

## Erratum: Orbital Hall effect as an alternative to valley Hall effect in gapped graphene [Phys. Rev. B **103**, 195309 (2021)]

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We report a missing phase factor in  $f(\vec{k})$  given in Eq. (16) of the original paper. The correct expression for  $f(\vec{k})$  is given below:

$$f(\vec{k}) = t \exp(-ik_y a) [1 + 2 \cos(\sqrt{3}k_x a/2) \exp(i3k_y a/2)]. \quad (1)$$

This consequently modifies the expressions of Berry curvature  $\Omega_v^z$  in Eq. (24) and the orbital moment  $m_v^z$  in Eq. (21), as given below,

$$\Omega_v^z = \frac{\sqrt{3}\Delta a^2 t^2 g(\vec{k})}{[\Delta^2 + |f(\vec{k})|^2]^{\frac{3}{2}}}, \quad (2)$$

$$m_v^z = -\frac{e}{\hbar} \frac{\sqrt{3}\Delta a^2 t^2 g(\vec{k})}{[\Delta^2 + |f(\vec{k})|^2]}, \quad (3)$$

where

$$g(\vec{k}) = \sin\left(\vec{k} \cdot \frac{\vec{\delta}_1 - \vec{\delta}_2}{2}\right) \sin\left(\vec{k} \cdot \frac{\vec{\delta}_2 - \vec{\delta}_3}{2}\right) \sin\left(\vec{k} \cdot \frac{\vec{\delta}_3 - \vec{\delta}_1}{2}\right). \quad (4)$$

where  $\vec{\delta}_1 = (a/2)(-\sqrt{3}\hat{i} + \hat{j})$ ,  $\vec{\delta}_2 = (a/2)(\sqrt{3}\hat{i} + \hat{j})$ , and  $\vec{\delta}_3 = -a\hat{j}$ . These formulas are manifestly  $C_3$ -invariant and are in agreement with the ones previously reported in Ref. [1].

The error did not affect the low-energy model Hamiltonian, and accordingly all the results of the original paper near the valley points, relevant to gapped graphene, remain unaffected. For larger values of  $\Delta$ , however,  $k$  points away from the valley also contribute, leading to a deviation in the quantitative results of orbital Hall conductivity (see Fig. 1). Qualitatively, the results do not change.

In addition to this, we report a typo in Eq. (15) of the original paper. The correct equation should be  $\mathcal{H}(\vec{k}) = \sum_{\vec{k}, \alpha, \beta} \hat{c}_{\vec{k}\alpha}^\dagger H_{\alpha, \beta}(\vec{k}) \hat{c}_{\vec{k}\beta}$ .

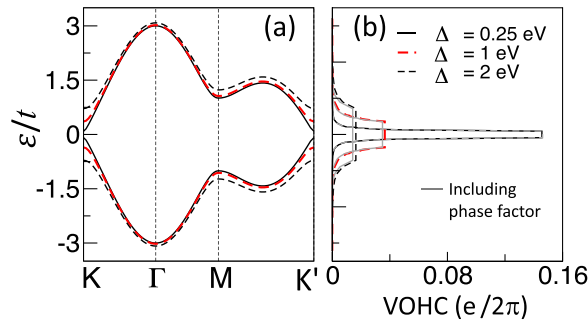


FIG. 1. The valley orbital Hall conductivity (VOHC) as a function of energy for three different values of  $\Delta$  including the effect of the missing phase factor in Eq. (1) shown in gray color. VOHC does not change for parameters relevant to gapped graphene. However, larger values of  $\Delta$  lead to small deviation.

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[1] J. N. Fuchs, F. Piéchon, M. O. Goerbig, and G. Montambaux, Topological Berry phase and semiclassical quantization of cy-

clotron orbits for two dimensional electrons in coupled band models, [Eur. Phys. J. B \*\*77\*\*, 351 \(2010\)](#).