

# Manipulating the electrical properties of conductive substoichiometric titanium oxides

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Conducting metal oxides offer many advantages for novel electronics applications, including sensors, fuel cells, piezoelectric devices, and microelectronic circuits, due to their conductivity, hardness, and chemically inert surfaces. In particular, their high electrical conductivity and mechanical properties make these materials suitable for microelectromechanical and nanoelectromechanical system (MEMS/NEMS) devices. NEMS switches have great potential for next-generation electronic computing associated with scalability to small dimensions, low power consumption, and (relatively) high speed. Oxygen-deficient Ti oxides with ordered planes of vacancies ( $\text{Ti}_n\text{O}_{2n-1}$ , Magnéli phases) are good candidates for NEMS applications because of their metallic conductivity, environmental resistance, and low cost, as compared with other conductive oxides like  $\text{RuO}_2$ . Although Ti suboxides have been produced in crystalline form, various synthesis methods may also produce amorphous material. In this paper, we focus on the structural and electrical transport properties of several Ti suboxides. In particular, we examine the effects of temperature, transition-metal dopants, and amorphization on these structural and electronic properties and the potential applicability of Magnéli phase Ti suboxides for NEMS switch applications.

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## I. INTRODUCTION

Transition-metal oxides exhibit electrical conductivities that vary over many orders of magnitude. This variability is attributable, in part, to a wide range of conduction mechanisms. Applications of these materials requiring high electrical conductivity focus, primarily, on those where electrical conduction is primarily metallic. While titanium dioxide ( $\text{TiO}_2$ ) is a wide band-gap semiconductor, many of its substoichiometric variants,  $\text{TiO}_{2-x}$  or  $\text{Ti}_n\text{O}_{2n-1}$  for  $n = 4 - 10$ , are metallic conductors [1,2]. These variants, known as Magnéli phases [3–5], are determined by the ordered O vacancy planes named as shear planes [5–9]. The electrical conductivity of these suboxides depends on the structural phase and on both point defect and dopant concentrations. The large number of Magnéli phases and the ability to manipulate defect/dopant concentrations allows for electrical conductivity engineering in this class of materials.

The properties of rutile titanium dioxide ( $\text{TiO}_2$ ) and its defect structures have been widely studied [10–22]. While rutile titanium dioxide is (nearly) a line compound, it remains stable when very slightly oxygen deficient ( $\text{TiO}_{2-x}$  for  $x < 0.008$ ); the nonstoichiometry is accommodated by oxygen vacancies and/or titanium interstitials. Annealing  $\text{TiO}_2$  under reducing conditions at high temperature (873–1273 K) can produce larger deviations from stoichiometry (larger  $x$ ), leading to the

formation of Magnéli phases (i.e., long-range ordered oxygen vacancy structures with  $n = 4, 5, \dots$ ) [3,10,23,24]. From a theoretical and modeling point of view, it is known that, while  $\text{TiO}_2$  is a typical semiconductor, Magnéli phases exhibit delocalized electrons in the  $d$  band, leading to a high electrical conductivity metallic conductor [6,7,25–27]. The Magnéli phases have a mixture of  $\text{Ti}^{+3}$  and  $\text{Ti}^{+4}$  oxidation states [26]. Electronic structure calculations have been used to understand mainly structural and electronic properties of crystalline  $\text{Ti}_4\text{O}_7$  [7,25,26],  $\text{Ti}_5\text{O}_9$  [27], and  $\text{Ti}_6\text{O}_{11}$  [8]. The concentration of delocalized charges is closely related to half the amount of O vacancies [22]. For example, the delocalized electrons in  $\text{Ti}_4\text{O}_7$  reside in  $\text{Ti}_2\text{O}_3$  planes (with  $\text{Ti}^{3.5+}$ ).

Among the Magnéli phase titanium suboxides,  $\text{Ti}_4\text{O}_7$  has attracted the most attention, including for applications in lithium-sulfur batteries [28], photocatalysis [29], electrocatalysis [30], and thermoelectrics [31]. This phase has the highest electrical conductivity amongst the Magnéli phases and is degradation-resistant over a wide range of chemical environments [32]. The electrical conductivity of  $\text{TiO}_x$  ( $0.75 \leq x \leq 1.45$ ), is reported to be  $1000\text{--}3000$  ( $\Omega \text{ cm}$ )<sup>−1</sup>. The reported room-temperature electrical conductivity of  $\text{Ti}_n\text{O}_{2n-1}$  is high for  $n = 4, 5$ , and 6 and decreases with increasing  $n$  [1]. The room-temperature electrical conductivity of bulk  $\text{Ti}_4\text{O}_7$  [1] is  $1605.3$  ( $\Omega \text{ cm}$ )<sup>−1</sup>; this is within an order of magnitude of that of high-cost transition metal oxides (Ru, Ir, Re, and Rh) [33,34].

In electrical contact applications, electric current flows through contacting asperities at the interface, leading to a significant rise in local temperature, which can produce significant electrical conductivity changes in the critical contact

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region. The conductivity of  $\text{Ti}_n\text{O}_{2n-1}$  ( $4 \leq n \leq 9$ ) varies with temperature (either increasing or decreasing with temperature, depending on the temperature range); this suggests that there may be conduction through different mechanisms [35]. For example,  $\text{Ti}_4\text{O}_7$  shows two successive transitions in electrical conductivity (by increasing with temperature): from a low-temperature insulating phase to an intermediate phase (semiconductor-semiconductor transition) at 130 K, and from the intermediate phase to a high-temperature metallic phase (semiconductor-metal transition) at 150 K [1,2,35]. Similarly, the conductivities of both  $\text{Ti}_5\text{O}_9$  and  $\text{Ti}_6\text{O}_{11}$  increase with increasing temperature for  $150 \text{ K} < T < 300 \text{ K}$  (semiconductor behavior) and decrease with increasing temperature above 450 K (metallic behavior).

Doping is an efficient strategy to modulate the electrical properties of metal oxides. For  $\text{TiO}_2$ , transition-metal (TM) dopants modify the structural and electronic properties (photocatalytic activity and electrical conductivity) [36–38] through modification of the band gap. For example, Ta and Nb doping shifts the Fermi level of  $\text{TiO}_2$  into the conduction band, while Ru doping introduces Ru  $d$  electrons into the gap [39]. The conducting metal-oxide  $\text{RuO}_2$  exhibits a large electrical conductivity [ $2.5 \times 10^6 (\Omega M)^{-1}$ ], which changes with TM doping [40]. TM doping of the highly conductive Magnéli phase of  $\text{Ti}_4\text{O}_7$  can further increase the electrical conductivity beyond  $\text{Ti}_4\text{O}_7$  (generating additional states near the Fermi energy). By doping  $\text{Ti}_n\text{O}_{2n-1}$ , properties relevant to its use as an electrode material can be altered, such as electrical [41,42], tribological [43,44], and electrocatalytic [45,46] characteristics. Doping  $\text{Ti}_4\text{O}_7$  with metal cations such as Zr, Nb, Mo, and W increases both thermodynamic and structural stability [41,42]. It has been demonstrated that V and Fe doping in  $\text{Ti}_4\text{O}_7$  stabilizes it against thermal oxidation and enhances the electrical conductivity due to an increase in the carrier concentration [41]. Doping  $\text{Ti}_4\text{O}_7$  with Ce can improve catalytic activity and electrochemical stability via formation of surface oxygen vacancies compared to pristine  $\text{Ti}_4\text{O}_7$  [45].

