# **Magnetic structure and exchange interactions of transition metal dihalide monolayers: First-principles studies**

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Based on the first-principles linear response method and Monte Carlo simulation, we investigate the magnetic properties of single-layer transition metal dihalides  $MX_2$  ( $M = V$ , Cr, Mn, Fe, Co, Ni;  $X = Cl$ , Br, I). We calculate the magnetic exchange interactions using the first-principles linear response method and find that the thirdnearest-neighbor interaction of most dihalides is very important. When spin-orbit coupling is included, Fe*X*<sup>2</sup> and  $CrX_2$  ( $X = Cl$ , Br, I) are predicted to be insulators. Using the parameters of the exchange interactions and magnetic anisotropy, the magnetic ground states of transition metal dihalide monolayers are obtained by Monte Carlo simulation. Moreover, we find that the magnetic ground states of ten *MX*<sup>2</sup> monolayers are independent of the *U* value. Among them, the magnetic ground state of CrI<sub>2</sub> is ferromagnetic, the magnetic ground state of CrCl<sub>2</sub> is collinear antiferromagnetic, the magnetic ground states of VCl<sub>2</sub>, FeI<sub>2</sub>, and Co $X_2$  (*X* = Br, I) are 120<sup>°</sup> antiferromagnetic, and the magnetic ground states of FeCl<sub>2</sub> and Ni $X_2$  ( $X = Cl$ , Br, I) are helimagnetic.

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# **I. INTRODUCTION**

Since the successful synthesis of atomically thin  $CrI_3$  [\[1\]](#page-5-0),  $Cr_2Ge_2Te_6$  [\[2\]](#page-5-0), and Fe<sub>3</sub>GeTe<sub>2</sub> [\[3\]](#page-5-0), two-dimensional (2D) van der Waals (vdW) magnetic materials have attracted extensive attention. These intrinsically magnetic 2D vdW magnets provide an ideal platform for exploring long-range magnetic ordering at the 2D limit and show promising applications in next-generation spintronics devices. However, few 2D vdW materials have been synthesized experimentally so far. Therefore first-principles calculations have attracted more and more attention in the search for 2D magnetic materials, and a large number of 2D magnetic materials have been predicted, such as transition metal halides  $(MX_3$  and  $MX_2$ , where *M* is a transition metal and  $X = \text{Cl}$ , Br, I)  $[4-8]$ , transition metal dichalcogenides (*MX*2, where *M* is a transition metal and *X*  $=$  O, S, Se, Te) [\[9–11\]](#page-5-0), ternary transition metal compounds (*ABX*3, where *A* and *B* are metal cations and *X* is an anion)  $[12–16]$ , etc.

The single-layer transition metal dihalides have been shown to be stable in previous studies [\[4\]](#page-5-0). The calculation of the formation energy of transition metal dihalides confirmed that their monolayers are not only stable but also potentially exfoliable [\[4\]](#page-5-0). In recent years, single-layer 3*d* transition metal dihalides  $MX_2$  have attracted increasing attention  $[17–22]$ . Although some advancements have been made in the study of 3*d* transition metal dihalide monolayers, there are still many issues that require in-depth investigation: (i) At present, magnetic analysis of monolayer  $MX_2$  is mainly focused on collinear magnetic ordering. However, since the transition metal atomic arrangements of monolayer *MX*<sup>2</sup> are triangular lattices, the geometric frustration of the spin configurations is inevitable. For example, many experimental [\[23–26\]](#page-5-0) and theoretical [\[21,22,27\]](#page-5-0) works show that the ground state magnetic order of  $\text{Nil}_2$  is helimagnetic. However, some other theoretical studies only consider the nearest-neighbor interaction and predict that the magnetic ground state of monolayer  $\text{Nil}_2$ is ferromagnetic  $(FM)$  [\[4,6,28\]](#page-5-0). (ii) Many theoretical studies have predicted a half-metallic state of monolayer  $FeCl<sub>2</sub>$ with a large spin gap [\[20,29–31\]](#page-5-0). In fact, however, Cai *et al.* demonstrated that monolayer  $FeCl<sub>2</sub>$  is a uniform insulator with a gap of the magnitude of an electron volt [\[32\]](#page-5-0). Recently, Lu *et al.* [\[33\]](#page-5-0) obtained a FM insulator solution by properly treating the spin-orbit coupling (SOC), but the estimated Curie temperature  $(T_C = 25 \text{ K})$  is much smaller than that of previous works  $(109-470 \text{ K})$  [\[4,6,20\]](#page-5-0). As a result, accurate prediction of magnetic properties of *MX*<sup>2</sup> requires the correct electron ground state and magnetic exchange Hamiltonian. The determination of the magnetic ground state of  $MX_2$  is still a significant problem.

