

## Layer-locked anomalous valley Hall effect in a two-dimensional A-type tetragonal antiferromagnetic insulator

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Antiferromagnetic (AFM) spintronics provides a route towards energy-efficient and ultrafast device applications. Achieving anomalous valley Hall effect (AVHE) in AFM monolayers is thus of considerable interest for both fundamental condensed-matter physics and device engineering. Here we propose a route to achieve an AVHE in A-type AFM insulator composed of vertically stacked monolayer quantum anomalous Hall insulators with strain and electric field modulations. Uniaxial strain and electric field generate valley polarization and spin splitting, respectively. Using first-principles calculations,  $\text{Fe}_2\text{BrMgP}$  monolayer is predicted to be a prototype hosting *valley-polarized quantum spin Hall insulators* in which AVHE and quantum spin Hall effect are synergized in a single system. Our findings reveal a route to achieve multiple Hall effects in two-dimensional tetragonal AFM monolayers.

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### I. INTRODUCTION

Utilizing valley degree of freedom to encode and process information, characterized as valleytronics, provides remarkable opportunities for developing next-generation minimized devices [1–7]. Valley refers to a local energy minimum/maximum in conduction/valence bands, where these energy extremes are robust against phonon and impurity scattering due to the large separation in the momentum space [1]. Recent proposals of valleytronics are mainly based on time-reversal-connected valleys, where valley polarization is induced by an external field, dynamically or electrostatically [8–13]. Intrinsic valleytronics materials with spontaneous valley polarization are more advantageous in terms of valley robustness, energy efficiency, and simplicity in operation, which are beneficial for practical device applications. Recently, the ferrovalley (FV) semiconductor has been proposed [14], which possesses spontaneous valley polarization induced by the combined effects of magnetic order and spin-orbit coupling (SOC). Valley-dependent Berry curvature in FV materials leads to the anomalous valley Hall effect (AVHE). FV materials thus offer an interesting platform to study valley-contrasting transport and Berry physics.

Achieving spontaneous valley polarization typically requires a ferromagnetic (FM) system as a basic premise [15]. Compared with ferromagnetism, antiferromagnetic (AFM)

materials with zero magnetic moment are inherently robust to external magnetic perturbation and possess ultrafast dynamics [16,17], thus offering enormous potential for both valleytronic and spintronic applications. However, spontaneous valley polarization in AFM materials is rarely reported [18–22], and the AVHE in AFM systems is undesirably suppressed [19,20]. Furthermore, topological states based on the valley-polarized quantum anomalous Hall insulator (VQAHI) have been recently proposed, in which valley polarization and the quantum anomalous Hall effect (QAHE) are combined in one material. Such a system is particularly interesting due to its compatibility with low-power electronics, spintronics, sensing, metrology, and quantum information processing applications [23–25]. Several VQAHI systems have been theoretically proposed by constructing complex heterostructure, layer-dependent proximity effects, and accurate regulation of strain and correlation strength [26–40]. It is worth noting that the quantum anomalous Hall state of the “valley-polarized quantum anomalous Hall state” in twisted graphene and transition-metal dichalcogenides occurs exactly because of the valley polarization, and the electron interactions spontaneously polarize the valleys at special fillings in these twisted systems [41–43]. The “valley-polarized quantum anomalous/spin Hall state” referred to in our work means that the valley polarization and quantum anomalous/spin Hall state can coexist, and the valley polarization is not the direct cause of the quantum anomalous/spin Hall state.

It should also be noted that previously reported valley-polarized AFM systems are mostly based on hexagonal symmetric lattice [18–22]. Whether FV can be achieved in