Compared to crystalline TM oxides, their amorphous counterparts exhibit some beneficial features, including lower deposition temperature, lower surface roughness, more uniform etching, increased flexibility, and the lack of grain boundaries and associated phenomena [47]. While crystalline metal oxides exhibit electronic and electrical properties that vary through the microstructure [48], this is not the case for the amorphous phases. The existence of several Ti-O Magnéli phases over a small composition range ( $\text{TiO}_{1.75-1.9}$ ) suggests that amorphization may occur readily in these systems. The conductivities of the amorphous analogues of the Magnéli phases have not been reported.

Electrically conductive oxides are excellent candidates for low-resistance contacts in NEMS because of their wear resistance and resistance to a wide range of environmental degradation mechanisms that plague more traditional metallic contacts [49–51]. O-deficient substoichiometric phases of  $\text{TiO}_2$  are appealing for such applications, owing to their metallic conductivity, environmental degradation resistance, and low cost compared with other conductive oxides [52,53].

In this paper, we examine the electrical and electronic behavior of several highly conductive Magnéli phases,  $\text{Ti}_n\text{O}_{2n-1}$

TABLE I. Structural parameters of  $\text{TiO}_2$ ,  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$  crystals. The atomic positions in  $\text{TiO}_2$  were optimized using an on-site Ti  $d$ -state Hubbard  $U$  correction ( $U=4.2$  eV) and a  $5 \times 5 \times 8$  k-point mesh was employed for bulk rutile  $\text{TiO}_2$  and a  $5 \times 4 \times 3$  k-point mesh for bulk  $\text{Ti}_n\text{O}_{2n-1}$  ( $n = 4, 5, 6$ ) with a 520 eV energy cutoff.

Parameter	Theory (this paper) / Experiment			
	r- $\text{TiO}_2$ [67]	$\text{Ti}_4\text{O}_7$ [68]	$\text{Ti}_5\text{O}_9$ [68]	$\text{Ti}_6\text{O}_{11}$ [68]
$a$ (Å)	4.70/4.58	5.56/5.59	5.61/5.62	5.60/5.58
$b$ (Å)	4.69/4.58	6.93/6.90	7.21/7.18	7.21/7.14
$c$ (Å)	3.04/2.95	7.21/7.13	8.55/8.56	9.80/9.69
$\alpha$ (°)	90.0/90.0	64.05/64.12	69.38/69.49	90.03/89.75
$\beta$ (°)	90.0/90.0	72.01/71.24	74.98/75.19	79.64/79.38
$\gamma$ (°)	90.0/90.0	75.28/75.65	71.17/71.31	71.23/71.5
Space group	$P4_2/mnm$	$P\bar{1}[2]$	$P\bar{1}[2]$	$P\bar{1}[2]$
Crystal system	tetragonal	trigonal	trigonal	trigonal

( $n = 4, 5$ , and 6). In addition, we examine the effects of dopants, temperature, and amorphization in these systems. We focus on these questions: (1) What are the electronic structures and electrical properties of the different phases? (2) How do impurities affect these phases? (3) Since there are so many competing phases over a very small composition range in the Ti-O system, amorphous structures are possible. What are the properties of amorphous analogues of the Magnéli phases relative to their crystalline counterparts? Our study is based upon first-principles methods and the Kubo-Greenwood formalism [54,55] for calculating the electrical conductivity. Finite-temperature data are obtained through *ab initio* molecular dynamics (AIMD).

## II. COMPUTATIONAL DETAILS

Our calculations were performed using the density functional theory package VASP [56,57] in the generalized gradient approximation with the Perdew-Burke-Ernzerhof exchange correlation energy functional [58]. The Fermi surface was Gaussian smeared, and the core electrons were treated via the projector augmented-wave method [59,60]. The energy cutoff was 520 eV for the plane-wave basis in the structure optimization with a relatively dense k-point mesh of  $3 \times 3 \times 3$  for the  $2 \times 2 \times 2$  supercells of  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$  and  $\text{Ti}_6\text{O}_{11}$  crystal structures. For the 216-atom supercell of crystalline rutile- $\text{TiO}_2$  (r- $\text{TiO}_2$ ), we used a  $2 \times 2 \times 3$  k-point mesh. The crystalline rutile  $\text{TiO}_2$  and  $\text{Ti}_n\text{O}_{2n-1}$  ( $n = 4, 5$ , and 6) structural parameters are in Table I.

To describe the strong electron correlation interactions in the  $d$  orbitals of the Ti  $3d$  electrons, the DFT+ $U$  method [61,62] is introduced for the inclusion of on-site Coulomb repulsion at atomic sites. The results presented in this paper are from GGA+ $U$  with  $U=4$  eV for Ti  $3d$  electrons. For clarity, we also compared the low-temperature (at 120 K) and high-temperature (at 300 K) phases of  $\text{Ti}_4\text{O}_7$ . The low  $T$  phase with GGA+ $U$  [63] (see Fig. S1 [64]) is shown to be a semiconductor (0.27 eV), in agreement with experimental band gap value of 0.25 eV [65]. The high  $T$  phase (same composition, different ion arrangement) with GGA+ $U$  is shown to be very

metallic (agrees with experiment) [1–4,25,26]. Our choice (GGA) is justified by previous experimental and theoretical results [63,65,66] showing the metallic behavior of Magnéli phases,  $\text{Ti}_n\text{O}_{2n-1}$  ( $n = 4, 5$ , and 6), at high temperatures.

A melt and quench method was used to generate 216, 176, 224, and 272 atom amorphous variants of the  $\text{TiO}_2$ ,  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$  crystal structures, respectively. The liquid Ti suboxide structures were generated computationally by AIMD of the crystal at 4000 K for times sufficient to lose structural memory, and then quenching to and annealing at 0, 300, 600, and 900 K for 45 ps (9000 5-fs AIMD time steps) under isothermal-isobaric conditions ( $NPT$ ) [69]. For computational efficiency, we employed relatively low-precision settings ( $\Gamma$  point only; plane-wave kinetic-energy cutoff of 400 eV) in the AIMD simulations. After equilibration, atomic structures were collected following 15 ps thermalization in 1.5 ps intervals. For each configuration, we performed static density functional theory calculations with a denser k-point grid ( $3 \times 3 \times 3$ ); we used Fermi-Dirac function smearing to obtain the frequency-dependent electrical conductivity (see Supplemental Material [64] for details). The widths of the Dirac delta function (Eq. (S1) [64]) were selected from average values of eigenvalue differences of each configuration.

