Axion electrodynamics without Witten effect in metamaterials

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Artificial media provide a unique playground to test fundamental theories, allowing one to probe the laws of electromagnetism in the presence of hypothetical axions. While some materials are known to realize this physics, here we propose the nonlocal extension of axion electrodynamics. Compared to the usual axion case, the suggested metamaterial features similar optical properties including Kerr and Faraday rotation. However, the external sources in this structure do not induce dyon charges, eliminating the well-celebrated Witten effect. We put forward the design of such a nonreciprocal nonlocal metamaterial and discuss its potential applications.

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I. INTRODUCTION

The axion field was proposed to resolve the strong *CP* problem in quantum chromodynamics (QCD) [1–3]. Since then, its fundamental quanta, axions, have attracted the interest of the physics community including such areas as high-energy physics, cosmology, and string theory [4–6]. Since the axion is anticipated to be extremely light and weakly interacting with ordinary matter, this hypothetical particle or its generalizations known as axionlike particles (ALPs) are viewed as promising candidates for cosmological dark matter [7–11].

The coupling between axions and electromagnetic fields is captured by the axion electrodynamics [12,13] which features an additional term in the Lagrangian density $\mathcal{L}_\theta \sim \kappa a(x)\mathbf{E}\cdot\mathbf{B}$, where $a(x)$ is the axion field and κ is the axion-photon coupling. An important prediction of the theory is the Witten effect [13–15]: magnetic charge in a vacuum bubble surrounded by the axion medium gives rise to effective dyon charges, i.e., a combination of electric and magnetic monopole fields [Fig. 1(a)].

Axion electrodynamics has found a number of realizations in condensed matter physics [16,17] and, recently, in metamaterials [18–20], since a low-energy effective description of those structures yields the axion Lagrangian. Alternatively, the same physics can be viewed through the prism of constitutive relations. In such case, the equations of axion electrodynamics correspond to the special nonreciprocal bianisotropic medium with the constitutive relations of the form $\mathbf{D} = \varepsilon\,\mathbf{E} + \theta(x)\mathbf{B}$ and $\mathbf{H} = \mu^{-1}\,\mathbf{B} - \theta(x)\mathbf{E}$, known in the photonics community as Tellegen media. Evidently, the θ term in these equations preserves $\mathcal{PT}$ symmetry, while breaking $\mathcal{P}$ and $\mathcal{T}$ symmetries separately.

In this paper, we make the next conceptual step showing that the metamaterial platform allows one not only to implement synthetic axion fields, but actually go beyond that probing nonlocal extensions of axion electrodynamics and associated exotic physics. While the suggested metamaterial responds to plane-wave excitation similarly to the standard axion case, its interaction with the external localized sources is fundamentally different. In particular, the Witten effect in this structure, further termed the ψ metamaterial, vanishes. Below, we explore the physics of the ψ medium, put forward its possible experimental implementation, and discuss potential applications.

II. ψ ELECTRODYNAMICS

We investigate a metamaterial described by the constitutive relations

$$\mathbf{D}_\psi = \varepsilon\mathbf{E} + \psi\nabla^2\mathbf{B}, \tag{1}$$

$$\mathbf{H}_\psi = \mu^{-1}\mathbf{B} - \psi\nabla^2\mathbf{E}. \tag{2}$$

The ψ term in these equations shares with the axion field θ the same symmetry properties with respect to $\mathcal{P}$ and $\mathcal{T}$; explicit breaking of $\mathcal{T}$ symmetry indicates nonreciprocity of the medium. However, differently from the axion case, the constitutive relations contain spatial derivatives of the field pointing towards electromagnetic nonlocality. If formulated in terms of the effective Lagrangian, which is typically an approximate description in the metamaterials context, this corresponds to the higher-order derivative extension of Maxwell-Chern-Simons electrodynamics [21]. Related extensions have been studied theoretically in high-energy physics [22–24].