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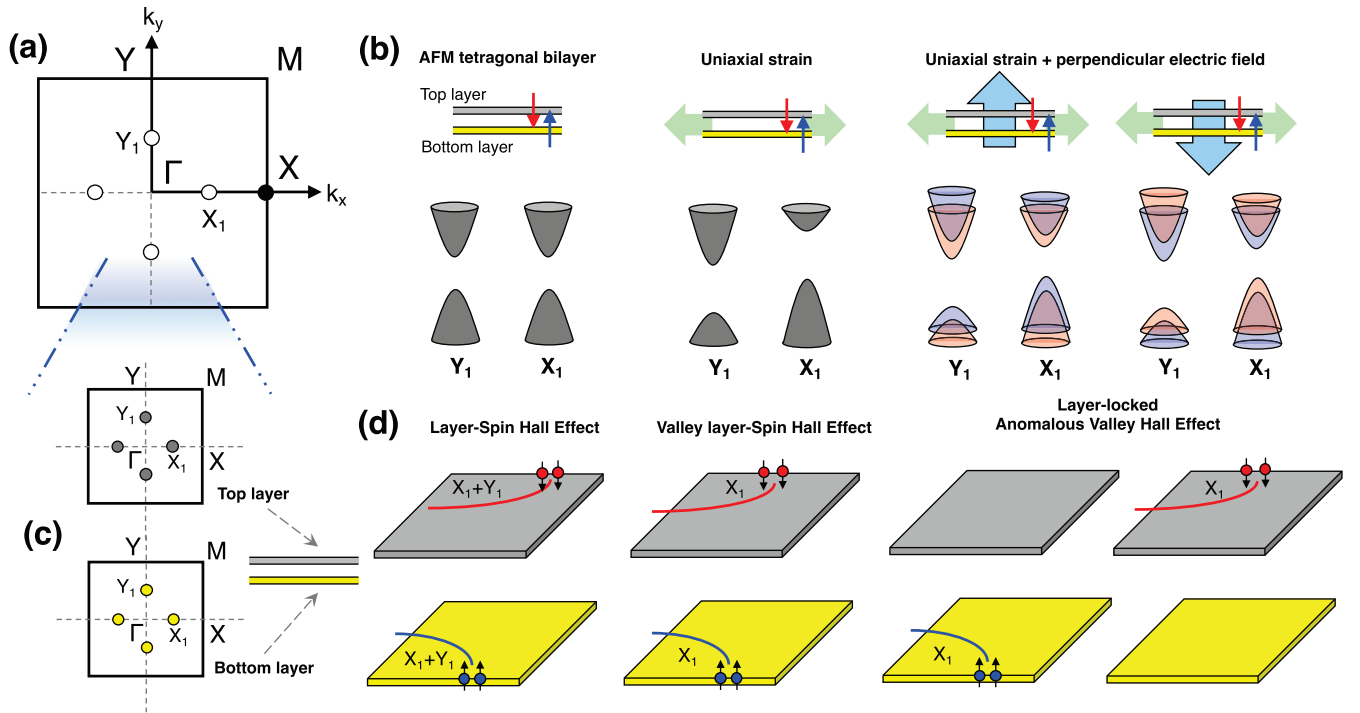


FIG. 1. Concept of layer-locked anomalous valley Hall effect in 2D tetragonal lattice. (a) 2D tetragonal lattice possesses equivalent valleys along  $\Gamma$ - $X$  and  $\Gamma$ - $Y$  lines with Berry curvature mainly occurring around  $Y_1$  and  $X_1$  valleys. (b) Applying uniaxial strain along the  $x$  direction makes the  $Y_1$  and  $X_1$  valleys become unequal but spin degeneracy is still preserved. Simultaneous application of uniaxial strain and the out-of-plane electric field further breaks the spin degeneracy. Reversing the electric field leads to opposite spin splitting at both  $Y_1$  and  $X_1$  valleys. The spin-up and spin-down channels are depicted in blue and red, respectively. (c) Superposition of two tetragonal QAHIs with equal but opposite magnetic moments (A-type AFM order) leads to spin-degenerate equivalent  $Y_1$  and  $X_1$  valleys with net-zero Berry curvature in momentum space. However, the Berry curvatures for the spin-up and spin-down channels are positive and negative, respectively, yielding nonzero layer-locked hidden Berry curvature in real space. (d) The layer-spin Hall effect, the valley layer-spin Hall effect, and the layer-locked anomalous valley Hall effect.

AFM system beyond hexagonal lattice symmetry remains an open question thus far. Can AVHE in AFM materials be achieved in a system beyond hexagonal lattices? Is it possible to achieve other valley-polarized topological states? Here we propose a way to realize AVHE in an A-type tetragonal AFM insulator composed of vertically-stacked monolayered quantum anomalous Hall insulators (QAHI) under both strain and electric field tuning. A peculiar valley-polarized quantum spin Hall insulator (VQSHI) can be achieved in the proposed systems, which is verified via first principles in  $\text{Fe}_2\text{BrMgP}$  monolayer as a prototype VQSHI. Our findings open up a previously unexplored concept of VQSHI in which valley polarization and the quantum spin Hall effect (QSHE) are synergized in a single system.

## II. ACHIEVING AVHE IN AFM INSULATORS

Firstly, a two-dimensional (2D) tetragonal FM QAHI is used as the basic building block, which has a layer of magnetic atoms and possesses equivalent valleys along the  $\Gamma$ - $X$  and  $\Gamma$ - $Y$  lines in the first Brillouin zone (BZ) due to  $C_4$  rotation symmetry [Fig. 1(a)]. The Berry curvatures mainly occur around the  $Y_1$  and  $X_1$  valleys with positive values.