In this paper, based on density functional theory (DFT), we systematically study the magnetic properties of single-layer 3*d* transition metal dihalides  $MX_2$  ( $M = V$ , Cr, Mn, Fe, Co, Ni;  $X = \text{Cl}$ , Br, I). We calculate the magnetic exchange constants based on the combining magnetic force theorem and linear response approach [\[34–](#page-5-0)[37\]](#page-6-0) and find that the third-nearestneighbor interaction  $J_3$  is crucial for most dihalides. When SOC is included, we estimate the magnetic anisotropy energy (MAE) and find that monolayer Fe $X_2$  and Cr $X_2$  ( $X = \text{Cl}$ , Br, I) change from semimetals to insulators. With the calculated magnetic exchange constants and MAE, we carry out Monte Carlo simulations and determine the magnetic phase diagram.

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<span id="page-1-0"></span>We find that when  $U = 2$  eV, only the magnetic ground state of monolayer  $\text{CrI}_2$  is FM and its Curie temperature is 100 K. In addition, we also study the magnetic ground states of dihalide monolayers with different *U* values and find that the magnetic ground states of  $V_{X_2}$  ( $X = Br$ , I), CrBr<sub>2</sub>, Mn<sub>X<sub>2</sub></sub> ( $X$  $=$  Cl, Br, I), FeBr<sub>2</sub>, and CoCl<sub>2</sub> depend on the value of the Hubbard *U*.

## **II. METHOD**

The electronic structure calculations were carried out by using the full potential linearized augmented plane-wave method as implemented in the WIEN2K package [\[38\]](#page-6-0). For the exchange-correlation potential, the generalized gradient approximation (GGA) is used for structural relaxation and optimization. The crystal structure of monolayer  $MX_2$  is fully optimized, while the vacuum space is set to be 15 Å to avoid interactions with other neighboring layers.  $GGA + U$  calculations are performed to include the effect of Coulomb repulsion in the 3*d* orbital [\[39\]](#page-6-0). We also consider various values of the effective parameter  $U_{\text{eff}}$  ( $U_{\text{eff}} = 2{\text{-}}6$  eV).  $U = 2$  eV is used in the following calculations, while the results of other *U* values are provided in the Supplemental Material [\[40\]](#page-6-0). Using the second-order variational procedure, we include the SOC interaction [\[41\]](#page-6-0). The basic functions were expanded to  $R_{mt} \times K_{\text{max}} = 7$ , where  $R_{mt}$  is the smallest of the muffin-tin sphere radii and  $K_{\text{max}}$  is the largest reciprocal lattice vector used in the plane-wave expansion. A  $15 \times 15 \times 1$  *k*-point mesh is used for the Brillouin zone integral. The convergence of *k*-point meshes has been tested to ensure the accuracy of magnetic exchange constants. The self-consistent calculations are considered to be converged when the difference in the total energy of the crystal does not exceed 0.001 mRy at consecutive steps.