Using Eqs. (1) and (2) with spatially varying ψ and assuming the CGS system of units, we present Maxwell's equations in the form

$$\nabla\cdot(\varepsilon\mathbf{E}) = 4\pi\rho - \nabla\psi\cdot\nabla^2\mathbf{B}, \tag{3}$$

$$\nabla\times(\mu^{-1}\mathbf{B}) - \frac{1}{c}\frac{\partial(\varepsilon\mathbf{E})}{\partial t} = \frac{4\pi}{c}\mathbf{J} + \nabla\psi\times\nabla^2\mathbf{E}, \tag{4}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{5}$$

$$\nabla\times\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = \mathbf{0}, \tag{6}$$

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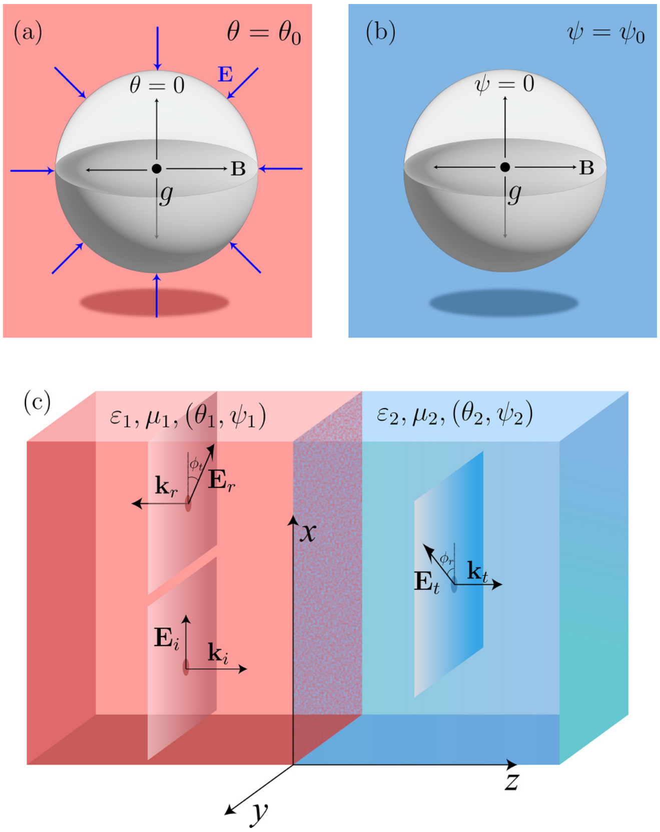

FIG. 1. Comparison of the electromagnetic properties between the θ and ψ response. (a) Witten effect: an electric field induced by the magnetic monopole g in a vacuum bubble surrounded by the constant axion field θ_0. Black and blue arrows denote the force lines of the magnetic and electric field, respectively. (b) In the case of ψ electrodynamics there is no induced electric field for magnetic charge in a vacuum bubble. (c) Light transmitted and reflected at θ or ψ interface experiences the rotation of the polarization plane.

where ε is the permittivity and μ the permeability of the metamaterial. Extra terms arising in Eqs. (3) and (4) can be regarded as effective charge and current densities: $\rho_\psi = -\frac{1}{4\pi}\nabla\psi \cdot \nabla^2\mathbf{B}$, $\mathbf{J}_\psi = \frac{c}{4\pi}\nabla\psi \times \nabla^2\mathbf{E}$. Thus, if ψ is constant, effective charge and current densities vanish, yielding the conventional Maxwell's equations.

While deriving Eqs. (3) and (4), we implicitly assume that $\nabla \cdot (\nabla^2 \mathbf{B}) = \nabla^2(\nabla \cdot \mathbf{B})$, which holds for smoothly varying material parameters, but requires careful analysis [21] in the presence of the interfaces.