To realize the AFM insulator, a superposition of two 2D tetragonal FM QAHIs with equal but opposite magnetic moments (A-type AFM order) is constructed [Fig. 1(b)], giving

rise to spin degeneracy and equivalent  $Y_1$  and  $X_1$  valleys. Here, we assume that the AFM system has a symmetry of a combination of inversion symmetry  $\mathcal{P}$  and time-reversal symmetry  $\mathcal{T}$  ( $\mathcal{PT}$ ), which leads to a spin-degenerate 2D system. To induce valley polarization, a natural way is to destroy  $C_4$  rotation symmetry via uniaxial strain along the  $x$  or  $y$  direction [Fig. 1(b)]. However, the spin degeneracy is still maintained under uniaxial strain, which prohibits the AVHE. An out-of-plane electric field  $E_{\perp}$  is introduced to break the  $\mathcal{PT}$  symmetry, which lifts the spin degeneracy of valleys [Fig. 1(b)]. Such spin-degeneracy breaking is due to the layer-dependent electrostatic potential  $\propto eEd$  ( $e$  and  $d$  denote the electron charge and the layer distance) created by the out-of-plane electric field, which causes the spin-up and spin-down bands in different layers to stagger, leading to the spin-splitting effect. A similar mechanism can be found in electric potential difference antiferromagnetism [44]. More interestingly, the spin orders at both  $Y_1$  and  $X_1$  valleys can be reversed through reversing the direction of out-of-plane electric field [Fig. 1(b)], thus offering electrostatic-field-tunable valley polarization.

The superposition of two tetragonal QAHIs leads to zero Berry curvature  $\Omega(k)$  in momentum space due to  $\mathcal{PT}$  symmetry [Fig. 1(c)]. However, each layer breaks the  $\mathcal{PT}$  symmetry *individually*, which gives rise to the layer-locked *hidden* Berry curvature, and the Berry curvatures for the spin-up and

spin-down channels are positive and negative-valued, respectively. Such layer-locked hidden Berry curvature leads to a peculiar layer-Hall effect not commonly found in other AVHE systems.

In the presence of a longitudinal in-plane electric field  $E_{\parallel}$ , the Bloch carriers acquire an anomalous transverse velocity  $v_{\perp} \sim E_{\parallel} \times \Omega(k)$  [4]. By shifting the Fermi level in the valence band via hole doping, various layer–spin Hall and layer-locked AVHE can occur [Fig. 1(d)]:

(i) The spin-up and spin-down electrons from  $Y_1$  and  $X_1$  valleys accumulate along opposite sides of different layers in the case of the 2D A-type tetragonal AFM system, resulting in the layer–spin Hall effect.

(ii) When a uniaxial strain is applied, the spin-up and spin-down electrons from only the  $X_1$  valley accumulate along the opposite sides of different layers, resulting in the valley layer–spin Hall effect.

(iii) The spin-up/spin-down electrons from only  $X_1$  valley accumulate along one side of bottom/top layer, resulting in the rarely explored *layer-locked anomalous valley Hall effect*.

### III. MATERIAL REALIZATION

Monolayer  $\text{Fe}_2XY$  ( $X = \text{or } \neq Y = \text{Cl, Br, and I}$ ) and  $\text{Li}_2\text{Fe}_2XY$  ( $X = \text{or } \neq Y = \text{S, Se, and Te}$ ) families [45–50] can be used as the basic building blocks. These monolayers are tetragonal QAHIs with equivalent valleys along the  $\Gamma$ - $X$  and  $\Gamma$ - $Y$  lines in the first Brillouin zone (BZ), and the extremes of Berry curvatures are located at the  $Y_1$  and  $X_1$  valleys. Instead of employing the vertical stacking of two identical monolayers via the van der Waals heterostructure approach, we consider an intercalation architecture in which two identical monolayers are intercalated to form an “ultra-thick”  $\text{Fe}_2XYP$  monolayer ( $X = \text{Br, Cl, and I}$ ;  $Y = \text{Mg and Be}$ ) [51]. We use  $\text{Fe}_2\text{BrMgP}$  as a prototype system to illustrate the concept of the layer-locked AVHE in a 2D tetragonal AFM system.