The magnetic exchange interactions are the basis for understanding the magnetic properties, and the first-principles calculation of exchange constants has attracted much interest. Commonly used methods include energy-mapping analysis [\[42\]](#page-6-0) and the spin-polarized Ruderman-Kittel-Kasuya-Yosida  $(RKKY)$  method  $[34,43-46]$  $[34,43-46]$ . Here, we use the method based on combining the magnetic force theorem and the linear response approach [\[34](#page-5-0)[–37\]](#page-6-0) to calculate the exchange interactions. The magnetic force theorem [\[34\]](#page-5-0) assumes a rigid rotation of atomic spin at sites  $R + \tau$  and  $R' + \tau'$  of the lattice (here *R* are the lattice translations, and  $\tau$  are the atoms in the basis). Then, the exchange constant *J* can be given as a second variation of the total energy induced by the rotation of atomic spin at sites  $R + \tau$  and  $R' + \tau'$  [\[35\]](#page-6-0),

$$
J_{R_l+\tau,R_{l'}+\tau'}^{\alpha\beta} = \sum_{nkn'k'} \frac{f_{nk} - f_{n'k'}}{\varepsilon_{nk} - \varepsilon_{n'k'}} \langle \psi_{nk} | [\sigma \times B_\tau]_{\alpha} | \psi_{n'k'} \rangle
$$
  
 
$$
\times \langle \psi_{n'k'} | [\sigma \times B_{\tau'}]_{\beta} | \psi_{nk} \rangle e^{i(k'-k)(R_l - R_{l'})}, \quad (1)
$$

where  $\sigma$  and *B* are the Pauli matrix and the effective local magnetic field, respectively.  $\varepsilon$  is the one-electron energy, while  $\psi$  is the corresponding wave function. *n* represents the band index, and *k* represents the *k* point index. This method directly computes the lattice Fourier transform  $J(q)$  of the exchange interaction  $J(R_l)$  and is convenient to obtain the exact long-range exchange interactions. This technique has



FIG. 1. (a) Top and (b) side views of the single-layer transition metal dihalide *MX*2. The red and green balls represent the *M* and *X* ions, respectively. The arrows define the magnetic exchange interactions.

been successfully applied to evaluate magnetic interactions in various magnetic materials [\[35–37,47–53.](#page-6-0) The algorithm of this method is now implemented in open-source software called WIENJ  $[54]$ , as an interface to WIEN2K  $[38]$ . Based on the calculated magnetic exchange interactions, we explore the magnetic phase diagrams by a replica-exchange Monte Carlo method [\[8](#page-5-0)[,55\]](#page-6-0).

### **III. RESULTS AND DISCUSSION**

### **A. Crystal structure**

Transition metal dihalide monolayers have two common crystal structures, namely the 2*H* phase with *P*-6*m*2 space group and the 1*T* phase with *P*-3*m*1 space group. Previous studies have demonstrated that all monolayer  $MX_2$  ( $M =$ V, Cr, Mn, Fe, Co, Ni;  $X = \text{Cl}$ , Br, I) tend to have 1-T structures at the monolayer level [\[4,7\]](#page-5-0). Therefore here we will focus only on the 1-*T* phase. The typical crystal structure of monolayer  $MX_2$  is shown in Fig. 1. For a 2D vdW system, careful treatment of long-range interactions is essential [\[56\]](#page-6-0). Since monolayer  $MX_2$  does not involve interlayer interactions, we do not consider the vdW interaction here, and we use the GGA scheme for structural relaxation and optimization as in previous studies [\[4,6\]](#page-5-0). The calculated lattice parameters of

TABLE I. Calculated lattice parameters (in angstroms) for monolayer  $MX_2$  within the GGA scheme.

a	Cl	Br	
V	3.65	3.83	4.11
Cr	3.64	3.82	4.09
Mn	3.70	3.88	4.15
Fe	3.53	3.73	4.02
Co	3.52	3.73	3.97
Ni	3.49	3.68	3.96

TABLE II. The magnetic exchange constants (in meV) of monolayer  $MX_2$  ( $M = V$ , Cr, Mn, Fe, Co, Ni;  $X = Cl$ , Br, I) evaluated from the GGA +  $U$  ( $U = 2$  eV) scheme. The calculated band gaps (in eV) and magnetic moments (in  $\mu_B$ ) are presented in the second and third columns. HM, half metal.