As a simplest nontrivial example, we consider a piecewise-constant function ψ that takes the constant value ψ_1 in a region R_1 and ψ_2 in a region R_2, defining the boundary of the two media. The gradient of the piecewise function ψ is

$$\nabla\psi = -\tilde{\psi}\,\delta(\Sigma)\hat{\mathbf{n}}, \quad \tilde{\psi} = \psi_1 - \psi_2, \tag{7}$$

where Σ is the boundary of the regions R_1 and R_2 and $\hat{\mathbf{n}}$ is a unit vector normal to the boundary pointing from R_1 to R_2 (Fig. 2). Hence, similarly to the axion case [25], the only modification of Maxwell's equations occurs at the boundary due to the additional boundary currents.

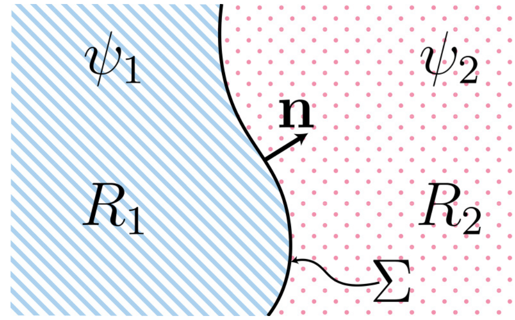

FIG. 2. Interface between two distinct ψ metamaterials with different permittivity and permeability.

However, since the material is nonlocal, the structure of the boundary conditions strongly depends on metamaterial termination and could be engineered without affecting the bulk properties. Examples of such termination-dependent boundary conditions include wire media [26] and multilayered metamaterials [27]. As a physically motivated choice [28,29], we consider the following boundary conditions (BCs) [21],

$$[\mathbf{D}]_n = \tilde{\psi}\,\nabla^2\mathbf{B} \cdot \hat{\mathbf{n}}|_\Sigma, \quad [\mathbf{H}]_\parallel = -\tilde{\psi}\,\nabla^2\mathbf{E} \times \hat{\mathbf{n}}|_\Sigma, \tag{8}$$

$$[\mathbf{B}]_n = \mathbf{0}, \quad [\mathbf{E}]_\parallel = \mathbf{0}, \tag{9}$$

where $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{H} = \mu^{-1}\mathbf{B}$, and $[\mathbf{A}]_n = (\mathbf{A}^{(2)} - \mathbf{A}^{(1)}) \cdot \hat{\mathbf{n}}$ and $[\mathbf{A}]_\parallel = (\mathbf{A}^{(2)} - \mathbf{A}^{(1)}) \times \hat{\mathbf{n}}$. Here, $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are the fields in the regions 1 and 2 close to the boundary Σ. To make the problem well defined, the Laplacian in Eq. (8) is understood as a half sum of the field Laplacians from both sides of the interface.

III. ELECTROMAGNETIC WAVES IN ψ METAMATERIAL

Having the set of BCs, we examine the propagation of electromagnetic waves through the planar interface $z = 0$ between two metamaterials with ψ_1, ε_1, μ_1 for $z < 0$ and ψ_2, ε_2, μ_2 for $z > 0$, respectively, using the conventional ansatz for the plane-wave solutions:

$$\begin{pmatrix} \mathbf{E}(t,\mathbf{r}) \\ \mathbf{B}(t,\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}(\omega,\mathbf{k}) \\ \mathbf{B}(\omega,\mathbf{k}) \end{pmatrix} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}. \tag{10}$$

The fields of the plane wave $E_{\alpha,\beta}(\omega,\mathbf{k})$, $B_{\alpha,\beta}(\omega,\mathbf{k})$ are labeled by the two indices α and β. The first index $\alpha = i, r, t$ distinguishes incident, reflected, and transmitted waves, while the second index $\beta = \perp, ||$ denotes transverse electric (TE) or transverse magnetic (TM) polarization of the wave, respectively.