### IV. COMPUTATIONAL DETAIL

The spin-polarized first-principles calculations are performed within density functional theory [52], as implemented in VASP code [53–55] within the projector augmented-wave method. We adopt the generalized gradient approximation of Perdew-Burke-Ernzerhof [56] as the exchange-correlation functional. The kinetic energy cutoff of 500 eV, the total energy convergence criterion of  $10^{-8}$  eV, and the force convergence criterion of  $0.001 \text{ eV } \text{\AA}^{-1}$  are set to obtain the accurate results. To account for the localized nature of  $3d$  orbitals of Fe atoms, a Hubbard correction  $U_{\text{eff}} = 2.5 \text{ eV}$  [51] is employed within the rotationally invariant approach proposed by Dudarev *et al.* [57], and the SOC is incorporated for investigation of electronic structures. To avoid adjacent interactions, the vacuum space along the  $z$  direction is set to more than  $16 \text{ \AA}$ . A  $12 \times 12 \times 1$  Monkhorst-Pack  $k$ -point mesh is used to sample the BZ for calculating electronic structures and elastic properties.

The elastic stiffness tensors  $C_{ij}$  are calculated by using the strain-stress relationship (SSR) method. The 2D elastic coefficients  $C_{ij}^{2D}$  have been renormalized by  $C_{ij}^{2D} = L_z C_{ij}^{3D}$ ,

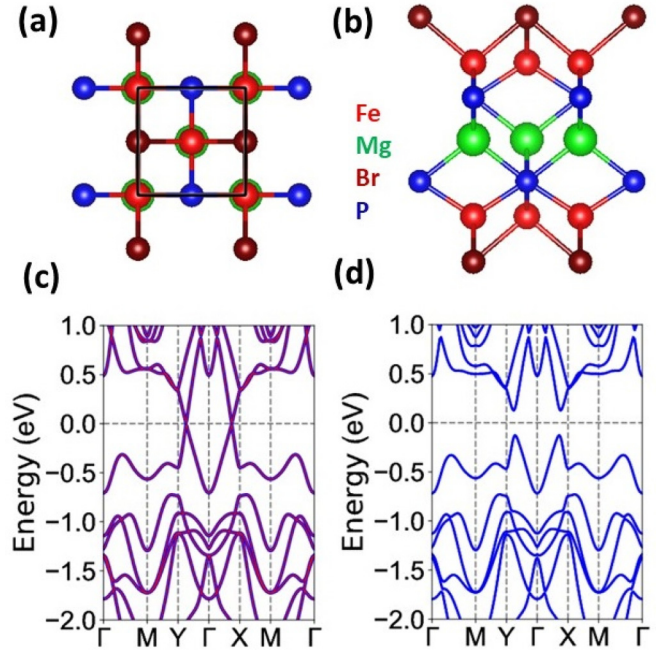


FIG. 2. Lattice and electronic structures of  $\text{Fe}_2\text{BrMgP}$  monolayer. (a) and (b) Top and side views of monolayer  $\text{Fe}_2\text{BrMgP}$ . Energy band structures of  $\text{Fe}_2\text{BrMgP}$  monolayer (c) without and (d) with SOC. In panel (c), the spin-up and spin-down channels are depicted in blue and red.

where  $L_z$  is the length of the unit cell along the  $z$  direction. The topological properties are studied with the maximal localized Wannier function tight-binding model by WANNIER90 and WANNIERTOOLS [58,59].

### V. LATTICE, MAGNETIC, AND ELECTRONIC PROPERTIES

$\text{Fe}_2\text{BrMgP}$  monolayer is dynamically, mechanically, and thermally stable [51]. The crystal structures of  $\text{Fe}_2\text{BrMgP}$  monolayer are plotted in Figs. 2(a) and 2(b), crystallizing in the  $P4/nmm$  space group (No. 129). The unit cell contains ten atoms with a seven-atom layer sequence of Br-Fe-P-Mg-P-Fe-Br. The optimized equilibrium lattice constants are  $a = b = 4.03 \text{ \AA}$  by GGA +  $U$  method. To determine the ground state of  $\text{Fe}_2\text{BrMgP}$ , we consider four magnetic configurations, including FM ordering, AFM1 ordering (A-type AFM ordering), AFM2 ordering, and AFM3 ordering (see Fig. S1 [60]). This A-type AFM ordering is predicted to be the ground state, and its energy is  $55/645/759 \text{ meV}$  per unit cell lower than that with the FM/AFM2/AFM3 ordering. The  $\text{Fe}_2\text{BrMgP}$  monolayer is centrosymmetric. However, when spin is considered, the inversion symmetry  $\mathcal{P}$  is missing for A-type AFM ordering, and the time-reversal symmetry  $\mathcal{T}$  is also lacking. A combination of inversion symmetry  $\mathcal{P}$  and time-reversal symmetry  $\mathcal{T}$  gives this  $\mathcal{PT}$  preserving: the  $\mathcal{T}$  operation reverses the spin direction of each Fe atom, followed by the  $\mathcal{P}$  operation which swaps the top and bottom Fe atoms. The different magnetic orientation can affect the symmetry of a system as well as the valley and topological properties [34–40,45–50]. For example, in monolayer  $\text{Fe}_2\text{Br}_2$ ,