	Gap	$\boldsymbol{m}$	$J_1$	$J_2$	$J_3$
$\text{VCl}_2$	2.55	2.70	4.91	0.03	0.21
VBr <sub>2</sub>	2.41	2.72	1.79	0.07	0.33
VI <sub>2</sub>	2.29	2.73	$-2.19$	0.15	0.48
CrCl <sub>2</sub>	HM	3.76	1.22	$-0.41$	$-6.40$
CrBr <sub>2</sub>	HМ	3.80	$-0.59$	$-0.65$	$-7.81$
CrI <sub>2</sub>	HМ	3.85	$-2.48$	$-1.05$	$-8.90$
MnCl <sub>2</sub>	2.94	4.62	1.50	0.08	0.22
MnBr <sub>2</sub>	2.61	4.60	1.18	0.13	0.32
MnI <sub>2</sub>	2.00	4.57	0.92	0.25	0.45
FeCl <sub>2</sub>	HМ	3.65	$-14.74$	$-0.17$	$-0.36$
FeBr <sub>2</sub>	HМ	3.62	$-11.48$	$-0.43$	$-0.35$
FeI2 <sub>2</sub>	HМ	3.55	$-7.18$	$-0.70$	$-0.13$
CoCl <sub>2</sub>	1.87	2.62	0.48	0.07	0.74
CoBr <sub>2</sub>	1.50	2.58	1.46	0.07	1.20
Col <sub>2</sub>	0.87	2.46	4.00	0.05	2.61
NiCl <sub>2</sub>	1.93	1.55	$-1.13$	0.0008	1.38
NiBr <sub>2</sub>	1.46	1.48	$-0.95$	$-0.006$	2.38
NiI <sub>2</sub>	0.88	1.35	$-0.68$	0.0003	4.76

monolayer  $MX_2$  are shown in Table [I](#page-1-0) and are consistent with theoretical and experimental bulk material data [\[4,6\]](#page-5-0).

## **B. Magnetic properties**

Based on the optimized crystal structure, we perform GGA  $+ U (U = 2$  eV) calculations with the FM configuration. The calculated band gaps and magnetic moments on the transition metal ions are shown in Table II. The results indicate that monolayer Cr $X_2$  and Fe $X_2$  ( $X = C1$ , Br, I) are half metals while other dihalide monolayers are insulators, which is consistent with previous theoretical studies [\[6\]](#page-5-0). The calculated magnetic moments of  $M^{2+}$  ions are in agreement with the previous theoretical results [\[4,6\]](#page-5-0), indicating the high spin state of  $M^{2+}$ ions.

As shown in Fig. [1,](#page-1-0) we depict the main magnetic interactions. Based on the calculated FM structure, we estimate the magnetic exchange constants for the first-nearest neighbor  $J_1$ , the second-nearest neighbor  $J_2$ , and the third-nearest neighbor  $J_3$  in Table II. We also calculate the exchange interactions of the fourth and more distant neighbors, and as shown in Fig. S2 of the Supplemental Material [\[40\]](#page-6-0), the exchange constants decrease rapidly with increasing distance. It can be seen from Table II that most dihalide monolayers have relatively strong  $J_3$ , including  $MCl_2$  ( $M = Cr$ , Co, Ni),  $MBr_2$  ( $M = Cr$ , Mn, Co, Ni), and  $M_2$  ( $M = Cr$ , Mn, Co, Ni). The  $J_1$ ,  $J_2$ , and  $J_3$  of  $CrX_2$  (*X* = Br, I) and Fe $X_2$  (*X* = Cl, Br, I) are FM; so their magnetic ground states are FM, and the results for Fe $X_2$  ( $X =$ Cl, Br, I) are consistent with previous theoretical calculations  $[4,6,7]$ . In contrast, the  $J_1$  and  $J_3$  of the eight dihalides are antiferromagnetic (AFM), and their magnetic ground states should be AFM, including  $VX_2$  (*X* = Cl, Br), MnX<sub>2</sub> (*X* = Cl, Br, I), and  $CoX_2$  ( $X = Cl$ , Br, I). In addition,  $VI_2$ ,  $CrCl_2$ , and



