + N^2|\lambda_1 - \lambda_2|^4 - e^{-N|\lambda_1 - \lambda_2|^2}}{1 - e^{-N|\lambda_1 - \lambda_2|^2}}. \tag{12}$$

The expectations above are with respect to the whole Ginibre matrix H. Recall that the typical distance between nearby eigenvalues is $\sim N^{-1/2}$, when the last line of (12) is actually independent of N. In the regime, $|\lambda_1 - \lambda_2| \gg N^{-1/2}$ (12) simplifies to $\mathbb{E} O_{11} O_{22} \to \mathbb{E} O_{11} \mathbb{E} O_{22}$.

To study the ETH we consider $\langle L_i|\,A\,|R_j\rangle$, and their correlations. In particular, we will compute the expectation and the variance for these quantities using Weingarten calculus, which provides closed form expressions for integrals of the unitary group with respect to the Haar measure [48]:

$$\int [dU] U_{i_1,j_1} \cdots U_{i_n,j_n} U^*_{i'_1,j'_1} \cdots U^*_{i'_n,j'_n} = \sum_{\sigma,\tau \in S_n} \delta_{i_1 i'_{\sigma(1)}} \cdots \delta_{i_n i'_{\sigma(n)}} \delta_{j_1 j'_{\tau(1)}} \cdots \delta_{j_n j'_{\tau(n)}} \mathrm{Wg}(N, \sigma\tau^{-1}). \quad (13)$$

Here, Wg are the so-called Weingarten functions, whose explicit form may be written in terms of the characters of the symmetric group. For our purposes, we will only need up to the fourth moment of the unitary group. We find the *Mathematica* package RTNI [49] to be helpful in explicit evaluations of the 576 terms in the sum when evaluating the fourth moment in (19) and (20). We will only present the leading order (in N) results.

Importantly, the expectations we compute are only with respect to the U's. The independent random variables in the T matrix are not averaged over, leaving a resulting distribution that aids in conjectures.

Diagonal ETH. We first consider the expectation values of operators, i.e., the diagonal portion of ETH. The Haar integral is straightforward, giving the normalized trace of the observable [32]

$$\mathbb{E}_U \langle L_i| A |R_i\rangle = \frac{\mathrm{Tr}(A)}{N}. \quad (14)$$

To estimate the average size, we instead need the square

$$\mathbb{E}_U |\langle L_i|A|R_i\rangle|^2 = \frac{1}{N^2}|\mathrm{Tr}(A)|^2 + \frac{O_{11}}{N^2}[\mathrm{Tr}(A^\dagger A) - N^{-1}|\mathrm{Tr}(A)|^2]. \quad (15)$$

From (9) we see that the second line is the same order as the first, so the fluctuations in the expectation values are not suppressed, strongly violating (1).

We may expect that at large N (for a motivation see the *Universality* section below)

$$\frac{N}{(1-|\lambda_i|^2)\mathrm{Tr}(A^\dagger A)} |\langle L_i|A|R_i\rangle - N^{-1}\,\mathrm{Tr}(A)|^2 \to \frac{|\xi|^2}{\gamma_2}, \quad (16)$$

in distribution, where ξ is a complex Gaussian random variable independent of γ_2. The probability density function of (16) is

$$p(x) = \frac{2}{(1+x)^3}. \quad (17)$$

Due to the γ_2 in the denominator of (16), there is a heavy tail in this distribution, a characteristic of non-Hermitian systems, also seen in the entanglement spectrum of typical states [12], which is in stark contrast with the Hermitian case. We verify this expectation numerically in Fig. 2.

Off-diagonal ETH. Off of the diagonal, the ETH implies that the matrix elements are exponentially suppressed in the entropy. Because there is a scaling ambiguity in (2) taking $|R_i\rangle \to c_i |R_i\rangle$ and $\langle L_i| \to c_i^{-1} \langle L_i|$, a well-defined (scale-independent) notion of the off-diagonal elements is $\langle L_i| A |R_j\rangle \langle L_j| A |R_i\rangle$. Averaging over unitaries, we

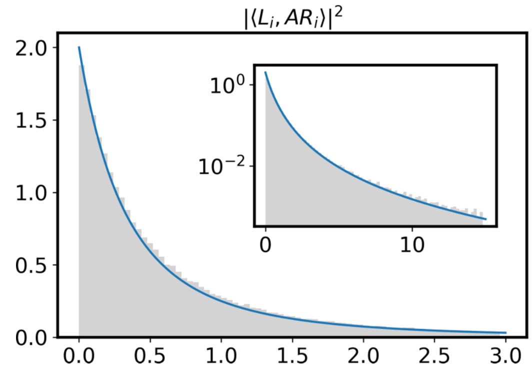

FIG. 2. The absolute value squared of the matrix elements weighted as in the left-hand side of (16) of a deterministic matrix. We take 1000 realizations for $N = 200$. The blue line is (17). The inset is the same data but with logarithmic scaling of the y axis. We have taken A to be independently Ginibre to speed up convergence. The result is identical for all choices of A.

