Letter

Critical behavior and duality in dimensionally reduced planar Chern-Simons superconductors

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The quantum electrodynamics of particles constrained to move on a plane is not a fully dimensionally reduced theory because the gauge fields through which they interact live in higher dimensions. By constraining the gauge field to the surface of the bulk, we obtain a fully reduced planar Abelian Chern-Simons Higgs model that can describe the vortex dynamics and second-order superconducting-normal phase transitions in planar Chern-Simons superconductors. Dual analyses performed before and after dimensional reduction yield the same Lagrangian for describing the vortex dynamics, indicating the self-consistency of our reduced theory. Compared to ordinary (2+1)-dimensional electrodynamics, we obtain anomalous fermion statistical vortices, consistent with results considering boundary effects. An additional electric charge constraint and different Chern-Simons parameter constraints are also found, which may help define a self-dual conformal field theory. Our renormalization group analysis shows that the quantized critical exponent depends on the Chern-Simons parameter. Quench disorder can bring more stable fixed points with different dynamical critical exponents. If we dimensionally reduce to a curved surface, our theory can also be extended to curved spacetimes, where geometric flow will be introduced and compete with vortex flow.

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Introduction. The discoveries of the quantum Hall effect [1–4] and topological planar materials [5–8] in the past 20 years have attracted attention to (2+1)-dimensional [(2+1)D] materials. Electromagnetic fields play an important role in these planar materials [9–16], prompting the investigation of effective theories that incorporate gauge fields. The direct application of quantum electrodynamics in (2+1)D (QED3) in order to describe the electromagnetic interaction in such planar materials leads to incorrect results because despite the fact that the quasiparticles are constrained to a plane, the gauge field is not. For this reason, the gauge fields in QED3 cannot be considered completely fully fledged dimensionally reduced from (3+1)D quantum electrodynamics. Pseudoquantum electrodynamics (PQED), which is also called reduced quantum electrodynamics, is the correct way for introducing the $U(1)$ gauge fields in planar materials, that in spite of being a fully (2+1)D theory, does describe a (3+1)D electromagnetic field interacting with planar particles. PQED was first proposed by Marino [17] and has attracted increasing attention both in theoretical [18–22] and numerical [23] research during these years.

PQED avoids the logarithmic divergent of a Coulomb potential in the plane which is associated with QED3. In the static limit, the Coulomb potential $V(r) \propto 1/r$ is obtained in PQED and it can correctly describe the electron-electron interactions in two-dimensional materials. In PQED, the normal Maxwell term $F_{\mu\nu}^2$, with gauge field strength $F_{\mu\nu}$, is changed into a nonlocal term $F_{\mu\nu}^2/\sqrt{\Box}$. It has been proved that the PQED is unitary [24] and the causality is preserved [25].

Planar systems contain many interesting phenomena, such as superconductivity [26–28], anyon statistics [29–34], quantum Hall effect [35,36], and quantum vortices [37,38]. They can all result from the Chern-Simons (CS) term, which can be obtained from the one-loop correction of the gauge field in Fermi quantum electrodynamics. It has no further corrections at higher loops [39], a result known as the Coleman-Hill theorem. Gauge invariance is restored by using Pauli-Villars regularization while leading to a parity anomaly [40,41]. Due to the natural introduction of the CS term through the integration of the electronic field in superconducting systems, and the intrinsic vortex excitations, we choose to study planar CS superconductors.