The electrical conductivity values ( $\sigma_0$ ) were obtained by averaging over structures generated using AIMD simulations. The  $\sigma_0$  values were extracted from  $\sigma(\omega)$  by calculating  $\sigma_0 = \lim_{\omega \rightarrow 0} \sigma(\omega)$ . We fit  $\sigma(\omega)$  to a Drude model [70,71] to obtain electrical conductivity values at 0 eV (see Fig. S2 [64]). The three diagonal components of the dc conductivity ( $\sigma_{0,xx}$ ,  $\sigma_{0,yy}$ ,  $\sigma_{0,zz}$ ) were calculated; the reported dc conductivity is the arithmetic average of these.

In addition to studying the defect-free crystal structures of the suboxides, we also considered the role of point defects. Specifically, the structure and energetics of dopant additions were examined, starting from crystalline  $\text{Ti}_4\text{O}_7$ . The effects of TM doping of  $\text{Ti}_4\text{O}_7$  were determined by substitution of a single dopant for a Ti atom in a  $2 \times 2 \times 2$  supercell (i.e.,  $\text{Ti}_6M_1\text{O}_{112}$ , where  $M = \text{V, Nb, Ta, Zr, Cr, Mo, and Sc}$ ), corresponding to a  $\approx 1.6\%$  doping concentration. The defect formation energy and the doping energy (heat of solution)  $H_F$  is defined as

$$H_F = E_d - E_s - \sum_i \mu n_i, \quad (1)$$

where  $E_d$  is the total energy of the defective or doped structure,  $E_s$  is the total energy of the pure, stoichiometric structure,  $n_i$  is number of dopant atoms in the unit cell, and  $\mu_i$  is the chemical potential of species  $i$ . The dopants considered were Nb, Ta, V, Mo, Zr, Sc, and Cr.

### III. CRYSTALLINE SUBOXIDES

Previous experimental studies [1] reported that the first  $\text{Ti}_4\text{O}_7$  Magnéli phase has the highest conductivity [ $1830 (\Omega \text{ cm})^{-1}$ ] and that the electrical conductivity of  $\text{Ti}_n\text{O}_{2n-1}$  decreases with increasing  $n$ . For example, the room temperature electrical conductivity of  $\text{Ti}_5\text{O}_9$  ( $n = 5$ ) and  $\text{Ti}_6\text{O}_{11}$  ( $n = 6$ ) are 1540 and 1100 ( $\Omega \text{ cm})^{-1}$ , respectively. Our predicted results (see Table II) slightly overestimate the experimental values, but the trend of the electrical conductivity is consistent

TABLE II. Electrical conductivities ( $\sigma_0$ ) of a supercell containing  $2 \times 2 \times 2$  trigonal simulation cells of  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$  at 0 K and 300 K. Prefixes  $c$ - and  $a$ - denote crystal and amorphous (see text), respectively.

Structure	0 K	300 K	600 K	900 K	300 K exp. [73]
$c$ - $\text{Ti}_4\text{O}_7$	2113	2093	1974	1850	1830
$a$ - $\text{Ti}_4\text{O}_7$	1340	1378	1497	1505	
$c$ - $\text{Ti}_5\text{O}_9$	1787	1526	1558	1537	1540
$a$ - $\text{Ti}_5\text{O}_9$	1153	1171	1247	1251	
$c$ - $\text{Ti}_6\text{O}_{11}$	1586	1410	1445	1436	1100
$a$ - $\text{Ti}_6\text{O}_{11}$	960	1067	1057	1097	

with previous reported values [1]. The small supercell sizes and discretization of the eigenvalues can be the main sources of error in the results [72].

Due to the metallic nature of the Ti  $n = 4$ –6 suboxides, increasing temperature from 0 K to 300 K leads to a decrease in electrical conductivity. For  $c$ - $\text{Ti}_4\text{O}_7$ , we found no significant effect of temperature on the predicted electrical conductivity below 300 K but 12% reduction upon increasing temperature from 300 K to 900 K (see Table II). This trend was only observed at low temperature for  $c$ - $\text{Ti}_5\text{O}_9$  and  $c$ - $\text{Ti}_6\text{O}_{11}$  (see Table II).

### IV. TRANSITION METAL DOPING

In the  $\text{Ti}_4\text{O}_7$  Magnéli phase (bulk crystal), there are two inequivalent Ti sites present as  $\text{Ti}^{3+}$  and  $\text{Ti}^{4+}$ , as seen in Fig. 1(a), and these two sites are bonded to six  $\text{O}^{2-}$  ions in

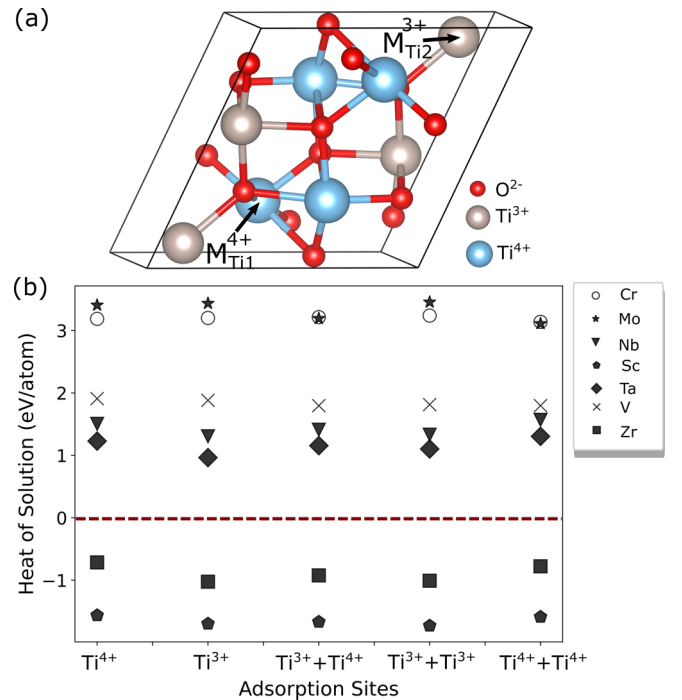


FIG. 1. (a) Doping sites in  $\text{Ti}_4\text{O}_7$  Magnéli phase. (b) Heat of solution,  $H_F$  (eV/atom) for transition-metal dopants in  $\text{Ti}_4\text{O}_7(M_{\text{Ti}}$ ,  $M = \text{Nb, Ta, Zr, V, Sc, Cr, and Mo}$ ) at  $\text{Ti}^{3+}$ ,  $\text{Ti}^{4+}$ ,  $\text{Ti}^{3+} + \text{Ti}^{3+}$  ( $2M_{\text{Ti}}^{3+}$ ),  $\text{Ti}^{4+} + \text{Ti}^{4+}$  ( $2M_{\text{Ti}}^{4+}$ ), and  $\text{Ti}^{3+} + \text{Ti}^{4+}$  sites.