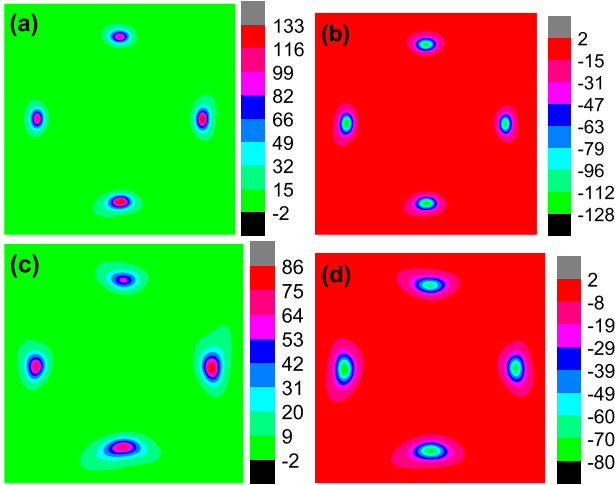


FIG. 3. Layer-dependent Berry curvature of  $\text{Fe}_2\text{BrMgP}$ . The Berry curvatures are shown for the spin-up (a, c) and spin-down (b, d) channels for  $a/a_0 = 1.00$  and  $E = 0.00 \text{ V/\AA}$  (a, b) and for  $a/a_0 = 1.04$  and  $E = 0.02 \text{ V/\AA}$  (c, d).

when the magnetic orientation is out-of-plane, the hot spots in the Berry curvature are around four gapped Dirac cone with the same signs, leading to Chern number  $C = 2$ ; while for in-plane magnetization, two of four main hot spots in the Berry curvature have the opposite sign of the other two, giving rise to a vanishing Chern number [49]. For our proposed system, an out-of-plane magnetic orientation is needed, and the magnetic orientation can be determined by the magnetic anisotropy energy (MAE). By the GGA +  $U$  + SOC method, the MAE can be calculated as  $E_{\text{MAE}} = E_{\text{SOC}}^{\parallel} - E_{\text{SOC}}^{\perp}$ , where  $\parallel$  and  $\perp$  denote the in-plane spin orientation and the out-of-plane spin orientation, respectively. The MAE is  $451 \mu\text{eV/Fe}$ , and the positive value indicates the out-of-plane easy magnetization axis of  $\text{Fe}_2\text{BrMgP}$ , which confirms our proposed design principles. The total magnetic moment per unit cell is strictly  $0.00 \mu_B$ , and the magnetic moments of bottom and top Fe atoms are  $3.06$  and  $-3.06 \mu_B$ , respectively.

The calculated energy band structures of  $\text{Fe}_2\text{BrMgP}$  without and with SOC are shown in Figs. 2(c) and 2(d), respectively. When neglecting SOC, two pairs of band-crossing points occur near the Fermi level along the  $\Gamma$ -X and  $\Gamma$ -Y lines. With SOC, a Dirac gap of  $252 \text{ meV}$  is introduced, yielding equivalent valleys along the  $\Gamma$ -X and  $\Gamma$ -Y lines due to the  $C_4$  rotation symmetry. The corresponding  $k$  points in the momentum space are marked by  $X_1$  and  $Y_1$  without valley splitting ( $\Delta E_C = E_{X_1}^C - E_{Y_1}^C$  and  $\Delta E_V = E_{X_1}^V - E_{Y_1}^V$ ) for both conduction and valence bands. Because of the  $\mathcal{PT}$  symmetry, the bands of  $\text{Fe}_2\text{BrMgP}$  are spin degenerate both without and with SOC.