In this Letter, we demonstrate that the planar Abelian CS Higgs model, which describes planar CS superconductors, exhibits a second-order superconducting-normal phase transition. This planar theory is significantly different from QED3, as the CS term is no longer a topological mass term and exhibits distinct critical behaviors. Considering the inevitable existence of impurities in a real experimental system, we conduct a renormalization group (RG) analysis in the presence of weak quenched disorder, and identify that the fixed points and critical exponents depend on the CS parameter, while disorder can yield more diverse critical behaviors. Finally,

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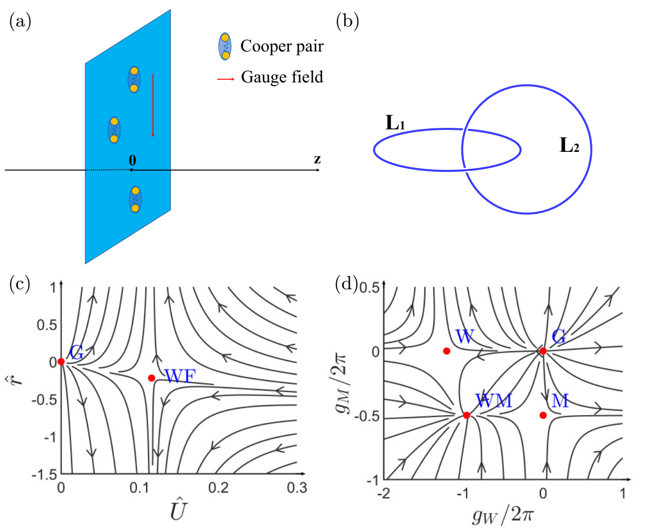

FIG. 1. (a) Schematic illustration of a planar Chern-Simons superconducting system in electromagnetic fields. Cooper pairs are constrained to the plane and the electromagnetic gauge field also lives in (2+1)D. (b) Linking of two vortex worldlines L_1 and L_2. (c) RG flow of the dimensionless couplings $\hat{U}$ and $\hat{r}$ in the absence of disorder, where $n = N = 1$. There is a Gaussian fixed point (G) and a Wilson-Fisher fixed point (WF) stable along the $\hat{U}$ direction. (d) RG flow in the $(g_W/2\pi, g_M/2\pi)$ plane with weak quenched mass disorder and current disorder, where $c = 25/558$ and all other couplings are set to zero. Besides the Gaussian fixed point (G), the other three fixed points (W, M, and WM) are stable along the directions of g_W, g_M, and both, respectively.

based on the aforementioned phase transition behaviors and statistical effects due to the CS term, it is feasible to discuss the properties of vortices. We obtain a unified effective action, which describes vortex loops, based on two different considerations from the surface and bulk, revealing that the anomalous fermionic statistics of vortices are consistent with the inclusion of boundary effects, and our results do not rely on the limit $e^2 \to \infty$.

Dimensionally reduced planar Chern-Simons superconductors. The dimensionally reduced planar Abelian CS Higgs model, which describes the superconducting-normal phase transition in a planar system, reads as

$$\mathcal{L} = \frac{1}{4e^2} F^{\mu\nu} \frac{2}{\sqrt{\Box}} F_{\mu\nu} + |(\partial_\mu - iA_\mu)\phi|^2 + r|\phi|^2 + \frac{U}{2}|\phi|^4 + \xi A^\mu \frac{\partial_\mu \partial_\nu}{\sqrt{\Box}} A^\nu + i\frac{\theta}{2}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with A_μ being the $U(1)$ gauge field. Note that, since A_μ lives in (2+1)D and interacts with the massive planar complex scalar field ϕ, as shown in Fig. 1(a), but corresponding to a $U(1)$ electromagnetic gauge field living in (3+1)D, we use the nonlocal PQED theory to describe it. e is the effective electric charge with zero scalar dimension ($[e] = 0$), $\Box$ is the d'Alembertian operator, and ξ is a gauge fixing parameter, where $\theta = n/2\pi$ is the CS parameter with zero scaling dimension ($[\theta] = 0$) and n must be an integer if the $U(1)$ gauge field is compact [42], and the charged excitations all have integer charge and are fully gapped. We note that the CS term is not gauge invariant except for the topologically trivial gauge transformation, while $e^{-S_{CS}}$ is always gauge invariant with integer CS level n.