TABLE III. Bader charge of dopant,  $\rho$  ( $|e|$ ), density of states at the Fermi energy,  $N(E_F)$  (states/eV) and predicted electrical conductivity,  $\sigma_0$  ( $(\Omega \text{ cm})^{-1}$ ) of  $\text{Ti}_{3.94}\text{M}_{0.06}\text{O}_7$  (doping concentration is 1.6%,  $\text{M} = \text{Nb, Ta, Zr, V, Sc, Cr, and Mo}$ ).

Dopant	$\rho$	$N(E_F)$	$\sigma_0$
undoped		74	2113.0
Nb	2.52	140.4	3046.1
Ta	2.51	133.41	3027.3
Zr	2.51	120.9	2974.0
V	1.62	94.8	2951.7
Sc	1.77	118.5	2914.2
Cr	1.38	103.7	2861.8
Mo	1.39	87.5	2765.9

the form of  $\text{TiO}_6$  octahedra that share corners with equivalent  $\text{TiO}_6$  octahedra. The bond distances range from 1.94–2.08 Å and 1.88–2.18 Å for  $\text{Ti}^{3+}-\text{O}^{2-}$  and  $\text{Ti}^{4+}-\text{O}^{2-}$ , respectively. To find the most favorable doping site in  $\text{Ti}_4\text{O}_7$ , we calculated the heat of solution for each doping element at  $\text{Ti}^{3+}$  and  $\text{Ti}^{4+}$  sites. The results are shown in Fig. 1(b).

For  $\text{M}_{0.5}\text{Ti}_{3.5}\text{O}_7$  ( $\text{M} = \text{Nb, Ta, Zr, V, Sc, Cr, and Mo}$ ), the  $\text{Ti}^{4+}$  site is the most favorable for Cr and Mo, while  $\text{Ti}^{3+}$  is the most favorable for Nb, Sc, Ta, V, and Zr [see Fig. 1(b)]. Overall, the most suitable TMs with which to dope  $\text{Ti}_4\text{O}_7$  are Sc, Zr, Nb, and Ta since they have the lowest heats of solution (and it is negative for Sc and Zr). For electrical conductivity enhancement, both  $\text{Ti}^{3+}$  and  $\text{Ti}^{4+}$  sites are very similar (see Table S1 [64]). For example, the predicted electrical conductivity value for  $\text{Nb}_{\text{Ti}^{3+}}$  is only 1% higher than  $\text{Nb}_{\text{Ti}^{4+}}$ . Moreover, we studied pairs of metal dopants in  $\text{Ti}_4\text{O}_7$  at different pairs of doping sites, namely,  $2M_{\text{Ti}^{3+}}$ ,  $2M_{\text{Ti}^{4+}}$ ,  $M_{\text{Ti}^{3+}} + M_{\text{Ti}^{4+}}$ . We found that the  $2M_{\text{Ti}^{3+}}$  site (doping concentration  $\approx 25\%$ ) is the most favorable of the three possibilities for Nb, Sc, Ta, and Zr, while the  $2M_{\text{Ti}^{4+}}$  site is the most favorable for Cr and Mo. The  $\text{Ti}^{3+} + \text{Ti}^{4+}$  site is preferable for only for V.

Our Bader charge analysis [74,75] for the Nb, Ta, Zr, V, Sc, Cr, and Mo doped structures (see Table III) suggests that the maximum charge donation occurs for Nb, Ta, and Zr doping. As a result of charge donation by these elements, the phase stability increases; this may be associated with the strengthening of the overall chemical bonding. On the contrary, Cr injects less charge, leading to reduced chemical bonding and hence decreased phase stability. Transferring and also sharing electrons between anions and cations (ionic and covalent bonding nature) leads to bond strengthening. While minor changes were observed in the vicinity of O atoms (e.g., metal-O bond length), the overall effect of these chemical bonding changes can be seen in the heat of solution data.

Although Ta doping fulfills the increased phase stability criterion (according to the heat of solution), a projected density of states analysis (PDOS) shows that Nb exhibits the largest DOS at the Fermi level (140.4 states/eV) when incorporated into  $c\text{-Ti}_4\text{O}_7$ . For the Ta- and Zr-doped structures, the  $N(E_F)$  values were found to be 133.4 and 120.9 states/eV, respectively. Compared to stoichiometric  $\text{Ti}_4\text{O}_7$ , Nb doping leads to 50% increase in the electrical conductivity, while Ta

and Zr doping lead to 44.7% and 42.1% increases, respectively. For Sc and Cr doping, we found that the magnitude of  $N(E_F)$  values are not consistent with the  $\sigma_0$  values. This may be associated with the effect of magnetization on the electrical properties (discussed further below). The results are summarized in Table III. Overall,  $N(E_F)$  and the conductivity  $\sigma_0$  are higher in the doped cases than for stoichiometric  $\text{Ti}_4\text{O}_7$ .

The PDOS for the undoped and all doped  $\text{Ti}_4\text{O}_7$  cases are shown in Fig. 2. The PDOS profiles for Nb-, Ta-, Zr-, and Sc-doped  $\text{Ti}_4\text{O}_7$  are very similar to one another. The TM dopant  $d$  states have a large spatial overlap with other states. Therefore, the contribution to the Fermi level is relatively smaller. For example, the contribution of Nb doping to the DOS at the Fermi level is higher than that for Zr doping due to a stronger overlap with states at the Fermi level. This may be evidence of delocalization of  $d$ -orbital electrons as a result of atomic distortion in the vicinity of the doping center (see Fig. S3 [64]). On the contrary, for Cr, Mo, and V doping, we observed localization of  $d$  states below the Fermi level and in the pseudogap (i.e., O  $2p$ -Mo  $4d$  hybridization). Although,  $N(E_F)$  is expected to decrease by this localization,  $N(E_F)$  increases, as summarized in Table III. This is associated with transferring additional electrons to the surrounding anions (O atoms) from dopant atoms in doped  $\text{Ti}_4\text{O}_7$ . As the distortion of the atomic structure around the doping center promotes delocalization of  $d$ -orbital electrons,  $N(E_F)$  increases compared with stoichiometric  $\text{Ti}_4\text{O}_7$ .

The classical theory of electrical conductivity of metals suggests that increasing  $N(E_F)$  gives rise to an increase in the predicted electrical conductivity. As just discussed and shown in Table III, this behavior is consistent with our simulation results. This is important because the states near the Fermi energy determine most electronic properties (e.g., conductivity); only electrons with energies close to the Fermi level participate in electrical conduction in metals. This suggests that increasing the number of free electrons leads to higher electronic density at the Fermi level, resulting in conductivity enhancement in metals. While the effects of TM doping on electrical conductivity (see Table III) is relatively weak as compared with doping effects on the conductivity in semiconductors [37,38], we do see that all of the TM dopants increase the electrical conductivity.