Because of the  $\mathcal{PT}$  symmetry, the Berry curvature of  $\text{Fe}_2\text{BrMgP}$  vanishes. However, each layer breaks the  $\mathcal{PT}$  symmetry locally, and such layer-specific symmetric breaking leads to a nonvanishing layer-locked hidden Berry curvature. Here the Berry curvatures of spin-up and spin-down channels are nonzero [Figs. 3(a) and 3(b)]. The Berry curvatures are opposite for spin-up and spin-down channels around the  $Y_1$  and  $X_1$  valleys, respectively. In the presence of a longitudinal

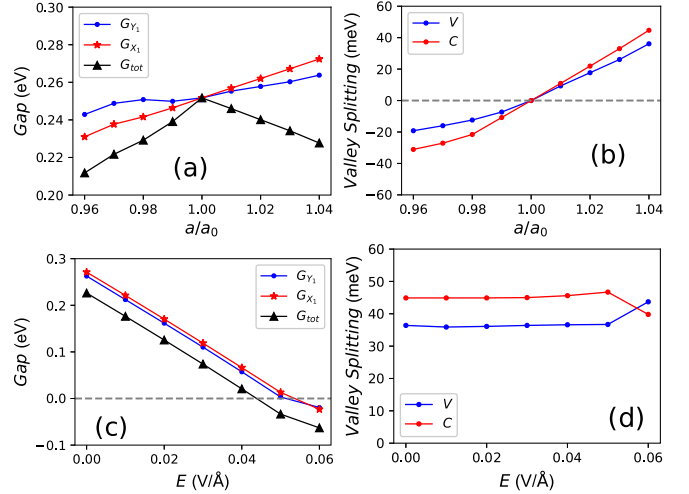


FIG. 4. Strain and electric field modulation. For  $\text{Fe}_2\text{BrMgP}$ , the related band gaps including the global gap ( $G_{\text{tot}}$ ) and gaps of  $Y_1$  and  $X_1$  valleys ( $G_{Y_1}$  and  $G_{X_1}$ ) (a, c), and valley splitting for both valence ( $V$ ) and conduction ( $C$ ) bands (b, d) as a function of  $a/a_0$  (a, b) and  $E$  (c, d) with  $a/a_0 = 1.04$ .

in-plane electric field  $E_{\parallel}$ , the spin-up and spin-down electrons from the  $Y_1$  and  $X_1$  valleys accumulate along opposite sides of the top and bottom Fe layers, resulting in the layer-spin Hall effect [Fig. 1(d)].

## VI. UNIAXIAL STRAIN INDUCES VALLEY POLARIZATION

To induce valley polarization in  $\text{Fe}_2\text{BrMgP}$ , a uniaxial strain along the  $x$  or  $y$  direction is applied which reduces  $C_4$  to  $C_2$  symmetry. The valleys along the  $\Gamma$ -X and  $\Gamma$ -Y lines become inequivalent, thus giving rise to valley polarization. We use  $a/a_0$  (0.96 to 1.04) to simulate a uniaxial strain along the  $x$  direction, and the lattice constant  $b$  along the  $y$  direction is optimized. The in-plane Young's modulus  $C_{2D}(\theta)$  as a function of the angle  $\theta$  relative to the  $x$  direction is plotted in Fig. S2 [60]. The obtained  $C_{2D}$  along the  $x$  direction is  $86 \text{ Nm}^{-1}$ . This is smaller than those of graphene ( $\sim 340 \pm 40 \text{ Nm}^{-1}$ ) and  $\text{MoS}_2$  ( $\sim 126.2 \text{ Nm}^{-1}$ ) [61,62], which indicates the better mechanical flexibility of  $\text{Fe}_2\text{BrMgP}$ , thus favoring the experimental realization of valley polarization by strain. The strained  $\text{Fe}_2\text{BrMgP}$  remains in the A-type AFM ground state with out-of-plane magnetic anisotropy within the considered strain range (see Fig. S3 [60]).

The electronic band structures of strained  $\text{Fe}_2\text{BrMgP}$  calculated using GGA + SOC are plotted in Fig. S4 [60]. The evolutions of the band gap and the valley splitting ( $\Delta E_V$  and  $\Delta E_C$ ) for both valence and conduction bands as a function of  $a/a_0$  are plotted in Figs. 4(a) and 4(b). The uniaxial strain induces valley polarization for both conduction and valence bands, and the valley polarization can be switched between  $X_1$  and  $Y_1$  valleys with strain transitioning from compressive to tensile cases. For common hexagonal FV systems, the valley polarization can be reversed by the magnetic field [34–39]. Therefore, the uniaxial strain can be regarded as a pseudomagnetic field for the tetragonal system [40]. For  $a/a_0 = 0.96/1.04$ , the corresponding valley splitting is