Our attention is drawn to the model above, which describes the planar superconducting system, motivated by the following considerations: (i) The $|\phi|^4$ theory, $\mathcal{L}_\phi = |\partial_\mu\phi|^2 + r|\phi|^2 + U|\phi|^4/2$, can capture the second-order superconducting-normal phase transition [43] without an electromagnetic field. The complex scalar field ϕ represents Cooper pairs. If we turn on the electromagnetic field and consider it interacts with Cooper pairs which are constrained on the plane, it can be described by the nonlocal planar Abelian Higgs model. (ii) Considering the massive electrons near the Dirac point on the plane interact with an electromagnetic field, the Lagrangian is given by $\bar{\psi}(\gamma^\mu\partial_\mu - i\gamma^\mu A_\mu - M)\psi$, where M is the mass of the electrons. Using Pauli-Villars regularization and integrating out the Fermi field, we get the CS term [40,44] $\mathcal{L}_F = i\,\mathrm{sgn}(M)\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho/8\pi$. We only consider the electrons near the Fermi surface and they always appear in pairs to form Cooper pairs, so only even multiples of $\mathcal{L}_F$ appear in Lagrangian (1) [45]. In addition, the CS term may also induce the fractional (anyon) statistics which attracts our attention. (iii) The reason why we do not consider the interaction $\lambda|\psi|^4$ is that the scaling dimension of λ is smaller than 0 ($[\lambda] = 2 - D$) and is irrelevant. The terms such as $g\phi\bar{\psi}\bar{\psi}$ will renormalize the mass term of ϕ after integrating out the Fermi field in a large momentum cut.

The bare propagator in the momentum space of the gauge field in Lagrangian (1) is

$$\Delta^0_{\mu\nu}(p) = \frac{2e^2}{4+\theta^2e^4}\frac{\delta_{\mu\nu}}{\sqrt{p^2}} - \frac{\theta e^4}{4+\theta^2e^4}\frac{\epsilon_{\mu\nu\alpha}p^\alpha}{p^2} - \left(\frac{2e^2}{4+\theta^2e^4} - \frac{e^2}{\xi}\right)\frac{p_\mu p_\nu}{p^2\sqrt{p^2}}. \quad (2)$$

Without the CS term ($\theta = 0$), the equation describes the Coulomb interaction in plane [46] and it has the same form as the leading-loop renormalization photon propagator at the limit of large-N in Fermi massless QED3 theory [47,48]. The CS term no longer acts as the topological mass term of the gauge field as in QED3 theory, instead playing a role in regulating the intensity of the Coulomb interaction.

Renormalization group analysis without disorder. We will use the Landau gauge ($\xi = \infty$) for convenience in this part. The one-loop RG β equations (details are presented in the Supplemental Material [45]),

$$\beta_{\hat{r}} = \mu\frac{d\hat{r}}{d\mu} = -\left(2 + \frac{2.9\hat{e}^2}{4\pi^2(4+\hat{\theta}^2\hat{e}^4)}\right)\hat{r} - 2(N+1)\hat{U},$$

$$\beta_{\hat{U}} = \mu\frac{d\hat{U}}{d\mu} = -\left(4 - D + \frac{5.8\hat{e}^2}{4\pi^2(4+\hat{\theta}^2\hat{e}^4)}\right)\hat{U} + 2(N+4)\hat{U}^2,$$

$$\beta_{\hat{e}^2} = \mu\frac{d\hat{e}^2}{d\mu} = -(3-D)\hat{e}^2 + (3-D)\frac{N}{32}\hat{e}^4, \quad (3)$$

are obtained by defining the dimensionless renormalized couplings $r/\mu^2 \to \hat{r}$, $S_D U/\mu^{4-D} \to \hat{U}$, electronic charge

$e_r^2/\mu^{3-D} \to \hat{e}^2$, and CS level $\theta\mu^{3-D} \to \hat{\theta}$, with $S_D = (4-D)\Gamma(2-D/2)/(4\pi)^{D/2}$ and spacetime dimension D.