The dopant effect on the electron density distribution is illustrated through delocalization of electron probability density in contour electron localization function (ELF) plots in Figs. 3(a) and 3(b). The O atoms are clearly seen as effective centers for localization of valence electrons from the reference bare structure in Fig. 3(a). Figure 3(b) (and for other TM doping of  $\text{Ti}_4\text{O}_7$  in Fig. S3 [64]) shows the ELF contour plot for the  $\text{Nb}_{\text{Ti}}$  structure. In this case, the electronic charge is mostly localized by the ELF value of nearly 0.7 in defect site. More importantly, the 0.3 ELF value between the Nb and nearest-neighbor O atoms due to the charge transfer from Nb to O is evidence of charge delocalization as a result of the atomic structure distortion around the impurity site, compared to the undoped  $\text{Ti}_4\text{O}_7$ . This additional delocalized electron density is consistent with the enhanced electrical conductivity observed for doped  $\text{Ti}_4\text{O}_7$ . This is because the local distortion of the atomic structure accompanying the electron delocalization is associated with the dopants donating their valence electrons.



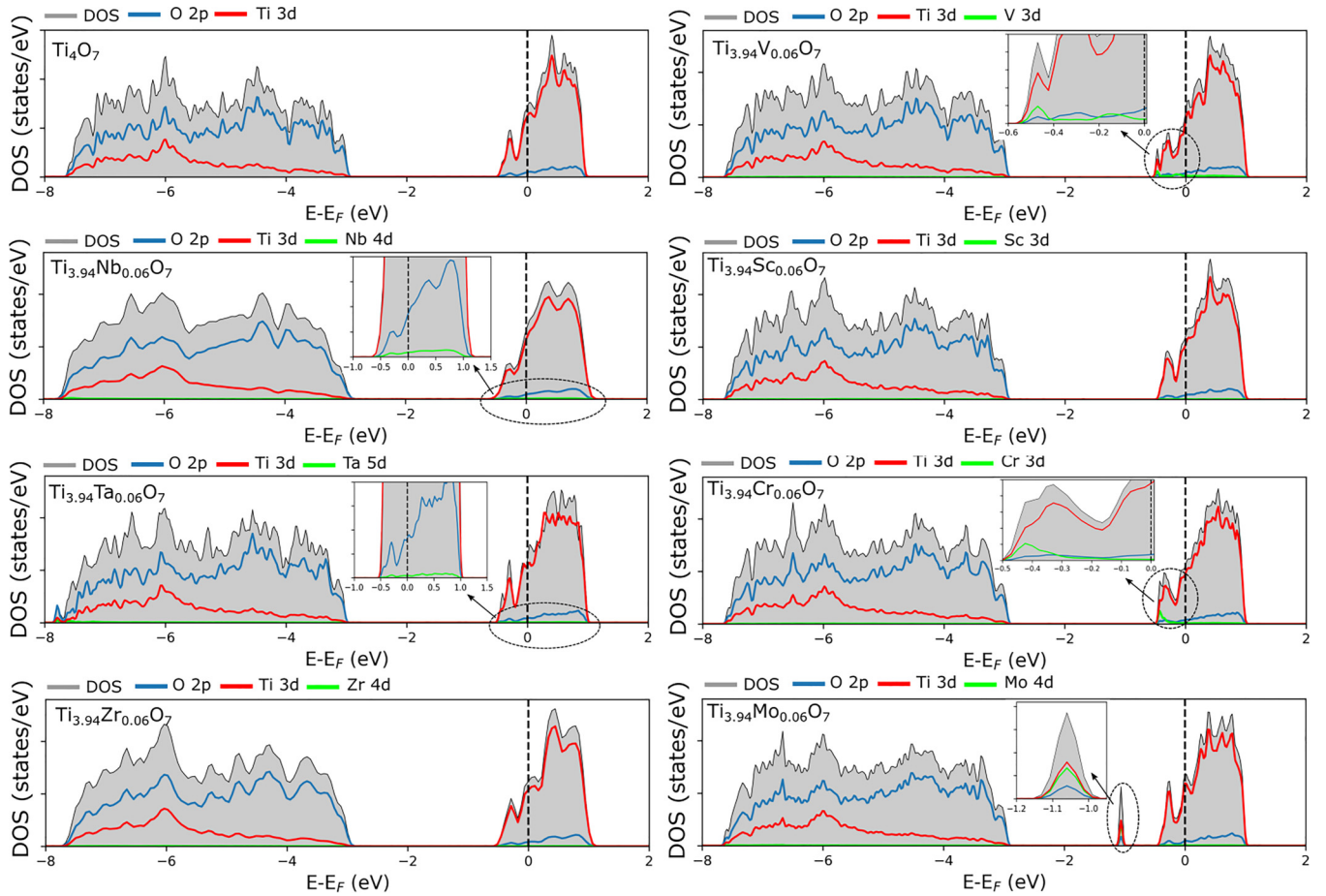


FIG. 2. The projected density of states (PDOS) of  $\text{Ti}_{3.94}\text{M}_{0.06}\text{O}_7$  (transition metal dopants,  $M = \text{Nb}, \text{Ta}, \text{Zr}, \text{V}, \text{Sc}, \text{Cr}, \text{Mo}$ ). The Fermi energy is set to zero. Insets are enlargements of the circled sections of the PDOS plots. The gray regions are the total density of states and the blue, red, and green curves correspond to the O  $2p$ , Ti  $3d$ , and  $M d$  states, respectively.

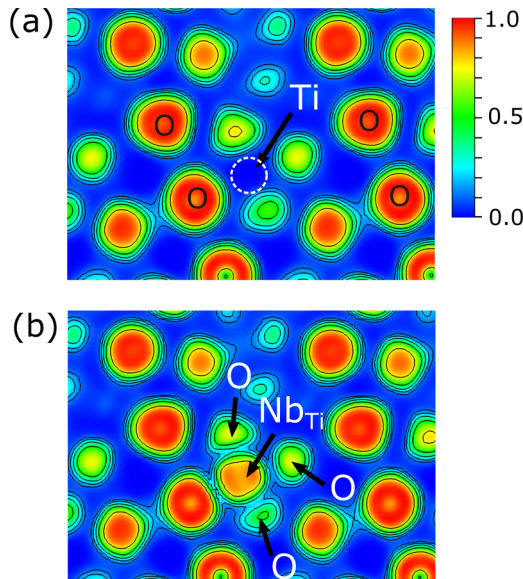


FIG. 3. The electron localization function (ELF) contours of (a) undoped and (b) Nb-doped  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_{3.98}\text{Nb}_{0.02}\text{O}_7$ . ELF values of 1, 0.5, and 0 correspond to perfect localization, metallic (free electron) regions, and regions of zero electron density, respectively.

These results suggest that Ta and Nb are the most promising transition-metal dopants for enhancing the electrical conductivity and phase stability of  $\text{Ti}_4\text{O}_7$ . We then calculated the change of the electrical conductivity ( $\sigma_0$ ) as a function of metal dopant (Nb, Ta, Zr, V, Sc, Cr, and Mo) concentration (Fig. 4 and Fig. S4 [64]). These results demonstrate that the conductivity values increase monotonically with increasing Ta and Nb concentration, while it decreases for Sc, Zr, V, Cr, and Mo.