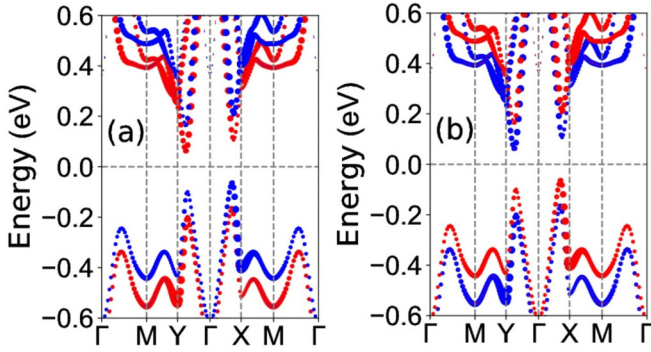


FIG. 5. Sign-reversible layer-locked anomalous valley Hall effect. For  $\text{Fe}_2\text{BrMgP}$  with  $a/a_0 = 1.04$ , the spin-resolved energy band structures for  $E = +0.02 \text{ V/\AA}$  (a) and  $E = -0.02 \text{ V/\AA}$  (b) are shown. The spin-up and spin-down channels are depicted in blue and red.

$-19$  ( $-31$ )  $\text{meV}/36$  ( $45$ )  $\text{meV}$  for the valence (conduction) band, which is close to or larger than the thermal energy of room temperature ( $25 \text{ meV}$ ). Using  $a/a_0 = 1.04$  as a representative, the Berry curvatures of the spin-up and spin-down channels are plotted in Fig. S5 [60]. The Berry curvatures are opposite for spin-up and spin-down channels around  $Y_1$  and  $X_1$  valleys. In this case, a longitudinal in-plane electric field  $E_{\parallel}$  can thus lead to the accumulation of the spin-up and spin-down electrons from only  $X_1$  valleys along the opposite sides of the top and bottom Fe layers, resulting in the valley layer-spin Hall effect as illustrated in Fig. 1(d).

### VII. ELECTRIC FIELD INDUCES SPIN SPLITTING

An out-of-plane electric field can break the  $\mathcal{PT}$  symmetry to lift the spin degeneracy of the valleys. Taking strained  $\text{Fe}_2\text{BrMgP}$  with  $a/a_0 = 1.04$  as an example, the electric field ( $E$ ) effects on the electronic structures are investigated. The difference between  $+E$  and  $-E$  is that the spin-splitting order is reversed. According to Fig. S6 [60], the ground state of  $\text{Fe}_2\text{BrMgP}$  remains in the A-type AFM ordering with out-of-plane magnetic anisotropy within the considered  $E$  range. The energy band structures of  $\text{Fe}_2\text{BrMgP}$  by using GGA + SOC at representative  $E$  are plotted in Fig. S7 [60], and the spin-resolved energy band structures at  $E = \pm 0.02 \text{ V/\AA}$  are shown in Fig. 5. The evolutions of related energy band gap and the valley splitting for both valence and conduction bands as a function of  $E$  are plotted in Figs. 4(c) and 4(d). In fact, by constructing the Janus structure  $\text{Fe}_4\text{BrMg}_2\text{P}_2$ , the external electric field can be equivalently replaced by the built-in electric field to achieve spin splitting (see Fig. S8 [60]).

The spin splitting induced by the out-of-plane electric field, and spin-polarization reversal via reversing the direction of electric field, can be observed in Fig. 5. The electric field can maintain the valley splitting amplitude and induce a semiconductor-metal phase transition [see Figs. 4(c) and 4(d)]. The sizes of spin splitting at  $X_1$  and  $Y_1$  valleys for both conduction and valence bands are plotted in Fig. S9 [60], which meets the layer-dependent electrostatic potential  $\propto eEd$  (The sizes of spin splitting can be calculated by  $eEd$ ). Using  $E = 0.02 \text{ V/\AA}$  as an example, the Berry curva-

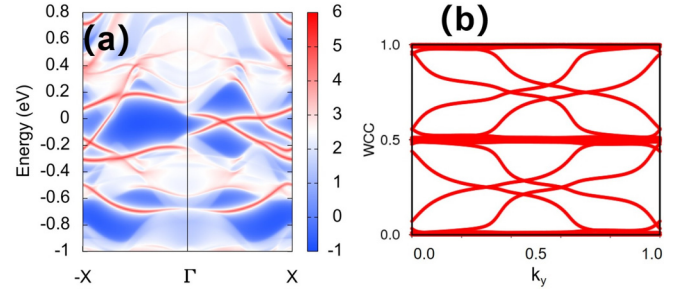


FIG. 6. Topological properties of  $\text{Fe}_2\text{BrMgP}$ . For  $\text{Fe}_2\text{BrMgP}$  with  $a/a_0 = 1.04$  at  $E = +0.02 \text{ V/\AA}$ : (a) the edge states along the  $[100]$  direction; (b) the evolution of WCCs along  $k_y$ .

ture calculations in Figs. 3(c) and 3(d) show that the Berry curvatures of spin-up and spin-down channels around  $Y_1$  and  $X_1$  valleys are opposite. In the presence of a longitudinal in-plane electric field  $E_{\parallel}$ , the spin-up electrons from only the  $X_1$  valley accumulate along one side of the bottom Fe layer, which leads to the layer-locked anomalous valley Hall effect of Fig. 1(d). When the direction of the electric field is reversed, the spin-down electrons from only the  $X_1$  valley accumulate along the other side of the top Fe layer, leading to a electric-field-effect-induced sign reversal of the AVHE previously predicted in the FV-FM system [63].