When $D > 3$, $\hat{e}_*^2 = 0$ is the IR stable fix point, and we get the same RG theory as without the gauge field. When $D < 3$, $\hat{e}_*^2 = 32/N$ is the IR stable fix point as the bare coupling $e^2 \to \infty$. Inserting the fixed point into the first two β functions in Eqs. (3), we get the Gaussian fixed point, $(\hat{r}_G, \hat{U}_G) = (0, 0)$, and the $O(2N)$ symmetric Wilson-Fisher fixed point, $(\hat{r}_{\rm WF}, \hat{U}_{\rm WF}) = [-(N+1)(1+\epsilon+\frac{5.8\hat{e}_*^2}{4\pi^2(4+\hat{\theta}^2\hat{e}_*^4)})/(N+4)(2+\frac{2.9\hat{e}_*^2}{4\pi^2(4+\hat{\theta}^2\hat{e}_*^4)}), (1+\epsilon+\frac{5.8\hat{e}_*^2}{4\pi^2(4+\hat{\theta}^2\hat{e}_*^4)})/2(N+4)]$, where $\epsilon = 3 - D$ and N is the total categories of the complex scalar field [49–52]. It can effectively describe superconductors when $N = 1$. The RG flow for the β function of the dimensionless coupling $\hat{r}$ and $\hat{U}$ is shown in Fig. 1(c). It has similar properties to the ϕ^4 theory, but with a different critical value. The nature of the critical behaviors [53,54] is described by the field theory of the Wilson-Fisher fixed point with a controllable CS level n. The critical temperature of the phase transition for one-loop at finite temperature is $T_c \sim (r_{\rm WF} - r)/U$ [45].

For the level 1 CS term, we can get the critical exponent $\eta \approx 0.08$ and the IR fixed point is $(-0.22.0.12)$ when we consider the superconductor model ($N = 1$). At long distances, the critical pair correlation function has the form $\langle\phi(x)\phi(0)\rangle \sim 1/|x|^{1.08}$. All calculations are performed in $D = 3 - \epsilon$ because the PQED and CS term restrict the dimension of spacetime to three, which is different from QED3, where the RG analysis should be conducted in $D = 4 - \delta$ with $\delta = 1$.

Renormalization group analysis with weak quenched disorder. Due to the influence of impurities, experimental systems inherently possess both disorders, so we need to take into account the effects of weak quenched disorders. We only consider disorders coupling to a gauge-invariant operator $\mathcal{O}$, $S_{\rm dis}[\mathcal{O}] = \int d^dx d\tau D(x)\mathcal{O}(x,\tau)$, where we have denoted the Euclidean spacetime coordinates as $r = (x, \tau)$. The quenched random coupling $D(x)$ is time independent, and is a Gaussian random variable, $\overline{D(x)D(x')} = \frac{g_D}{2}\delta^d(x-x')$ with zero average. g_D is the variance of $D(x)$ and controls the strength of the disorder. We use the replica trick to treat disorder questions.

Based on the RG analysis above, it has been determined that the phase transition is primarily controlled by the mass term. Therefore, we can first consider the mass disorder $\phi^*\phi$ while neglecting the mass term for convenience. The global $U(1)$ symmetry has a conserved current $J_\mu = i(\phi \overleftrightarrow{\partial_\mu} \phi^*) \equiv i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$ and we can also have flux disorder. The final disorder action takes the form

$$S_{\rm dis} = \int d^dx d\tau [V(x)|\phi(x,\tau)|^2 + iW(x)\phi(x,\tau)\overleftrightarrow{\partial_0}\phi^*(x,\tau) + iM^i(x)\phi(x,\tau)\overleftrightarrow{\partial_i}\phi^*(x,\tau)]. \tag{4}$$

Since disorders break the Lorentz invariance, we need to separate space and time. We use the roman letters i, j, k, etc., indicating the sum is over the spatial coordinates, and the greek letters μ, ν, δ, etc., including time as well, where $V(x)$, $W(x)$, $M^i(x)$ are Gaussian random variables with a vanishing mean, and the variance is g_V, g_W, g_M, respectively. The average of two of them is vanishing.