FIG. 2. The calculated band gaps of monolayer  $MX_2$  ( $M = Cr$ , Fe;  $X = \text{Cl}$ , Br, I) evaluated from the GGA + SOC + *U* scheme.

Ni $X_2$  ( $X = \text{Cl}$ , Br, I) have different signs for  $J_1$  and  $J_3$ , and the magnetic ground states need to be determined by Monte Carlo simulations.

# **C. SOC and MAE**

It is worth mentioning that although monolayer  $FeCl<sub>2</sub>$  was predicted to be a semimetal in previous theoretical studies  $[20,29-31]$ , experiments demonstrated that it is a uniform insulator with a gap of  $1.2 \text{ eV}$  [\[32\]](#page-5-0). Recent theoretical studies have shown that SOC can lead to metal-insulator transitions for FeCl<sub>2</sub> [\[6,33,](#page-5-0)[57\]](#page-6-0). Therefore we also perform  $GGA + SOC$  $+ U$  calculations for monolayer Cr*X*<sub>2</sub> and Fe*X*<sub>2</sub> (*X* = Cl, Br, I) and find that they are all insulators. The calculated band gaps of monolayer  $CrX_2$  and  $FeX_2$  ( $X = Cl$ , Br, I) are shown in Fig. 2. The band gap of FeCl<sub>2</sub> calculated by the GGA  $+$  SOC  $+ U (U = 2$  eV) scheme is 0.98 eV, which is close to the experimental value  $(1.2 \text{ eV})$   $[32]$ . Moreover, we also use the Vienna *ab initio* simulation package (VASP) with the revised Heyd-Scuseria-Ernzerhof (HSE06) hybrid functional to get a more accurate electronic structure prediction of monolayer  $FeCl<sub>2</sub>$  and obtain a band gap of 3.4 eV. The calculation details and results are described in Secs. A and C of the Supplemental Material [\[40\]](#page-6-0).

Using the first-principles linear response (FPLR) method, we also calculate the exchange interactions including SOC. We find that the exchange couplings of monolayer  $CrX_2$  and Fe $X_2$  ( $X = C1$ , Br, I) change significantly due to the opening of a band gap as a result of the metal-insulator transition, while the exchange constants of other dihalide monolayers change little (less than 0.01 meV). It can be seen from Table [III](#page-3-0) that after SOC is included, the  $J_1$  of CrBr<sub>2</sub> changes from FM to AFM, while the values of  $J_2$  and  $J_3$  of CrX<sub>2</sub> (X = Cl, Br, I) are much smaller. For Fe $X_2$  ( $X = \text{Cl}$ , Br, I), all exchange interactions change from FM to AFM except for  $J_1$  of FeCl<sub>2</sub>. Our  $J_1$  of FeCl<sub>2</sub> (−0.92 meV) agrees well with the results of other calculations  $(-0.7 \text{ meV})$  [\[19,33\]](#page-5-0).

To determine the MAE, we perform  $GGA + SOC + U$ calculations with spin orientations perpendicular to the *ab* plane and lying in the *ab* plane, i.e., the spin orientations are along the (001) and (100) directions. We list the calculated

<span id="page-3-0"></span>TABLE III. The magnetic exchange constants (in meV) of monolayer  $MX_2$  ( $M = Cr$ , Fe;  $X = Cl$ , Br, I) evaluated from the GGA +  $SOC + U (U = 2$  eV) scheme. The calculated band gaps (in eV) and magnetic moments (in  $\mu_B$ ) are presented in the second and third columns.