Our DOS analysis shows that the DOS at the Fermi level increases with increasing TM doping. For example,  $N(E_F)$  increases from 59.4 to 101.2 to 147.1 states/eV when the Nb concentration is increased from 0 to 1.6 to 12.5%. Correspondingly, the electrical conductivity increases by  $\approx 44\%$  and  $\approx 64.8\%$  for the same Nb-doping concentrations. This clearly shows that increasing  $N(E_F)$  values leads to an increase in the predicted electrical conductivity.

Interaction of metal-doped  $\text{Ti}_4\text{O}_7$  with oxygen vacancies and interstitials was also considered to regulate the electronic structure of  $\text{Ti}_4\text{O}_7$ . Our results show that different metal substitutions could lead to different thermodynamic stability depending on the oxidation states of defect site and the metal dopant. When the  $\text{Ti}^{3+}$  and/or  $\text{Ti}^{4+}$  sites are replaced by a metal dopant (Nb, Ta, Zr, V, Sc, Cr, and Mo), oxygen vacancies or interstitials will be generated to retain the charge

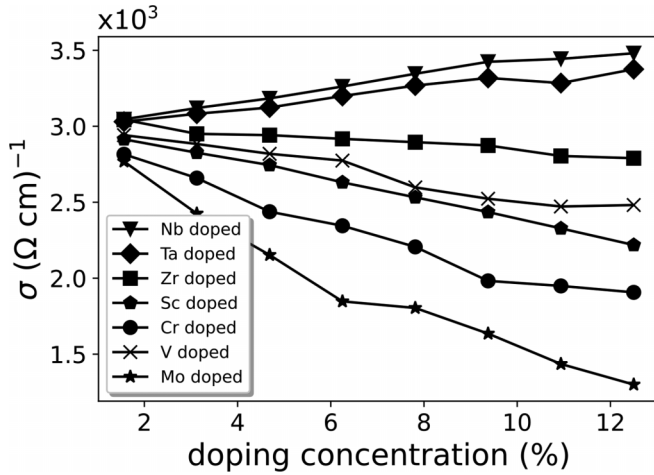


FIG. 4. The electrical conductivity as a function of dopant concentration in  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_{4-n}\text{M}_n\text{O}_7$ ,  $M=\text{Nb, Ta, Zr, V, Sc, Cr, and Mo}$ .

neutrality in doped  $\text{Ti}_4\text{O}_7$ . Then, the local lattice distortion will trigger the electron redistribution in doped  $\text{Ti}_4\text{O}_7$  to optimize the electronic structure. To identify the most stable site for vacancy and interstitial formation, multiple oxygen sites of the metal-doped structures were considered, and we show  $\text{Ti}^{3+}$  and  $\text{Ti}^{4+}$  sites with oxygen vacancy and interstitial sites in bulk  $\text{Ti}_4\text{O}_7$  (see Fig. S5 [64]). Gibbs formation energies were calculated from the following equations:

$$E_{\text{form}} = E(M_x\text{Ti}_{4-x}\text{O}_{7\mp y}) - E(M_x\text{Ti}_{4-x}\text{O}_7) \pm \frac{N_O}{2}(E_{\text{O}_2} - \Delta\mu_{\text{O}_2}), \quad (2)$$

where  $E(M_x\text{Ti}_{4-x}\text{O}_{7-y})$  and  $E(M_x\text{Ti}_{4-x}\text{O}_{7+y})$  are the total energy of dopants in  $\text{Ti}_4\text{O}_7$  with oxygen vacancy and interstitial;  $E(M_x\text{Ti}_{4-x}\text{O}_7)$  is the energy of dopant in  $\text{Ti}_4\text{O}_7$ .  $\pm N_O$  is the number of oxygen vacancy/ interstitial atom in  $M_x\text{Ti}_{4-x}\text{O}_{7\mp y}$ ,  $E_{\text{O}_2}$  is the DFT energy of the  $\text{O}_2$  molecule, and  $\Delta\mu_{\text{O}_2}$  is applied to convert the chemical potential of molecular oxygen at 300 K and 1 atm. The formation energies of oxygen vacancy and interstitial for each dopant are plotted in Fig. S6 [64].

The O incorporation thermodynamically stabilizes the Nb, Ta, Zr, and Sc dopants. It can be achieved by one O interstitial per one metal atom introduced for Ta, Zr, and Sc, while there are two O interstitials for Nb. At very low chemical potential of O, the O vacancy becomes thermodynamically favorable. We found that the V, Cr, and Mo doping will not promote the incorporating of O in  $\text{Ti}_4\text{O}_7$  compared to the undoped  $\text{Ti}_4\text{O}_7$ , while the O vacancy formation is favorable at low oxygen chemical potential region. We then calculated the change of the electrical conductivity ( $\sigma_0$ ) as a function of oxygen vacancy/interstitial concentration for doped  $\text{Ti}_4\text{O}_7$  (Fig. S7 [64]). These results demonstrate that the conductivity values decrease with oxygen incorporation, while the oxygen vacancy slightly increase the conductivity. Our results support the conclusion that minor Nb doping provides a promising path for enhancing the electrical conductivity of  $\text{Ti}_4\text{O}_7$ . However, incorporation of extra oxygen in Nb-, Ta-, Zr-, and Sc-doped  $\text{Ti}_4\text{O}_7$  decreases the conductivity.

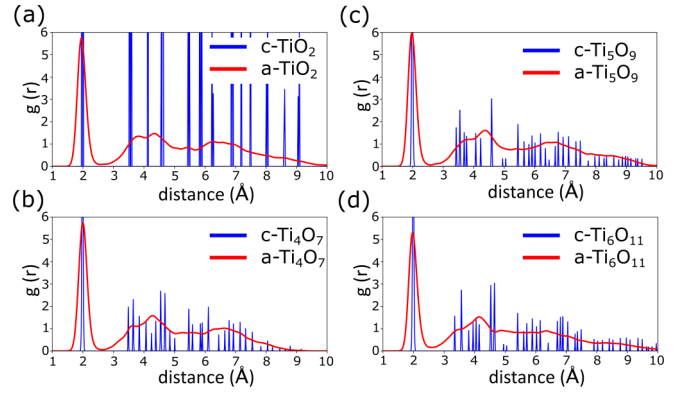


FIG. 5. The Ti-O radial distribution functions  $g(r)$  for amorphous (a)  $\text{TiO}_2$ , (b)  $\text{Ti}_4\text{O}_7$ , (c)  $\text{Ti}_5\text{O}_9$ , and (d)  $\text{Ti}_6\text{O}_{11}$  at 300 K and their crystalline counterparts at 0 K.

## V. AMORPHOUS SUB-OXIDES

The existence of several Magnéli Ti suboxides with very similar formation energies (within 0.01 eV/atom) in a very small composition range suggests that disorder is expected to be very common. This disorder could include amorphous phase formation or disordered vacancies on the (crystalline) oxygen sublattice. The extent of disorder will be sensitive to synthesis conditions. Here we examine and contrast two well-defined cases: ordered Magnéli suboxide phases and disordered suboxides due to rapid computational quenching of the  $\text{TiO}_2$ ,  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$  suboxides from the liquid state to low temperature.