We obtained the RG flow equations in one-loop order with dimensional regularization and a Feynman gauge (details are presented in the Supplemental Material [45]),

$$\begin{aligned}
z &= 1 + \frac{1}{4\pi}(2g_W - g_M),\\
\beta_{g_V} &= -g_V\left(2 + \epsilon + \frac{16e^2}{3\pi^2(4+\theta^2e^4)} + \frac{g_M + g_W}{2\pi}\right),\\
\beta_{g_W} &= -g_W\left(\epsilon + \frac{54e^2}{15\pi^2(4+\theta^2e^4)} + \frac{g_M + 2g_W}{2\pi}\right),\\
\beta_{g_M} &= -g_M\left(\epsilon + \frac{78e^2}{15\pi^2(4+\theta^2e^4)} + \frac{7g_M}{2\pi}\right),\\
\beta_U &= -U\left(1 + \epsilon - \frac{8e^2}{3\pi^2(4+\theta^2e^4)} - \frac{g_M}{\pi}\right),\\
\beta_{e^2} &= -e^2\left(\epsilon + \frac{Ne^2g_M}{32\pi}\right),
\end{aligned} \tag{5}$$

where $g_V, g_W, g_M, U, e^2, \theta$ are all the dimensionless renormalized couplings and z is the dynamical critical exponent. The quenched disorders bring richer critical behaviors [55,56]. The RG flow in the (g_W, g_M) plane is shown in Fig. 1(d) with $c = e^2/15\pi^2(4+\theta^2e^4) = 25/558$ and all other couplings are set to zero. There are three IR stable fixed points. One of them, $(g_M, g_W, z) = (-156\pi c/7, 0, 1 + 39c/7)$, has a dynamical critical exponent larger than 1 and the others, $(g_M, g_W, z) = (0, -54\pi c, 1 - 27c)$ and $(g_M, g_W, z) = (-156\pi c/7, -300\pi c/7, 1 - 111c/7)$, have a dynamical critical exponent smaller than 1. If we consider an effective theory near the Dirac point and view the scalar field as an order parameter [45], the damping of quasiparticles $\sim\omega^{1/z}$ at the critical point. Then the change in the dynamic critical exponent will lead to the transition of the scaling from a Fermi liquid to a non-Fermi liquid [57–60]. Although high-order diagrams can be calculated to determine the fixed point of e^2, we can also adjust the CS level to modify the critical value. β_U indicates that the interaction U can be adjusted to make it go from being relevant to being irrelevant, compared to the scenario in the absence of disorder where U can only be relevant, which means that the gauge invariant disorder can suppress interactions. However, mass disorder g_V can only be relevant according to β_V.

Vortices and duality. To discuss the properties of vortices, we change the complex scalar field in Lagrangian (1) into polar coordinates $\phi = \rho e^{i\theta}$ with a superfluid density $|\rho|^2$ and phase θ. By a dual analysis in (2+1)D [surface of the (3+1)D bulk] or electromagnetic duality in the bulk (pull back to the surface) as shown in the Supplemental Material [45], we can get the effective Lagrangian describing the insulator (vortices condense to give an insulator [56]),

$$\mathcal{L}_{\rm eff} = \frac{1}{16\pi^2|\rho|^2}(\epsilon^{\mu\nu\lambda}\partial_\mu a_\lambda)^2 + \frac{e^2}{4\pi^2(4+\theta^2e^4)}\frac{(\epsilon^{\mu\nu\lambda}\partial_\mu a_\lambda)^2}{\sqrt{\Box}} - ia^\lambda J_\lambda^a - i\frac{\theta e^4}{8\pi^2(4+\theta^2e^4)}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda, \tag{6}$$

where a_μ is the dual $U(1)$ gauge field and the J_μ^a is the vortex current. The vortices are minimally coupled to a_μ. The fact that these two methods yield the same results demonstrates the validity of our reduced nonlocal theory. One notable difference from QED3 is that our effective vortex Lagrangian contains both a normal Maxwell term and a nonlocal one [the first and second terms in effective Lagrangian (6), respectively].