In the plane-wave scenario, the Laplacian term in Eq. (8) acts as

$$\nabla^2 \to -k_0^2\,\overline{\varepsilon\,\mu}, \tag{11}$$

where $k_0 = \omega/c$ and $\overline{\varepsilon\,\mu} = (\varepsilon_1\mu_1 + \varepsilon_2\mu_2)/2$. As a result, the BCs are identical to those in axion electrodynamics with

$$\theta_1 - \theta_2 = -(\psi_1 - \psi_2)\,k_0^2\,\overline{\varepsilon\,\mu}\,. \tag{12}$$

Hence, the response of the metamaterial to the plane-wave excitation is indistinguishable from the axion case. In particular, transmitted and reflected light experience the rotation of the polarization plane, $\tan\phi_\alpha = E_{\alpha,\perp}/E_{\alpha,\parallel}$, known as Faraday and Kerr rotations, respectively,

$$\tan\phi_t = \frac{\tilde{\psi}\, k_0^2\, \overline{\varepsilon\,\mu}}{\sqrt{\frac{\varepsilon_1}{\mu_1}} + \sqrt{\frac{\varepsilon_2}{\mu_2}}}, \tag{13}$$

$$\tan\phi_r = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}\,\tilde{\psi}\, k_0^2\, \overline{\varepsilon\,\mu}}{\frac{\varepsilon_2}{\mu_2} - \frac{\varepsilon_1}{\mu_1} + \tilde{\psi}^2 k_0^4\, \overline{\varepsilon\,\mu}^2}, \tag{14}$$

where normal incidence is considered and $E_{i,\perp} = 0$. These expressions coincide with those in the axion electrodynamics [30–33] with the geometry of the problem depicted in Fig. 1(c).

The parallel with the axion electrodynamics can be extended further by examining the surface waves at the interface of the two media with distinct values of ψ. Surface waves are superpositions of TE and TM polarizations with the dispersion given by

$$\left(\frac{q}{\mu_1} + \frac{p}{\mu_2}\right)\left(\frac{q}{\varepsilon_1} + \frac{p}{\varepsilon_2}\right) + \tilde{\psi}^2\, k_0^4 (\overline{\varepsilon\,\mu})^2 \frac{pq}{\varepsilon_1\varepsilon_2} = 0, \tag{15}$$

where $q^2 = \beta^2 - k_0^2\mu_1\varepsilon_1 > 0$, $p^2 = \beta^2 - k_0^2\mu_2\varepsilon_2 > 0$, and β is the propagation constant along the interface. This dispersion relation is also consistent with the axion case [34].

IV. WITTEN EFFECT

However, the situation changes when the metamaterial interacts with the localized sources. To illustrate this physics, we consider a magnetic monopole g inside a spherical volume with the radius a surrounded by a ψ metamaterial with nonzero and constant $\psi = \psi_0$ [Fig. 1(b)]. Chosen geometry with a vacuum bubble ensures that higher-order spatial dispersion contributions to the constitutive relations are suppressed and the effective medium analysis remains adequate.

Because of the ψ term in Eq. (3), the shell carries a charge density $-\nabla\psi \cdot \nabla^2\mathbf{B}/(4\pi)$. However, $\nabla^2\,\mathbf{B}$ is only nonzero at the magnetic monopole location and vanishes at the surface of the shell, yielding zero-induced electric charge density. Hence, in contrast to the conventional Witten effect, there are no induced effective dyon charges in the ψ metamaterial. By the same logic, any static distribution of magnetic charges or multipoles inside the vacuum bubble does not induce respective electric charges or multipoles [21]. In this regard, our proposal contrasts also with Weyl semimetals which exhibit a modification of the Witten effect [35,36] due to the gradient of effective axion field. Furthermore, the ψ sphere subjected to the constant electric field does not acquire any magnetic dipole moment [21] which also distinguishes ψ electrodynamics from the conventional axion case.

V. ENGINEERING ψ RESPONSE

While the constitutive relations Eqs. (1) and (2) and the properties of the suggested material look exotic, we demonstrate below that the metamaterial platform can enable such physics.