	Gap	m	$J_1$	$J_2$	$J_3$
CrCl <sub>2</sub>	0.76	3.71	3.01	0.02	$-0.73$
CrBr <sub>2</sub>	0.47	3.75	0.73	0.006	$-1.25$
CrI <sub>2</sub>	0.03	3.79	$-1.29$	$-0.07$	$-2.59$
FeCl <sub>2</sub>	0.98	3.64	$-0.92$	0.11	0.36
FeBr <sub>2</sub>	1.02	3.61	0.10	0.13	0.56
FeI <sub>2</sub>	0.60	3.55	1.50	0.27	0.98

MAE in Table IV and find that the MAEs of monolayer  $CrX_2$ and Fe $X_2$  ( $X = C1$ , Br, I) are very large (even larger than the exchange constant). For  $FeCl<sub>2</sub>$ , the (001) direction has a higher energy (36 meV) than the (100) direction, consistent with previous theoretical calculations (55 meV) [\[19\]](#page-5-0). We also use VASP to confirm the results of magnetic anisotropy, and the calculation results are listed in Table S8 in the Supplemental Material [\[40\]](#page-6-0). Our calculations show that the (001) direction of  $CrX_2$  (*X* = Cl, Br, I), MnI<sub>2</sub>, and NiI<sub>2</sub> is the easy axis, while the (001) direction of Fe $X_2$  ( $X = \text{Cl}$ , Br, I), Co $X_2$  ( $X = \text{Cl}$ , Br, I), and NiCl<sub>2</sub> is the hard axis. In addition,  $VX_2$  (*X* = Cl, Br, I),  $MnX_2$  ( $X = Cl$ , Br), and  $NiBr_2$  have different easy axis directions calculated by WIEN2K and VASP.

It is worth mentioning that due to the inversion symmetry,  $J_{ij}^{\alpha\beta} = J_{ij}^{\beta\alpha}$ , there is no Dzyaloshinskii-Moriya interaction in monolayer *MX*2. Considering the potential significance of higher-order interactions in layered magnetic materials [\[58\]](#page-6-0), we investigate the biquadratic exchange interactions of some monolayer dihalides in Sec. H of the Supplemental

TABLE IV. MAE (in meV) of monolayer  $MX_2$  ( $M = V$ , Cr, Mn, Fe, Co, Ni;  $X = \text{Cl}$ , Br, I) with different *U* values. When no value is given, this indicates that the calculated MAE is less than 0.01 meV.

	$U = 2$ eV	$U = 4$ eV	$U = 6$ eV
VCl <sub>2</sub>	$-0.04$	$-0.07$	$-0.09$
VBr <sub>2</sub>	$-0.04$		
VI2			$-0.03$
CrCl <sub>2</sub>	3.06	1.63	1.01
CrBr <sub>2</sub>	6.16	3.58	2.24
CrI <sub>2</sub>	10.9	10.6	7.68
MnCl <sub>2</sub>	$-0.23$	$-0.22$	$-0.23$
MnBr <sub>2</sub>			
MnI <sub>2</sub>	0.57	0.58	0.53
FeCl <sub>2</sub>	$-36.0$	$-40.8$	$-40.1$
FeBr <sub>2</sub>	$-35.4$	$-37.8$	$-36.1$
FeI	$-36.0$	$-37.0$	$-34.1$
CoCl <sub>2</sub>	$-0.13$	$-0.10$	$-0.08$
CoBr <sub>2</sub>	$-0.66$	$-0.49$	$-0.34$
CoI <sub>2</sub>	$-1.16$	$-0.95$	$-0.68$
NiCl <sub>2</sub>	$-0.03$	$-0.03$	$-0.03$
NiBr <sub>2</sub>	0.03		0.04
NiI2	0.66	0.57	0.36



FIG. 3. The phase diagram obtained by the Monte Carlo simulations. The insets show schematic diagrams of the spin configuration.