The four crystalline oxides were computationally heated to 4000 K, melted, and equilibrated in their liquid form, followed by quenches to 900, 600, and 300 K within 40 ps, annealed at those temperatures for 20 ps, and then cooled to 0 K (each in 10 ps) using AIMD. While it is difficult to estimate quench rates during film deposition, the ones employed here are surely fast compared with most experimental synthesis techniques [76]; high quench rates were employed for computational efficiency. The evolution of the system energies (per atom) during the quench and anneal processes are shown in Fig. S8 [64]; the absence of abrupt drops in the energy and examination of the atomic structure suggest that no crystallization occurs during cooling/annealing. The Ti-O radial distribution functions  $g(r)$  of the amorphous structures (compositions corresponding to  $\text{TiO}_2$ ,  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$ ) at the end of the 300 K thermal schedule along with their crystalline counterparts at 0 K are shown in Fig. 5. Smooth  $g(r)$  distributions were obtained by time averaging the atomic coordinate data. These results suggest that these suboxides formed by rapid quenching from the melt are amorphous rather than crystalline with oxygen vacancy disorder.

The Ti-Ti, O-O, and Ti-O bond nearest-neighbor distances and  $g(r)$  in the amorphous structures (see Fig. S9 [64]) are very closely related to those in their crystalline counterparts.

In addition, the nearest-neighbor Ti-O distances in all of the amorphous sub-oxides show sharp peaks centered at approximately 2.0 Å, which is the single-crystal  $\text{TiO}_2$  nearest-neighbor separation [see Fig. 5(a)]. Nonetheless, there is a

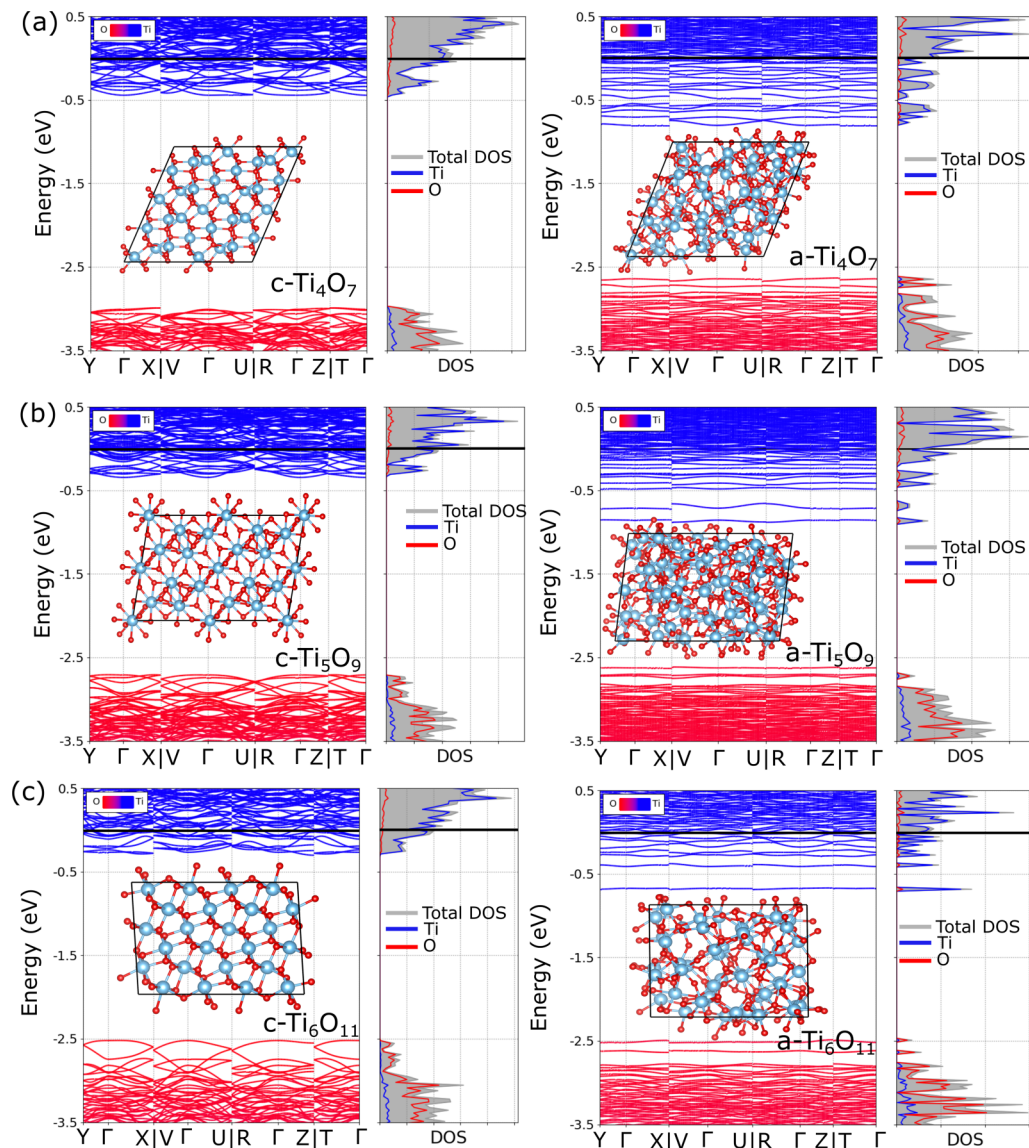


FIG. 6. Electronic band structure and density of states of crystalline and amorphous (a)  $\text{Ti}_4\text{O}_7$  (176-atom supercell), (b)  $\text{Ti}_5\text{O}_9$  (224-atom supercell), and (c)  $\text{Ti}_6\text{O}_{11}$  (136-atom supercell). The Fermi energy is set to zero. Insets show the crystal structures. Blue and red spheres represent titanium and oxygen, respectively. The gray regions in the density of state plots correspond to the total density of states, while the blue and red curves represent Ti and O states, respectively.

very small increase in the nearest-neighbor Ti–O distance (2.01 Å) in the amorphous suboxides as compared with their crystalline counterparts (1.98 Å). The second major Ti–O peak in  $g(r)$  for the amorphous suboxides occur at  $\approx 4.2$  Å. These observations are consistent with the similarities between the atomic structures of amorphous versions of  $\text{TiO}_2$ ,  $\text{Ti}_4\text{O}_7$ ,  $\text{Ti}_5\text{O}_9$ , and  $\text{Ti}_6\text{O}_{11}$ .