find (for $i \neq j$)

$$\mathbb{E}_U \langle L_i| A |R_j\rangle \langle L_j| A |R_i\rangle = \frac{1}{N^2}[\mathrm{Tr}(A^2) - N^{-1}\mathrm{Tr}(A)^2]. \quad (18)$$

There is no O_{12} dependence because we have not taken the complex conjugate of the second term. While the above expression is suppressed in N, the average size is large which can be seen from the absolute value squared

$$\mathbb{E}_U |\langle L_i|A|R_j\rangle\langle L_j|A|R_i\rangle|^2 = \frac{O_{11}O_{22}\mathrm{Tr}[A^\dagger A]^2 + |O_{12}|^2\mathrm{Tr}[A^\dagger A]^2}{N^4}, \quad (19)$$

where the O_{ij}'s are random variables defined in (7). From (9) and (12), we see that the expectation of the above equation is $O(1)$, so there is no entropic suppression off the diagonal, our second exhibition of a strong violation of the ETH. The second term is only the same order as the first when the eigenvalues are very close.

Correlation for nearby energies. The final feature in non-Hermitian Hamiltonians that we explore is the large correlation between nearby eigenvalues. This means that even though there are large fluctuations in the diagonal elements [see (15)], these fluctuations are correlated. For simplicity, we present the answer for traceless observables

$$\mathbb{E}_U |\langle L_i|A|R_i\rangle\langle L_j|A|R_j\rangle|^2 - \mathbb{E}_U |\langle L_i|A|R_i\rangle|^2 \mathbb{E}_U |\langle L_j|A|R_j\rangle|^2 = \frac{|O_{12}|^2\,\mathrm{Tr}[A^\dagger A]^2}{N^4}. \quad (20)$$

As seen from (11), the correlation is large when the eigenvalues are very close ($\lambda_i - \lambda_j \sim N^{-1/2}$) but quickly decays as the eigenvalues are separated.

Universality. We expect that the results that we have derived for Ginibre matrices are universal. In particular, if we consider a matrix H with i.i.d. entries but not necessarily with Gaussian distribution (or even with some specific correlation structure), then the same convergence results should hold.

To see this we rely on the *Hermitization trick* from Ref. [50] and used in Ref. [12]. The following discussion is similar to the one presented in Ref. [12], but we repeat it here for the reader's convenience. More precisely, we define the Hermitian matrix

$$H^z := \begin{pmatrix} 0 & H-z \\ (H-z)^\dagger & 0 \end{pmatrix}, \tag{21}$$

with $z \in \mathbb{C}$. Observing that

$$\lambda \in \mathrm{Spec}(H) \Leftrightarrow 0 \in \mathrm{Spec}(H^\lambda), \tag{22}$$

one can use H^z to study spectral properties of H itself when z is very close to one of its eigenvalues. We point out that H^z satisfies a chiral symmetry which induces a symmetric spectrum about zero. As a consequence, the eigenvectors $|\mathbf{w}^z_{\pm i}\rangle$ are of the form $|\mathbf{w}^z_{\pm i}\rangle = (|\mathbf{u}^z_i\rangle, \pm |\mathbf{v}^z_i\rangle)$, with $|\mathbf{u}^z_i\rangle, |\mathbf{v}^z_i\rangle \in \mathbb{C}^N$. Here, $|\mathbf{u}^z_i\rangle, |\mathbf{v}^z_i\rangle$ denote the left and right singular vectors of $H - z$, i.e.,

$$(H-z)\left|\mathbf{v}^z_i\right\rangle = E^z_i \left|\mathbf{u}^z_i\right\rangle, \quad (H-z)^\dagger \left|\mathbf{u}^z_i\right\rangle = E^z_i \left|\mathbf{v}^z_i\right\rangle, \tag{23}$$

with $E^z_i \geqslant 0$ the corresponding singular values. Since the singular values coincide with the eigenvalues of H^z (in absolute value), these two representations are equivalent. We choose to present both since in the following discussion it may be more convenient to refer to one or the other.