To come up with a suitable design, we recall that the Tellegen medium, i.e., an artificial structure with the conventional axion response, was originally suggested as a composite where electric and magnetic dipoles are attached to each other so that the incident electric field can orient magnetic dipoles and the incident magnetic field orients electric dipoles [37]. Physically, this can be achieved by engineering the overlapping electric and magnetic dipole resonances of the meta-atom while properly breaking $\mathcal{P}$ and $\mathcal{T}$ symmetries as has been recently elaborated in Ref. [20].

In the same spirit, we suggest to compose a ψ metamaterial from the meta-atoms with overlapping and hybridized higher-order electric and magnetic multipole resonances. Excitation of higher-order multipoles affects the constitutive relations which take the following form [38],

$$\mathbf{D} = \mathbf{E} + 4\pi\left(\mathbf{P} - \tfrac{1}{2}\nabla\cdot\overline{\overline{Q}} + \tfrac{1}{6}\nabla\cdot\nabla\cdot\overline{\overline{\overline{O}}}\right), \tag{16}$$

$$\mathbf{H} = \mathbf{B} - 4\pi\left(\mathbf{M} - \tfrac{1}{2}\nabla\cdot\overline{\overline{S}} + \tfrac{1}{6}\nabla\cdot\nabla\cdot\overline{\overline{\overline{\Xi}}}\right), \tag{17}$$

where $\mathbf{P}$ and $\mathbf{M}$ are polarization and magnetization, respectively, $\overline{\overline{Q}}$ and $\overline{\overline{S}}$ are electric and magnetic quadrupole moments, and $\overline{\overline{\overline{O}}}$ and $\overline{\overline{\overline{\Xi}}}$ are electric and magnetic octupole moments.

Such higher-order multipole moments $\overline{\overline{Q}}$, $\overline{\overline{S}}$, $\overline{\overline{\overline{O}}}$, and $\overline{\overline{\overline{\Xi}}}$ are induced by the incident fields or their gradients. Inspecting Eqs. (16) and (17), we recover that the desired type of the constitutive relations can be obtained, in particular, if the electric quadrupole moment $\overline{\overline{Q}}$ is induced by the gradient of the magnetic field and the magnetic quadrupole moment $\overline{\overline{S}}$ originates from the gradient of the electric field.

The relevant polarizabilities governing the strength of such a response can be calculated using the perturbation theory and read [21]

$$s^e_{\alpha\beta\gamma\sigma} = q^m_{\gamma\sigma\alpha\beta} = \sum_j \mathcal{E}_{jn}\,\mathrm{Re}\,\langle n|S_{\alpha\beta}|j\rangle\,\langle j|Q_{\gamma\sigma}|n\rangle\ . \tag{18}$$

Here, $|n\rangle$ denote the eigenmodes of the particle, $Q_{\gamma\sigma}$ and $S_{\alpha\beta}$ are the operators of the respective components of electric and magnetic quadrupole moments, ω is the frequency of excitation, $\mathcal{E}_{jn} = (2/\hbar)\,\omega_{jn}/(\omega_{jn}^2 - \omega^2)$ is the Lorentz factor, and ω_{jn} are the frequencies of the transitions between the different modes.

This expression, presented in a form resembling a quantum-mechanical one, has two immediate implications. First, to maximize the desired response, electric and magnetic quadrupoles should resonate at the same or close frequencies which allows to neglect the contribution of other off-resonant polarizabilities. Second, the eigenmodes of the particle should be hybrid, containing a mixture of electric and magnetic quadrupoles, thus rendering the two matrix elements simultaneously nonzero.