### VIII. VALLEY-POLARIZED QUANTUM SPIN HALL INSULATOR

In a 2D AFM insulator, one cannot define a  $Z_2$  invariant, which can be defined in a  $\mathcal{T}$ -preserved quantum spin Hall insulator (QSHI). To realize an AFM QSHI, a way is to create a  $z$ -component spin ( $s_z$ ) conserved superposition of two quantum anomalous Hall insulators with equal but opposite magnetic moments [64]. If the constructed AFM systems have a symmetry of a combination of  $\mathcal{P}$  and  $\mathcal{T}$  ( $\mathcal{PT}$ ), the states with different  $s_z$  in one branch are orthogonal to each other and belong to different irreducible representations, and then the spin Chern number  $C_s$  can still be well-defined [51].

To follow this proposal, the  $\text{Fe}_2\text{BrMgP}$  has been predicted to be an AFM QSHI with high spin Chern numbers, as confirmed by the gapless edge states and the topological invariant spin Chern numbers  $C_s$  [51]. By applying uniaxial strain and the electric field simultaneously, the VQSHI can be achieved in  $\text{Fe}_2\text{BrMgP}$ , which combines AVHE and QSHE in one material, providing a path towards integrating valleytronics, topological quantum effects, and spintronics in a single system. To confirm this aspect, for  $\text{Fe}_2\text{BrMgP}$  with  $a/a_0 = 1.04$  at  $E = +0.02 \text{ V/\AA}$ , the edge states along the  $[100]$  direction and the evolution of the Wannier charge centers (WCCs) along  $k_y$  are plotted in Fig. 6. Based on the evolution of WCCs, the spin Chern number  $|C_s|$  is 2, which is further determined by two pairs of gapless edge states with opposite chiralities appearing in the bulk gap.

It is noteworthy that an out-of-plane electric field can break  $\mathcal{PT}$  symmetry by breaking  $\mathcal{P}$  symmetry, which can mix the spin-up and spin-down states, and the gaps for edge states will be induced (The AFM QSHI is not well-defined.). However, an out-of-plane electric field  $0.02 \text{ V/\AA}$  is applied

in  $\text{Fe}_2\text{BrMgP}$ , and only a very small gap of about 5 meV for edge states is produced. After applying the electric field in  $\text{Fe}_2\text{BrMgP}$ , because the calculated edge states have almost no energy gap, it still maintains relatively good quantum spin characteristics. So, our proposed system  $\text{Fe}_2\text{BrMgP}$  can be approximated as a VQSHI.

## IX. CONCLUSION

In summary, we propose a paradigm for achieving the anomalous valley Hall effect in AFM tetragonal monolayers by external field and strain engineering. The proposed concept is confirmed by a prototype monolayer  $\text{Fe}_2\text{BrMgP}$  using first-principles calculations. Uniaxial strain induces valley polarization by breaking  $C_4$  symmetry, and an out-of-plane electric field gives rise to spin splitting via layer-dependent electrostatic potential. The concept of VQSHI is demonstrated, which is similar to VQAH. Our analysis can be readily extended to the broader family of  $\text{Fe}_2XY$  ( $X = \text{or } \neq$

$Y = \text{Cl, Br, and I}$ ) and  $\text{Li}_2\text{Fe}_2XY$  ( $X = \text{or } \neq Y = \text{S, Se, and Te}$ ) bilayers as they share the same Fe-dominated low-energy states as  $\text{Fe}_2\text{BrMgP}$ . The energy band structures for some of these families are shown in Fig. S10 [60], like  $\text{Fe}_2\text{Br}_2$ ,  $\text{Fe}_2\text{I}_2$ ,  $\text{Fe}_2\text{BrI}$ ,  $\text{Li}_2\text{Fe}_2\text{S}_2$ ,  $\text{Li}_2\text{Fe}_2\text{Se}_2$ , and  $\text{Li}_2\text{Fe}_2\text{SSe}$ . Our results reveal a route towards energy-saving and fast-operating spintronic-valleytronic devices based on 2D antiferromagnetic materials.

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