Material [\[40\]](#page-6-0). The biquadratic exchange of  $MCl_2$  ( $M = V$ , Cr, Mn, Fe, Co, Ni) calculated from noncollinear DFT simulations by VASP is shown in Table S10.

### **D. Magnetic ordering**

Based on the calculated magnetic exchange constants, we use the following Heisenberg model and carry out Monte Carlo simulations to estimate the magnetic transition temperature and the magnetic phase diagram for monolayer *MX*2:

$$
H = \sum_{i < j} J_{ij} S_i \cdot S_j - \sum_i A \left( S_i^z \right)^2,\tag{2}
$$

where  $A$  is the MAE.  $S_i$  is the spin vector (of unit length) at site *i*. We adopt  $L \times L \times 1$  supercells with  $L = 36$  in our Monte Carlo simulations.

Since most dihalides have very small values for  $J_2$ , and to simplify the model, we first set  $J_2 = 0$  and  $A = 0.1$  meV in the following simulations. Subsequently, we obtain the magnetic phase diagram of  $J_1$  and  $J_3$  in Fig. 3. As expected, when both  $J_1$  and  $J_3$  are negative, the magnetic ground state is FM; when both  $J_1$  and  $J_3$  are positive, the magnetic ground state is 120° AFM; when  $J_1 < 0$  and  $J_3 > 0$ , if  $J_3 < |J_1|/4$ , the magnetic ground state is FM, and otherwise the magnetic ground state is helimagnetic [\[59,60\]](#page-6-0); and when  $J_1 > 0$  and  $J_3$  < 0, if  $|J_3|$  <  $J_1/9$ , the magnetic ground state is 120 $\degree$  AFM, and otherwise the magnetic ground state is collinear AFM. Therefore the magnetic ground states of  $V_{X_2}$  ( $X = Cl$ , Br),  $MnX_2$  (*X* = Cl, Br, I), and Co*X*<sub>2</sub> (*X* = Cl, Br, I) are 120<sup>°</sup> AFM. For the five dihalides with different signs of  $J_1$  and  $J_3$ by the GGA +  $U$  ( $U = 2$  eV) scheme,  $VI_2$  is FM, CrCl<sub>2</sub> is collinear AFM, and  $NiX_2$  ( $X = Cl$ , Br, I) are helimagnetic. Furthermore, when SOC is included,  $CrX_2$  ( $X = Cl$ , Br) are collinear AFM,  $\text{CrI}_2$  is FM,  $\text{FeCl}_2$  is helimagnetic, and  $\text{FeX}_2$  $(X = Br, I)$  are 120 $\degree$  AFM.

Next, we also perform Monte Carlo simulations using the value of *A* given in Table IV for the model containing  $J_2$ . The obtained magnetic ground states are summarized in the second column of Table [V.](#page-4-0) The results show that the magnetic ground state of VI<sub>2</sub> changes from FM to helimagnetic, and the

<span id="page-4-0"></span>TABLE V. Magnetic ground state of monolayer  $MX_2$  from Monte Carlo simulations with different *U* values. FM, FM state; AFM, collinear AFM state; 120, 120◦ AFM state; H, helimagnetic state. The magnetic ground states of monolayer  $CrX_2$  and  $FeX_2$  ( $X = Cl$ , Br, I) including SOC are shown in parentheses.