To meet those two requirements, we design the meta-atom depicted in Fig. 3(a) consisting of two ferrite cylinders with the opposite directions of magnetization. Nonzero magnetization breaks $\mathcal{T}$ symmetry and renders the meta-atom nonreciprocal. In addition, both magnetization vectors change their sign upon mirror reflection in the Oxy plane which guarantees breaking of the $\mathcal{P}$ symmetry allowing the mixing

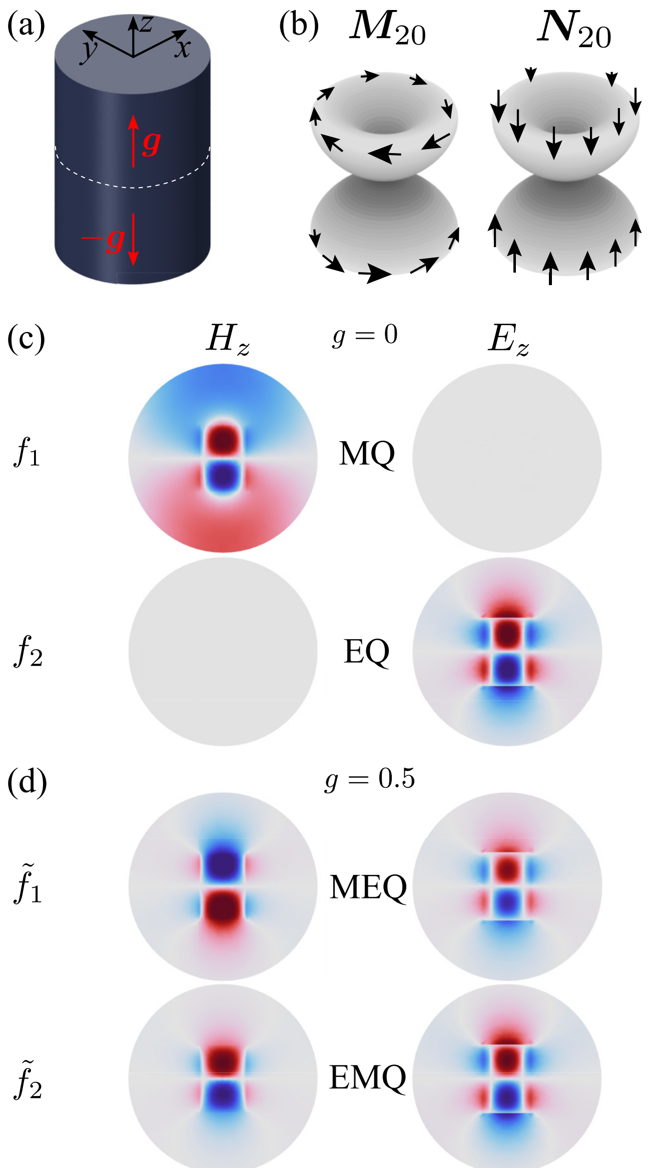

FIG. 3. Design of the meta-atom featuring the ψ response based on two oppositely magnetized ferrites stacked together. (a) Sketch of the proposed meta-atom with the following parameters: permittivity of both cylinders $\varepsilon = 14$, radius $r_0 = 8.2$ mm, and total height $h_0 = 24$ mm. Red arrows show the direction of magnetization, and the white dashed line shows the boundary between the cylinders. (b) Schematic representation of the vector spherical harmonics corresponding to magnetic, $\boldsymbol{M}$, and electric, $\boldsymbol{N}$, quadrupoles ($l = 2$) with $m = 0$. Black arrows denote mutually orthogonal electric field distributions. (c) Field distributions corresponding to the magnetic quadrupole (MQ, $H_z \neq 0$, $E_z = 0$) mode with eigenfrequency $f_1 = 4.59 - 0.01i$ GHz and the electric quadrupole (EQ, $H_z = 0$, $E_z \neq 0$) mode with eigenfrequency $f_2 = 4.68 - 0.04i$ GHz when cylinders are not magnetized ($g = 0$). (d) Field distributions of the eigenmodes with frequencies $\tilde{f}_1 = 4.63 - 0.03i$ and $\tilde{f}_2 = 4.73 - 0.02i$ GHz in the presence of magnetization ($g = 0.5$). Both modes contain a mixture of electric and magnetic quadrupoles. The strength of the magnetoelectric coupling is quantified by the complex ratio $a_{\mathrm{E}}(2,0)/a_{\mathrm{M}}(2,0)$, which is $0.38 + 0.04i$ and $-0.92 - 0.06i$ for the modes $\tilde{f}_1$ and $\tilde{f}_2$, respectively.

of electric and magnetic eigenmodes [39]. Note, however, that the combined $\mathcal{PT}$ symmetry is preserved for this design, which is exactly the symmetry properties needed for the effective axion as well as the effective ψ response.