In the amorphous structures of the Magnéli and rutile phase compositions, most Ti atoms are surrounded by six oxygen atoms (as in their crystalline form) and most O atoms have three titanium atom neighbors (in the crystalline Magnéli phases, O atoms have two or three Ti neighbors). However, some Ti have five and seven O neighbors, and some O have two or four Ti neighbors. This observation is consistent with previous experimental and theoretical studies for amorphous

$\text{TiO}_2$  (we know of no such data for Ti suboxide compositions) [77]. The lower oxygen coordination number suggests both variations in local short-range order and is also consistent with the observed reduction in the Ti–O distance in the amorphous structures.

The electrical conductivity reduces significantly upon amorphization, as shown in Table II. For example, the conductivity of a- $\text{Ti}_4\text{O}_7$  is  $\approx 33\%$  lower than its corresponding crystalline form. Nonetheless, the band structures and the density of states plots in Fig. 6 suggest that the conductivity is metallic (the Fermi energy lies midband) in both the crystalline and amorphous forms.

According to the electronic band structures of amorphous phases in Fig. 6, there is a set of flat bands corresponding to localized electronic states as a result of disordering. These flat



bands below the Fermi level have a very small dispersion, as evidenced by the shapes of the corresponding peaks in the DOS. In addition to these flat bands, dispersive bands were observed around the Fermi level, corresponding to the delocalized states in crystalline structure. The DOS plots in Fig. 6 showed that amorphization leads to a small broadening of the DOS and the development of states within the pseudogap below the Fermi level. These localize O and Ti states lie below the Fermi energy level, thereby lowering the electrical conductivity of the amorphous phases.

The localization associated with disordering gives rise to a decrease in the predicted electrical conductivity as summarized in Table II. Some electrons located on the Ti cation transfer to the O anion due to its larger electronegativity by preserving the energy differences between the Ti-3*d* and the O-2*p* states. Therefore, the disorder-induced defect levels upon amorphization, which appear below the rest of the *d* bands near Fermi level, can trap electrons, leading to small changes in the electrical conductivity [78]. Note that amorphization leads to drops of up to 40% in the electrical conductivity.

## VI. DISCUSSION

Defective  $\text{TiO}_{2-x}$  has a higher ionic contribution to electrical current compared to stoichiometric  $\text{TiO}_2$  [79,80]. For example, oxygen vacancies in  $\text{TiO}_{2-x}$  are ionized and serve as mobile space charges due to the delocalized vacancy states. In that case, reordering/creation of defects in the structure can switch between the different available phases like conductive Magnéli phases. For this reason, understanding the relationship between defects, oxide phase behavior, and conductivity induced by changes in growth conditions will be a key factor. For example, the Magnéli phases can be synthesized using scalable methods like atomic layer deposition of amorphous  $\text{TiO}_x$  on a substrate; this allows highly precise thickness control, high uniformity, and low defect density [81,82]. Alternatively, the crystalline Ti oxides can be heated to higher temperatures ( $> 1000$  K) under a controlled atmosphere to synthesize Ti suboxides with a Magnéli phase [83,84]. For device applications, we expect that Magnéli phases can be grown by precisely controlling the oxygen pressure and temperature in the atmosphere [83,85–88] using  $\text{TiO}_2$  and/or amorphous  $\text{TiO}_x$  as a source.

The results presented above suggest that the electrical properties of the Magnéli phases of Ti suboxides can be manipulated by structural phase changes and transition-metal doping. In particular, TM-doping was found to be an efficient strategy to increase the electrical conductivity.

Although trends in the predicted electrical conductivity in the crystalline materials are consistent with previously reported values, they slightly overestimate those reported experimentally. This may be due to defects (e.g., impurities or high conductivity paths in the microstructure) in the experimental samples or the discretization of the eigenvalues resulting from the small supercell sizes in the simulations. Our preliminary investigation suggests that the supercell size effect is not significant for the conductivity; e.g., the  $\text{Ti}_4\text{O}_7$   $\sigma_0$  of a  $2 \times 2 \times 1$  unit-cell size (88-atoms) supercell was found

to be only  $\approx 9\%$  lower than that of the  $2 \times 2 \times 2$  (176-atoms) supercell.

The existence of several distinct titanium suboxide conductive Magnéli crystal structures ( $\text{Ti}_n\text{O}_{2n-1}$ ;  $n = 4, 5, 6, \dots$ ) of very similar energies and compositions suggest the strong possibility that amorphous structures will readily form, depending on synthesis conditions. Since our motivation to consider these Magnéli phases is based upon their high electrical conductivity, it is notable that amorphization leads to a significant reduction of their electrical conductivity. The defect energy levels formed up on complete amorphization lead to free electron trapping and localization that decrease the electrical conductivity. Nonetheless, electrical conduction in the amorphous structures, like their crystalline counterparts, is primarily metallic (based on electronic structural analysis). Amorphous metallic conductors generally have a higher resistivity than the corresponding crystalline structures due to the shorter mean free path of electrons [89–92]. Hence, the predicted electrical conductivity values would be expected to decrease as temperature increases.

In general, since oxides tend to have high melting points (1940 K for  $\text{Ti}_4\text{O}_7$  [93]), the electrical conductivity is less temperature dependent than pure metals [94]. Moreover, this is particularly important for MEMS/NEMS switches where the electrical current density is very high across asperity contacts; high current density leads to high temperatures. Our findings suggest that the changes in the electrical conductivity for both crystalline (i.e., +12.5% for c- $\text{Ti}_4\text{O}_7$ ) and especially amorphous (i.e., –12.3% for a- $\text{Ti}_4\text{O}_7$ ) Ti suboxides are small over the entire temperature range examined  $0 \leq T$  (K)  $\leq 900$ . Hence, we consider both amorphous and crystalline Ti suboxides of potential interest for MEMS/NEMS switch applications. Overall, our expectations are that, while lower, the electrical conductivity of amorphous Ti suboxides is still sufficiently high for use in NEMS contact applications. For these applications, mechanical properties and chemical activity are important. Therefore, further study is required to evaluate these properties for the Magnéli phases and amorphous Ti suboxides.

## VII. SUMMARY AND CONCLUSION

We presented a detailed *ab initio* study of the structural, electronic, and electrical transport properties of the Magnéli Ti suboxides ( $\text{Ti}_n\text{O}_{2n-1}$ ). We demonstrated how the electrical properties may be manipulated through phase amorphization, substitutional doping, and temperature changes. To predict the finite temperature electrical conductivity properties, we coupled the Kubo-Greenwood approach with atomic structures obtained via AIMD simulations. Although the predicted electrical conductivity for the stoichiometric Ti sub-oxides are higher than experimental measurement, the conductivity trends are consistent with experimental observations. TM doping was found to provide an effective means of manipulating the electrical properties of  $\text{Ti}_4\text{O}_7$ . We found that even small doping ( $\approx 1\%$ ) leads to large electrical conductivity enhancements ( $\approx 40\%$ ); larger doping concentrations have little effect on electrical conductivity. This small doping effect is associated with electron delocalization. On the contrary, higher doping concentrations promote electron localization which



decreases electrical conductivity. We expect that the same trend applies for  $\text{Ti}_5\text{O}_9$  and  $\text{Ti}_6\text{O}_{11}$ .

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