In real space the vortex current can be written as [61–63]

$$J^a_\mu(x) = \epsilon_{\mu\nu\lambda}\partial^\nu v^\lambda(x) = \sum_c n_c \oint_{L_c} dy^c_\mu \delta^3(x - y^c), \tag{7}$$

with a vortex charge $n_c \in \mathbb{Z}$ and the vortex loop L_c. Under the limit of $|\rho|^2 \gg 1$ or $p^2 \to 0$, we can ignore the normal Maxwell term and integrate over the dynamical gauge fields a_μ. This yields the effective action for the vortex current J^a_μ, i.e.,

$$S_{\text{eff}} = \int d^3x d^3x' \bigg[-\frac{4\pi^2}{e^2}\frac{J^a_\mu(x)J^{\mu a}(x')}{2\pi^2|x-x'|^2} + i2\pi^2\theta\frac{\epsilon^{\mu\alpha\nu}J^a_\mu(x)(x-x')_\alpha J^a_\nu(x')}{4\pi|x-x'|^3}\bigg]. \tag{8}$$

Substituting Eq. (7) into action (8), we find that the second term on the right-hand side can characterize $2\pi^2\theta$ times the linking number [61,63] of two different vortex loops as shown in Fig. 1(b). Such an effective self-statistical angle of the unit vortex is $2\pi^2\theta$ and equal to πn, satisfying the Fermi statistics at level 1. It is just the statistical anomaly after considering the bulk effect as analyzed in Refs. [64,65]. In QED3, this result is obtained under the limit of $e^2 \to \infty$ [61,64], but ours does not rely on this limit.

The conservation of current J^a_μ, $\partial^\mu J^a_\mu = 0$, indicates we can add the term $i\partial^\mu J^a_\mu \xi$ as a constraint setting. At the same time, introducing a convergence factor [66] $tJ^2_\mu/2$ can be viewed as the chemical potential of the vortex loops [67]. We can get the same formal structure as the original Lagrangian (1) at long wavelengths after integrating over J_μ:

$$\tilde{\mathcal{L}}_{\text{dual}} = \frac{1}{4\tilde{e}^2}\tilde{F}^{\mu\nu}\frac{2}{\sqrt{\Box}}\tilde{F}_{\mu\nu} + |(\partial_\mu - i\tilde{A}_\mu)\tilde{\phi}|^2 + \tilde{r}|\tilde{\phi}|^2 + \frac{\tilde{U}}{2}|\tilde{\phi}|^4 + i\frac{\tilde{\theta}}{2}\epsilon^{\mu\nu\rho}\tilde{A}_\mu\partial_\nu\tilde{A}_\rho. \tag{9}$$