Each of the constituent cylinders is described in terms of the gyrotropic constitutive relations $\boldsymbol{B} = \mu\boldsymbol{H} + i\boldsymbol{H}\times\boldsymbol{g}$, where $\boldsymbol{g}$ is the gyration vector. In the case of $\boldsymbol{g} = \pm g\hat{z}$, ferrite cylinders have the following antisymmetric permeability tensor,

$$\hat{\mu} = \begin{pmatrix} \mu & \pm i\,g & 0 \\ \mp i\,g & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{19}$$

where + and − signs correspond to upward and downward magnetization, respectively. To prove the feasibility of the ψ-type response, we perform full-wave numerical eigenmodes simulations with $\mu = 2$ and $g = 0.5$ that correspond to the realistic ferrites such as spinels in the frequency range of several GHz. Since we are interested only in a relatively narrow spectral range of quadrupole resonances, the frequency dispersion of μ and g is neglected.

Fields of electric and magnetic quadrupoles are defined by the vector spherical harmonics with indices $l = 2$ and $m = -2, \ldots, 2$ [40]. To avoid degeneracies, we focus on the modes with $m = 0$ as depicted schematically in Fig. 3(b).

As a first step, we simulate the nonmagnetized version ($g = 0$) of the meta-atom. In this case, $\mathcal{P}$ and $\mathcal{T}$ symmetries are not broken and thus electric and magnetic quadrupole modes are perfectly decoupled. Their field profiles are shown in Fig. 3(c).

Next, we assume nonzero magnetization $g = 0.5$ and calculate the mode structure once again. Due to the breaking of time-reversal and spatial inversion symmetries, electric and magnetic modes mix, i.e., field distributions of the modes contain both electric and magnetic quadrupoles [Fig. 3(d)]. To quantify the strength of mixing and relative phase shift, the ratio between the multipole coefficients $a_{\mathrm{M}}(2,0)$ and $a_{\mathrm{E}}(2,0)$ is calculated. Both modes experience significant mixing (MQ/EQ ~ 0.4 and MQ/EQ ~ 1, respectively) and the relative phase shifts between the electric and magnetic quadrupole moments are close to 0 and π, respectively. Strong mixing of electric and magnetic quadrupole modes ensures that the array of such meta-atoms exhibits a strong and resonant ψ-type response. Simulations of plane-wave scattering on the designed meta-atom allow us to estimate maximal values of the ψ response at the level $\psi\,k_0^2 \approx 0.1$.

VI. DISCUSSION AND CONCLUSIONS

In summary, we have put forward a nonlocal generalization of axion electrodynamics featuring exotic electromagnetic properties. While the suggested material responds to the plane-wave excitation in a way similar to the usual axion case, its response to the localized sources is profoundly different exhibiting a vanishing Witten effect. While the respective constitutive relations may seem exotic at the first glance, we prove that they can be implemented experimentally using the platform of photonic metamaterials and optimizing the resonances of the constituent meta-atoms.

We believe that our findings open an exciting research avenue. From the fundamental perspective, of special interest might be the realization of dynamic ψ fields and the possibility to couple them to dark matter axions using such metamaterials in axion search experiments. On the other hand, ψ metamaterials can be of interest on their own, opening routes in the implementation of topologically protected photonic modes, nonreciprocal transmission of light on a chip, nonreciprocal optical elements, or wireless communication [41–43].

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