	$U = 2$ eV	$U = 4$ eV	$U = 6$ eV
VCl <sub>2</sub>	120	120	120
VBr <sub>2</sub>	120	120	H
VI <sub>2</sub>	H	<b>FM</b>	<b>FM</b>
CrCl <sub>2</sub>	AFM (AFM)	FM (AFM)	FM (AFM)
CrBr <sub>2</sub>	FM (AFM)	FM (FM)	FM (FM)
CrI <sub>2</sub>	FM (FM)	FM (FM)	FM (FM)
MnCl <sub>2</sub>	120	120	H
MnBr <sub>2</sub>	120	H	H
MnI <sub>2</sub>	120	FM	FM
FeCl <sub>2</sub>	FM(H)	FM(H)	FM(H)
FeBr <sub>2</sub>	FM(H)	FM (120)	FM(H)
FeI <sub>2</sub>	FM (120)	FM (120)	FM (120)
CoCl <sub>2</sub>	120	120	H
CoBr <sub>2</sub>	120	120	120
Col <sub>2</sub>	120	120	120
NiCl <sub>2</sub>	H	H	H
NiBr <sub>2</sub>	Н	H	H
$\mathrm{Nil}_2$	Н	Н	Н

magnetic ground state of FeBr<sub>2</sub> changes from 120 $\textdegree$  AFM to helimagnetic.

With the calculated magnetic exchange constants and anisotropy energy, we also simulate the thermal dependence of magnetic properties and the magnetic transition temperature. Figure 4 depicts the magnetization and magnetic susceptibility as functions of temperature for  $\rm{CrI_2}$ , the only FM insulator at  $U = 2$  eV. The calculated magnetic transition temperature of  $CrI<sub>2</sub>$  is 100 K.

### **E. The influence of Hubbard** *U*

As changes in *U* can modify the magnetic states and properties of 3*d* metal compounds, theoretically it is necessary to calculate the value of *U* for each material [\[61,62\]](#page-6-0). Here, we have varied the value of *U* from 2.0 to 6.0 eV, and the calculated magnetic exchange constants of  $U = 4$  and 6 eV are provided in Tables S4, S5, S6, and S7 in the Supplemental Material [\[40\]](#page-6-0). As shown in Table [IV,](#page-3-0) changing *U* has little effect on the MAE and does not change the easy axis. Moreover, we also investigate the effect of different *U* values on the magnetic ground state. It can be seen from Table V that  $V_{12}$  $(X = Br, I)$ ,  $CrBr_2$ ,  $MnX_2$   $(X = Cl, Br, I)$ ,  $FeBr_2$ , and  $CoCl_2$ will change the magnetic ground state with different *U* values. Among them, when the value of *U* increases above 4 eV, the magnetic ground states of  $VI_2$ , CrBr<sub>2</sub>, and MnI<sub>2</sub> become FM.

## **IV. CONCLUSIONS**

In conclusion, the magnetic properties of single-layer transition metal dihalides are further studied based on the magnetic interaction calculated by the first-principles linear



FIG. 4. Monte Carlo simulations of the magnetization and magnetic susceptibility for monolayer CrI2.

response method. When spin-orbit coupling is handled properly, monolayer Fe $X_2$  and Cr $X_2$  ( $X = \text{Cl}$ , Br, I) change from half metals to insulators. Based on the obtained magnetic exchange constants and anisotropy energy, Monte Carlo simulations are performed, and magnetic phase diagrams are generated. The results indicate that when  $U = 2$  eV, only the magnetic ground state of monolayer  $CrI<sub>2</sub>$  is ferromagnetic and its Curie temperature is 100 K. Moreover, there are ten dihalide monolayers whose magnetic ground states do not depend on the value of the Hubbard  $U$ . In addition to  $\text{CrI}_2$ having a ferromagnetic ground state, CrCl<sub>2</sub> has a collinear antiferromagnetic ground state;  $VCl_2$ ,  $FeI_2$ , and  $CoX_2$  ( $X = Br$ , I) have  $120^\circ$  antiferromagnetic ground states; and FeCl<sub>2</sub> and  $NiX_2$  ( $X = Cl$ , Br, I) have helimagnetic ground states. These results emphasize the importance of properly considering the effects of the Hubbard *U* and spin-orbit coupling to obtain the correct magnetic ground state. This study demonstrates accurate calculations of magnetic exchange constants for transition metal dihalides, which will help us to more deeply understand their magnetic properties and support further study of them.

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