Using the relation (23) and the definition of left and right eigenvectors $\langle L_i|, |R_i\rangle$, we write

$$\langle L_i| A |R_i\rangle = \frac{\langle \mathbf{u}| A |\mathbf{v}\rangle}{\langle \mathbf{u}|\, \mathbf{v}\rangle}, \tag{24}$$

Here, $|\mathbf{u}\rangle = |\mathbf{u}^{\lambda_i}_1\rangle$, $|\mathbf{v}\rangle = |\mathbf{v}^{\lambda_i}_1\rangle$. The key feature of the equality (24) is that we can express the non-Hermitian quantity in the left-hand side (LHS) in terms of Hermitian singular vectors, which are much better understood. For fixed z, the distribution of $|\mathbf{u}^z_i\rangle, |\mathbf{v}^z_i\rangle$ can be understood using the Dyson Brownian motion (DBM) for eigenvectors introduced in Ref. [51] (see also Refs. [36,52–57] for a detailed explanation of the DBM analysis in the mathematics literature). More precisely, in order to apply DBM, for fixed z we write

$$\langle \mathbf{u}^z_1| A |\mathbf{v}^z_1\rangle = \langle \mathbf{w}^z_1| \mathcal{A} |\mathbf{w}^z_1\rangle, \quad \mathcal{A} := \begin{pmatrix} 0 & A \\ 0 & 0 \end{pmatrix}, \tag{25}$$

and notice that (25) consists of a quadratic form with Hermitian eigenvectors $|\mathbf{w}^z_1\rangle$ for which the Hermitian DBM techniques from Refs. [36,51–57] can apply. We refer the interested reader to Ref. [12] and the Supplemental Material [32] for a gentle introduction to the DBM analysis for Hermitian eigenvectors. In particular, as a consequence of the representation (25) we can conclude that $N^{-2}(\mathrm{Tr}[A^\dagger A])^{-1}|\langle \mathbf{u}^z_1| A |\mathbf{v}^z_1\rangle|$ is (the absolute value of) a standard Gaussian [see the argument above in Ref. [12], Eq. (13)]. This gives an insight to motivate the convergence in (16), i.e., to conclude that the numerator of (24) converges to a Gaussian; the fact that the square of the denominator in (24) converges to a χ_2 random variable follows similarly. However, here there is a caveat; the equality in (24) holds only for $z = \lambda_i$ in (23), i.e., when z is not fixed independently but rather equal to an eigenvalue. Even though computing the distribution of (25) for fixed z is not enough to compute the distribution of the LHS of (24), we expect that this gives the correct answer. Indeed, we also confirm this universality phenomenon numerically in Fig. 3 where we consider two non-Hermitian random matrix ensembles obeying the circular law (eigenvalues uniformly distributed in unit circle). These are the complex Bernoulli ensemble (entries are independent $\pm$ real and imaginary parts) and random complex uniform ensemble (entries are uniformly drawn from the complex unit circle).

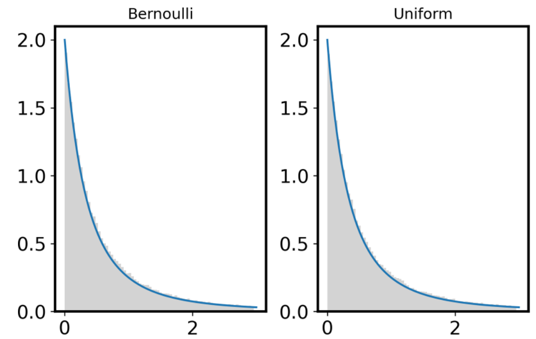

FIG. 3. The absolute value squared of the matrix elements weighted as in (16) are shown for the complex Bernoulli and random complex uniform matrices. The blue line is (17), motivating universality. We take 1000 realizations for $N = 200$ and a fixed deterministic matrix A.

Returning to the non-Hermitian SYK model, we note that all of the qualitative phenomena observed in Fig. 1 have now been explained from the Ginibre ensemble. However, we do not expect the equations for the Ginibre ensemble to quantitatively agree with the SYK model because the eigenvalues of the SYK model do not obey the circular law. We do still expect that, after performing the proper z rescaling the LHS of (16), the expectation values of operators are distributed as in (17). This is analogous to the Hermitian case where analytical results in the Gaussian ensembles only qualitatively agree with chaotic Hamiltonians because the eigenvalues of generic Hamiltonians do not obey the semicircle law. A certain unfolding is necessary.

Discussion. In this Letter, we have shown that the eigenstate thermalization hypothesis is strongly violated in non-Hermitian many-body systems. While initially observed in the SYK model, we gained an analytical understanding of this general phenomenon by modeling quantum chaotic non-Hermitian Hamiltonians using random matrix theory. We rigorously proved various features of the matrix elements of simple operators. These include (1) large fluctuations along the diagonal in the energy eigenbasis, of the same order as the mean, (2) fluctuations off the diagonal that were just as large as those on the diagonal, and (3) strong correlations between matrix elements with nearby energies. None of these phenomena are seen in quantum chaotic Hermitian Hamiltonians.

Non-Hermitian Hamiltonians possess strikingly different features from their Hermitian counterparts. The developing field of non-Hermitian random matrix theory presents a powerful tool set, which has only just begun to be applied, for uncovering universal phenomena in the emerging field of dissipative quantum chaos. We hope to make connections to dissipative dynamics in the near future.

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