A significant difference from the QED3 is that the nonlocal kinetic energy of the gauge field in the dual Lagrangian (9) is independent of the kinetic energy of the bosonic field ϕ in the original Lagrangian (1). In the long-wavelength limit, the related normal Maxwell term is much smaller than the nonlocal one and is discarded. The duality transformations show it satisfies $\tilde{\theta} = -\theta e^4/[4\pi^2(4+\theta^2e^4)]$, which is different from the QED3-Abelian Higgs model [68,69], $\tilde{\theta}\theta = -1/4\pi^2$. However, in the limit of $e^2 \to \infty$, the two results match. In addition, we have another constraint, $\tilde{e}^2e^2 = \pi^2(4+\theta^2e^4)$. It is possible [70] to set both $e^2 = \infty$ and $\tilde{e}^2 = \infty$ because the kinetic terms of the dual gauge fields $\tilde{A}_\mu$ are not dual to the kinetic energy of the ϕ particle compared with QED3, reflecting the self-duality of the planar Abelian CS Higgs model. Then the remaining CS term can induce a $2\pi/n$ flux bound to each ϕ bosonic particle worldline and $-2\pi n$ for dual bosonic field $\tilde{\phi}$. It is worth noting that these dualities are only valid in the infrared. As listed in Table I, the superconducting phase of Lagrangian (1) is dual to the insulator phase (with vortex condensation) of Lagrangian (9), while the normal phase is dual to the superfluid phase (with vortex excitation) [45].

TABLE I. Table showing the duality between Lagrangian (1) and (9). The central row gives the constraints of the couplings e^2 and θ. The last row gives the phases of a Lagrangian dual to the other.

	$\mathcal{L}[A_\mu, \phi]$	$\tilde{\mathcal{L}}[\tilde{A}_\mu, \tilde{\phi}]$
Couplings	e^2	$\tilde{e}^2 = \pi^2(4+\theta^2e^4)/e^2$
	θ	$\tilde{\theta} = -\theta e^4/4\pi^2(4+\theta^2e^4)$
Phases	Superconductor	Insulator
	Normal	Superfluid

It is remarkable that we can also get $O(2N)$ and $O(2N-2k)\times O(2k)$, $0<k<N$, symmetric fixed points [61,71,72] if we change the interaction from $U\sum_i|\phi_i|^4$ to $U_1\sum_{i<k+1}|\phi_i|^4 + 2U_2\sum_{i<k+1,\,j>k}|\phi_i|^2|\phi_j|^2 + U_3\sum_{i>k}|\phi_i|^4$, where different values of N can correspond to different theories. It can develop a lot of interesting phenomena, such as the deconfined quantum critical point [73], and the duality between the QED3–Gross-Neveu theory [74]. On the other hand, different scalar fields can be seen as distinct order parameter fields, and their competition can lead to diverse vortex dynamics [75].

In (3+1)D, $F_{\mu\nu}$ can be interpreted as the primary field in free Maxwell theory and it is conformally invariant [76]. However, this is not the case in other dimensions and we can replace it by a conformal gauge action $F^2_{\mu\nu}/\sqrt{\Box}$ in dimension three [77,78]. It is just the result after considering the one-loop fermion vacuum polarization diagram in QED3 theory as mentioned above. Marino [17] looked at it from another perspective, where particles conserved current j^μ equal to zero when $\mu = 3$ and $j^\mu\delta(x^3)$ otherwise, and obtained the same result. We can simply generalize this result to $j^\mu\delta(f(x^i))$, where particles constrain at any (2+1)-dimensional closed (or one-point compactifiable) submanifold (t, x^i) with $f(x^i) = 0$. At this time, covariant derivatives with a geometric connection will appear in the Lagrangian and can induce a geometric current in the effective vortex action [79,80].

Conclusion. We have studied an Abelian Chern-Simons Higgs model that undergoes a dimensional reduction to the plane, and obtained its quantized critical behaviors. Disorder can be introduced to adjust the critical behavior of the interaction from relevant to irrelevant. If dimensionally reduced to a curved surface, we may obtain a curved PQED model to investigate the critical behaviors of the theory as well as the dynamic behaviors of vortices. Furthermore, self-duality imposes constraints on the charge and CS parameter, which also apply to the flow-flow correlation functions near the critical points [45,68,69,81,82]. These constraints can even be extended to nonzero temperature cases. In summary, our reduced theory provides a good idea for the further study of self-duality near the quantum critical point, and can even be extended to nonzero temperature and nonflat spacetime.

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